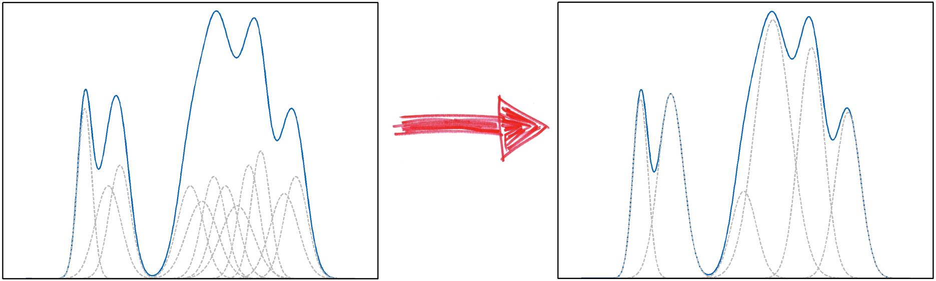
Gaussian Mixture Reduction via Clustering

Dennis Schieferdecker – *schiefer@ira.uka.de*
Marco Huber – *marco.huber@ieee.org*

GRK 1194: Self-organizing Sensor-Actuator-Networks



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Introduction

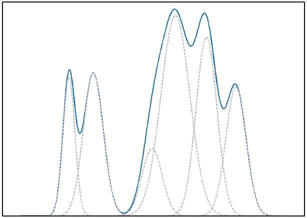
Gaussian Mixtures

Gaussian Mixture Density

- weighted sum of Gaussians

$$f(x;\underline{\eta}) = \sum_{i=1}^{N} \omega_i \cdot \mathcal{N}(x;\mu_i,\sigma_i^2)$$

- universal function approximator
- used to model probability density functions in estimation algorithms
  - target tracking,
  - machine learning,
  - speaker recognition,
  - ...



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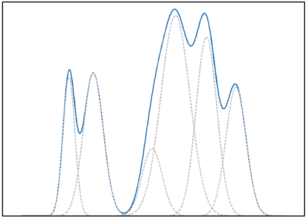

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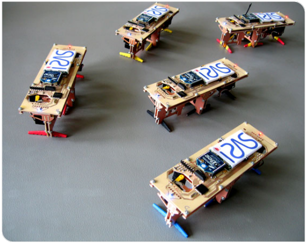

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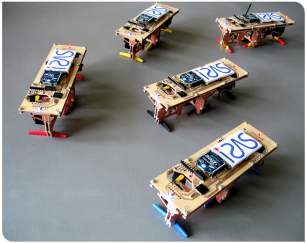

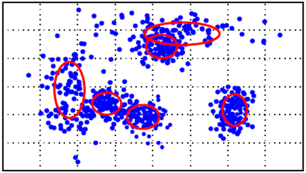

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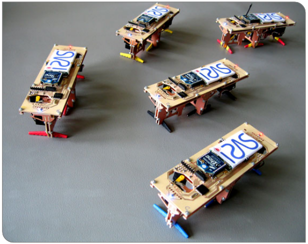

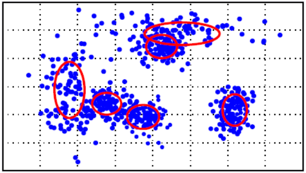

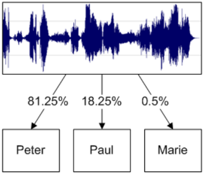

Problem Description

Gaussian Mixture Reduction

Problem in Applications

- recursive processing
  - multiplication of Gaussian mixtures
  - convolution of Gaussian mixtures
- number of components grows exponentially

Solution

- given a mixture $\tilde{f}(x;\underline{\tilde{\eta}})$ with N components (original mixture),
- find a mixture $f(x;\underline{\eta})$ with $K<N$ components (reduced mixture),
- so that a deviation measure $d(\tilde{f}(x;\underline{\tilde{\eta}}), f(x;\underline{\eta}))$ is minimized.

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Reduction Methods

Existing Algorithms

top-down approaches

- greedy methods
- start with full mixture
- iteratively replace a set of Gaussians with a smaller set
- sets can be chosen, using different deviation measures (local, global, hybrid)

bottom-up approaches

- constructive methods
- start with one component
- adaptively add/remove components as required
- progressive convergence towards full mixture
- PGMR

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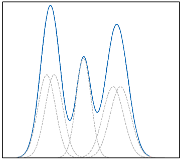

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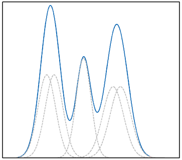

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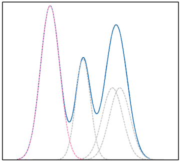

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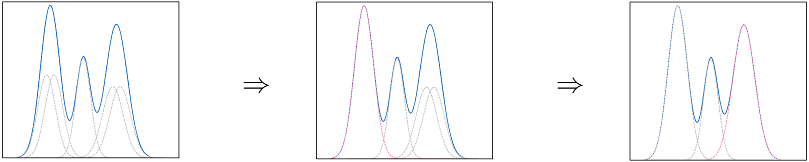

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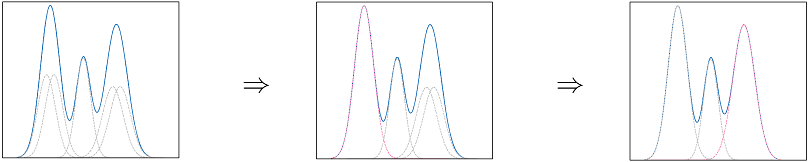

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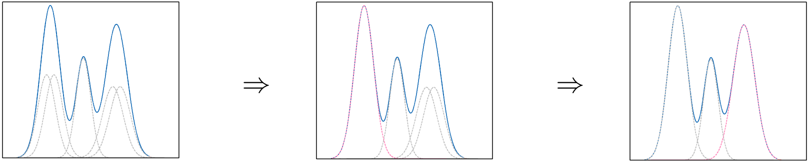

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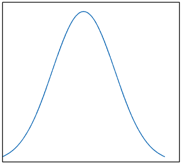

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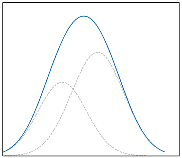

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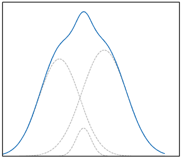

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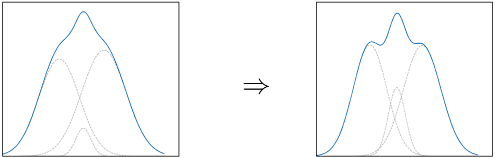

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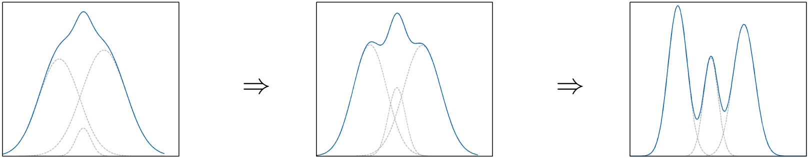

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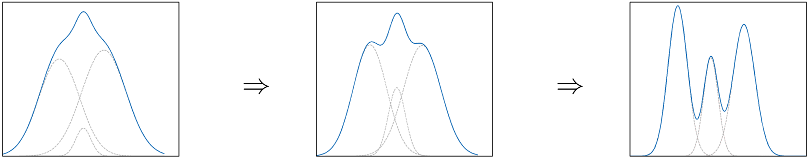
- progressive convergence towards full mixture
- PGMR - state-of-the-art



Clustering Method

Overview

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Gaussian Mixture Reduction via Clustering (GMRC)

- top-down approach using a global deviation measure
- three-step algorithm:

Preprocessing → LocalSearch → Refinement

Basic Operation

- quickly determine a rough initial solution
- push solution towards a good local optimum by local search
- refine solution using numerical methods

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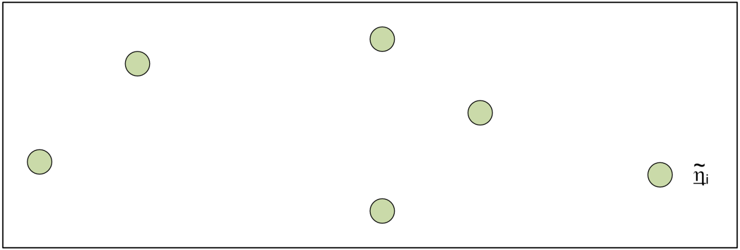
Clustering Method

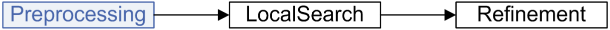
Preprocessing → LocalSearch → Refinement

Conception

- each component $\tilde{\eta}_i$ of a mixture can be mapped to a point (site) in a two-dimensional space
- distances between points correspond to the selected deviation measure

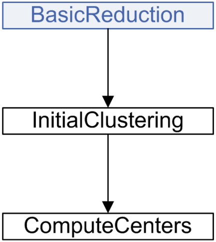


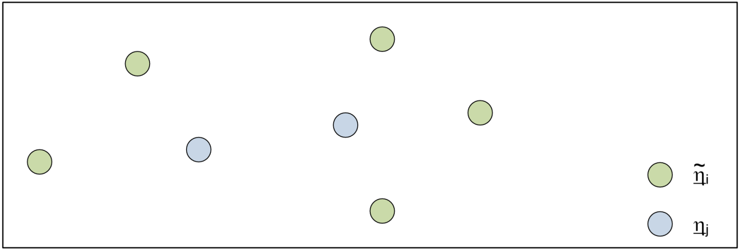
Clustering Method



BasicReduction

- compute an initial solution $\underline{\eta}$ for our problem (i.e. using West's or Runnalls' algorithm)
- the components $\underline{\eta}_j$ of the reduced mixture correspond to preliminary cluster centers





Clustering Method

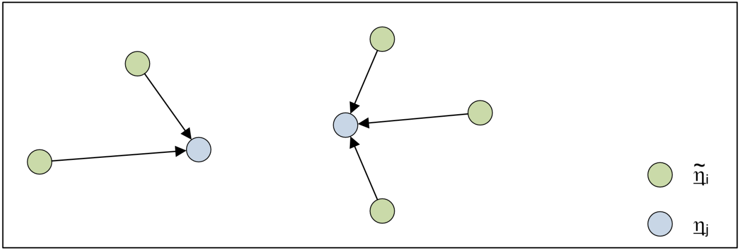
Preprocessing → LocalSearch → Refinement

BasicReduction → InitialClustering → ComputeCenters

InitialClustering

- associate each original component (site) $\underline{\tilde{\eta}}_i$ with the nearest one $\underline{\eta}_j$ of the reduced mixture (preliminary cluster center),
- using the Integrated Squared Distance (ISD):

$$d_{ISD}(\tilde{f}(x;\underline{\tilde{\eta}}_i), f(x;\underline{\eta}_j)) = \int_{\mathbb{R}} \left(\tilde{f}(x;\underline{\tilde{\eta}}_i) - f(x;\underline{\eta}_j)\right)^2 dx$$

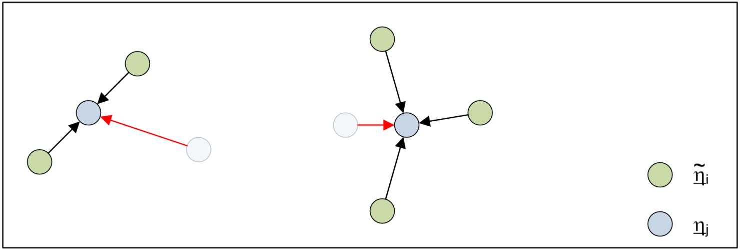


Clustering Method

BasicReduction → InitialClustering → ComputeCenters

ComputeCenters

- replace each reduced component η_j with a new one retaining mean and variance of the sum of the associated original components $\tilde{\eta}_i$
- equivalent to computing the center-of-mass of the sites associated to each center



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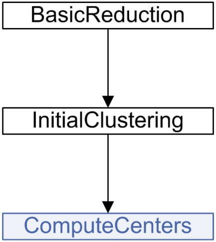

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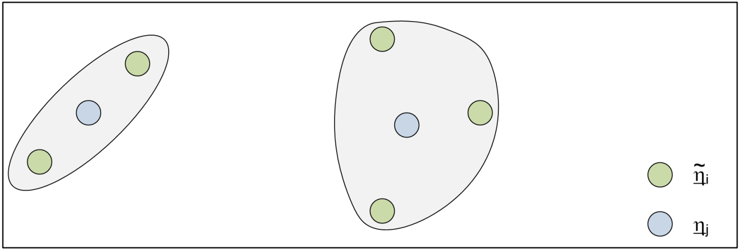


Clustering Method

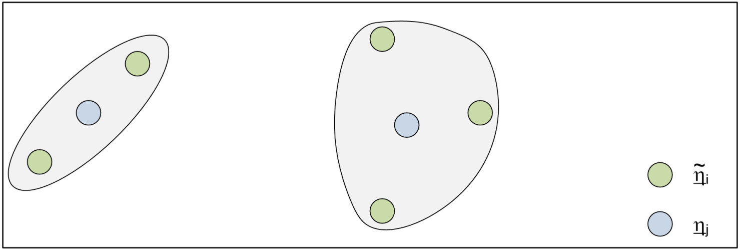
Preprocessing → LocalSearch → Refinement

Local Search

- greedy approach
- based on Lloyd's algorithm (k-means)

Basic Operation

- iteratively find the best association for each site $\tilde{\eta}_i$ to a center η_j,
- minimizing the selected deviation measure



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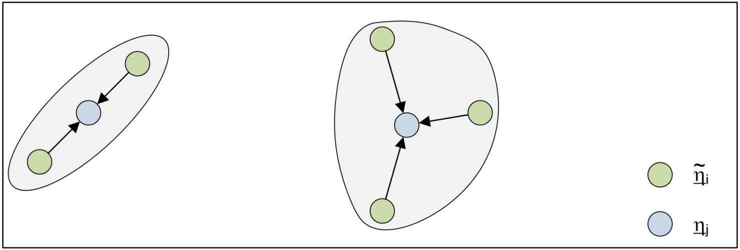


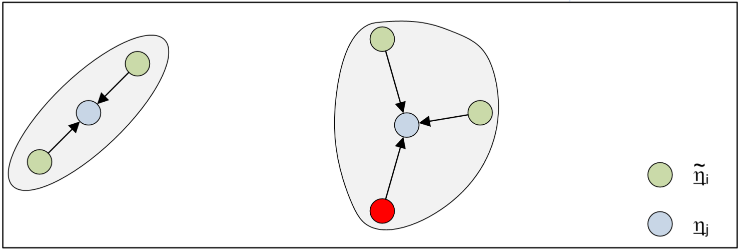
Clustering Method

Preprocessing → **LocalSearch** → Refinement

Finding the Best Association

- associate a site $\underline{\tilde{\eta}}_i$ with one of the current centers $\underline{\eta}_j$
- recompute and temporarily replace the affected centers $\underline{\eta}_{tmp}$
- determine normalized ISD (nISD) between the original and the current temporary reduced mixture:

$$d_{nISD}(\tilde{f}(x;\underline{\tilde{\eta}}), f(x;\underline{\eta}_{tmp})) = \frac{\int_{\mathbb{R}} \left(\tilde{f}(x;\underline{\tilde{\eta}}) - f(x;\underline{\eta}_{tmp})\right)^2 \mathrm{d}x}{\int_{\mathbb{R}} \tilde{f}(x;\underline{\tilde{\eta}})^2 \mathrm{d}x + \int_{\mathbb{R}} f(x;\underline{\eta}_{tmp})^2 \mathrm{d}x}$$



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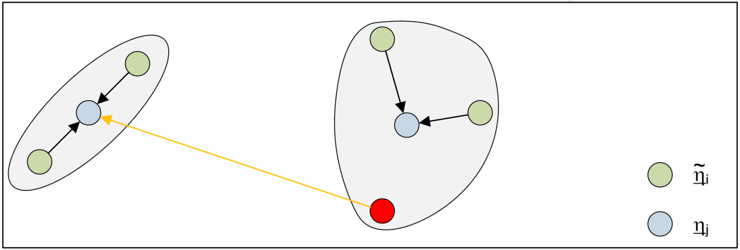

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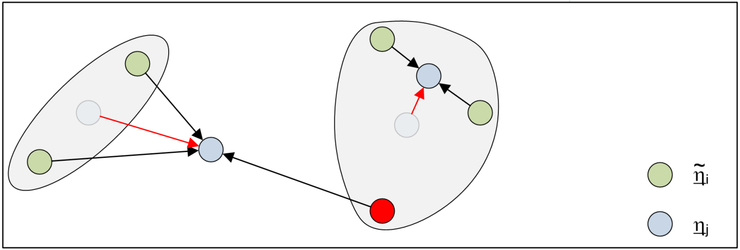

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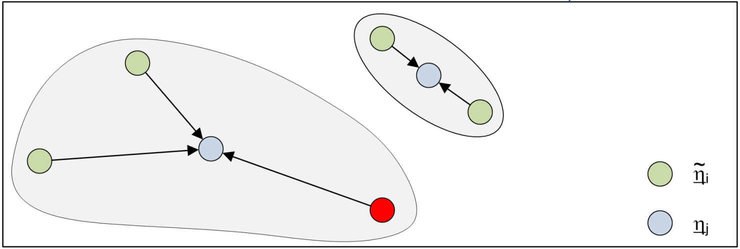
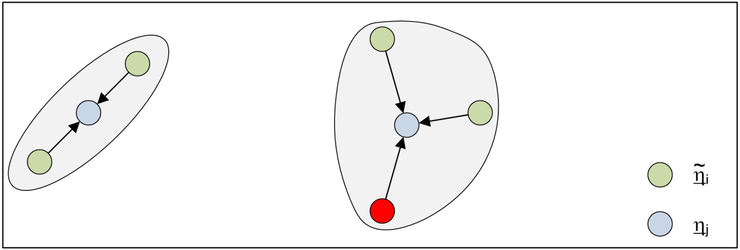


Clustering Method

Preprocessing → LocalSearch → Refinement

Finding the Best Association – continued

- revert temporary association
- repeat for all possible associations of site $\tilde{\eta}_i$ to a center
- retain the association with the smallest deviation (i.e. the reduced mixture which is closest to the original one)



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Finding the Best Association – continued

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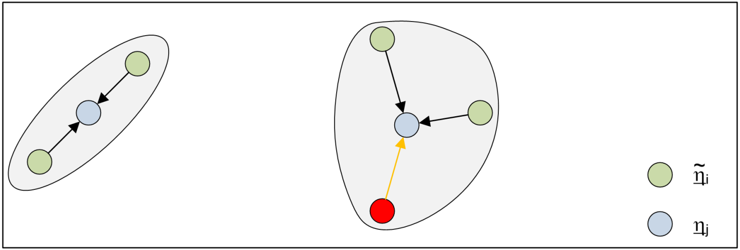
- repeat for all possible associations of site $\tilde{\eta}_i$ to a center
- retain the association with the smallest deviation (i.e. the reduced mixture which is closest to the original one)



Clustering Method

Preprocessing → LocalSearch → Refinement

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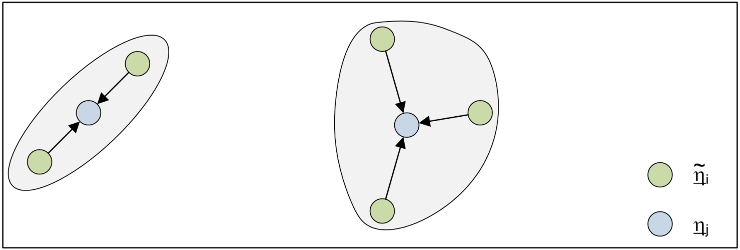

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Clustering Method

Preprocessing → LocalSearch → Refinement

Parameter Optimization

- optimize parameter vector $\underline{\eta}$ w.r.t. ISD

$$\min_{\underline{\eta}} \int_{\mathbb{R}} \left(\tilde{f}(x;\underline{\tilde{\eta}}) - f(x;\underline{\eta}) \right)^2 \mathrm{d}x$$

- non-linear optimization problem
  → Newton approach
- finds local optimum

OptimizeParams → OptimizeWeights

Weight Optimization

- solve system of linear equations

$$\begin{bmatrix} \int_{\mathbb{R}} \mathcal{N}_1^2 \mathrm{d}x & & \\ & \ddots & \\ & & \int_{\mathbb{R}} \mathcal{N}_L^2 \mathrm{d}x \end{bmatrix} \underline{\omega} = \sum_{j=1}^{N} \tilde{\omega}_j \begin{bmatrix} \int_{\mathbb{R}} \tilde{\mathcal{N}}_j \mathcal{N}_1 \mathrm{d}x \\ \vdots \\ \int_{\mathbb{R}} \tilde{\mathcal{N}}_j \mathcal{N}_L \mathrm{d}x \end{bmatrix}$$

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Results

Overview

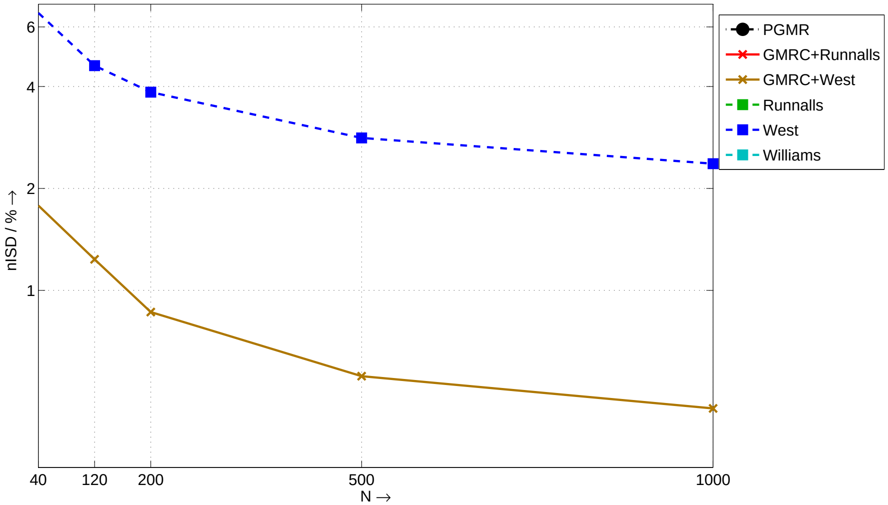
Simulation Setup

- Office PC (Intel Core2 Duo E8400)
- OpenSUSE 11.0
- Matlab 7.7.0 (R2008b)

- reduction of mixtures with $N \in \{40, 120, 200, 500, 1000\}$ components down to $K = 10$
- each evaluated with 1000 simulation runs

Results

Approximation Quality



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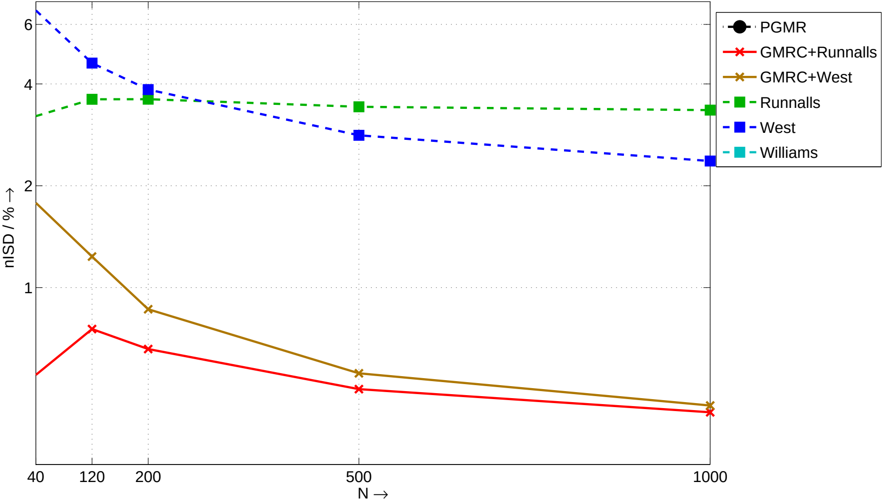

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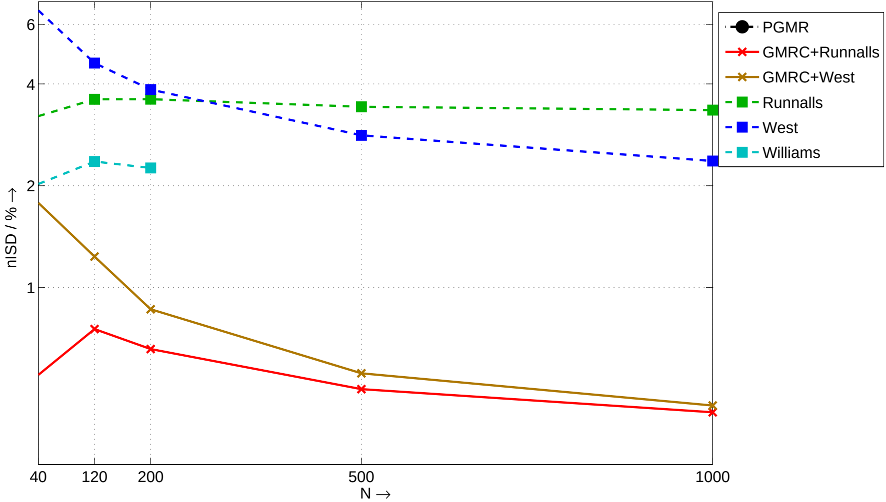

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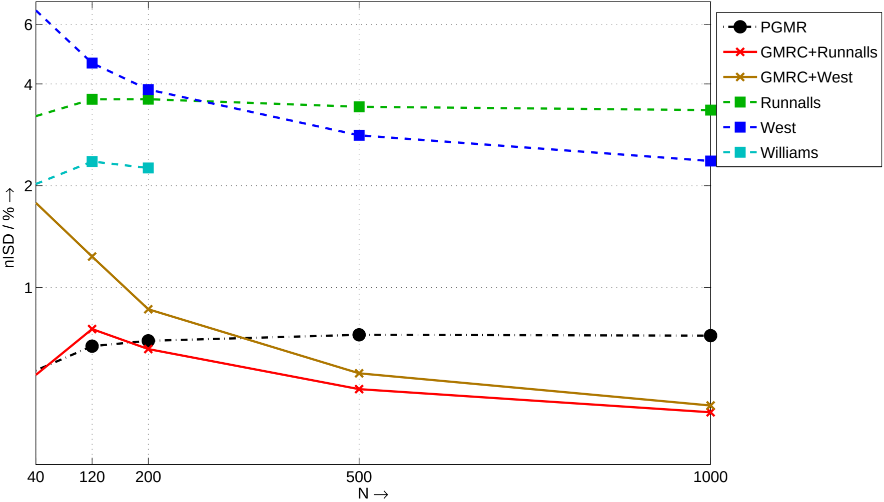
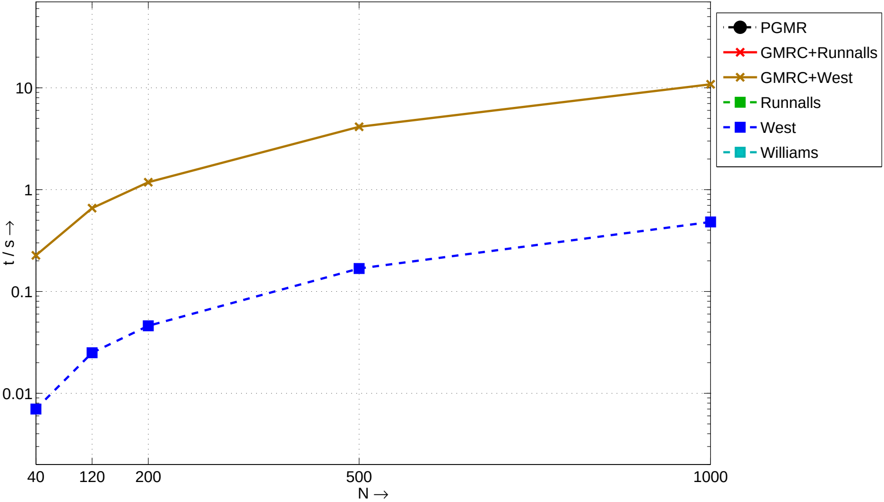


Results

Running Time



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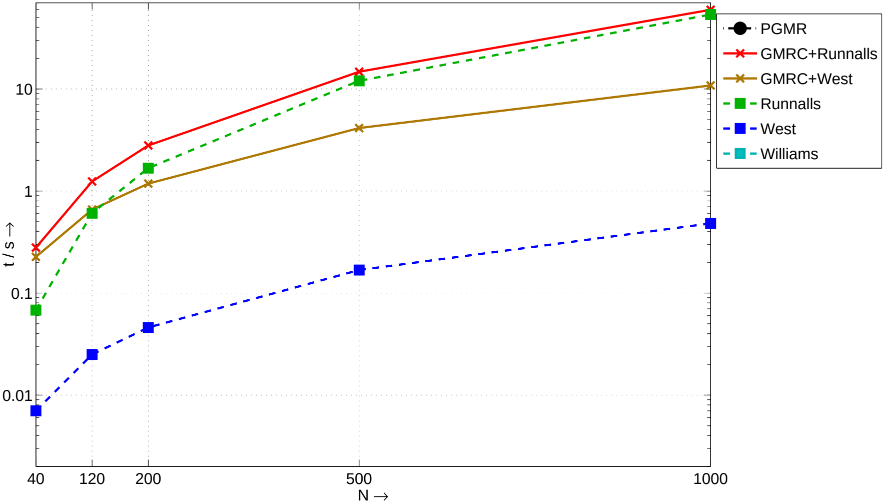

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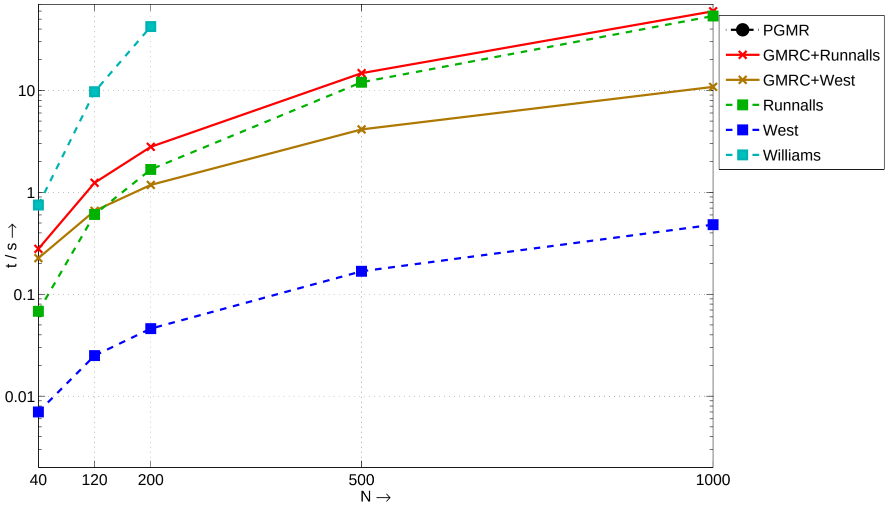

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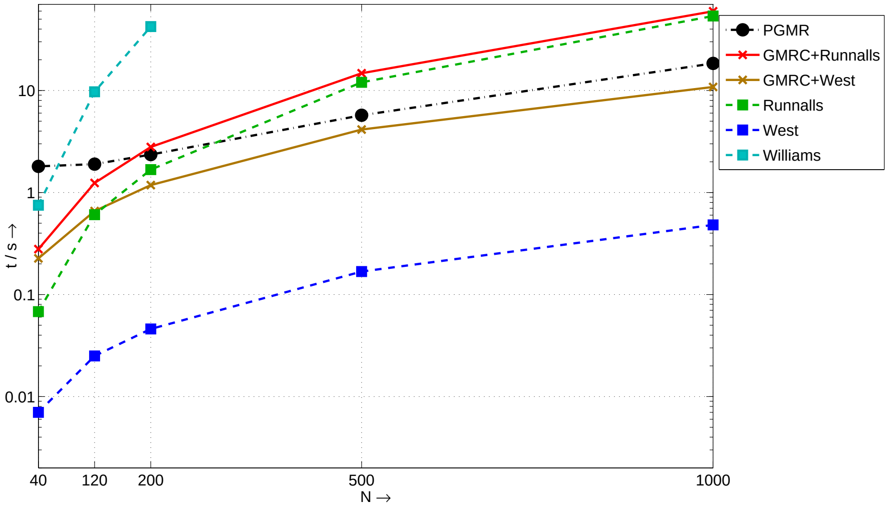

Results

Impact of Individual Steps

algorithm		running time	norm. ISD
GMRC	complete	2.793 ± 0.052s	0.658 ± 0.494
	w. random init.	1.135 ± 0.045s	1.272 ± 1.561
	w/o local search	1.742 ± 0.043s	0.774 ± 0.872
	w/o refinement	2.737 ± 0.036s	1.697 ± 0.432
Runnalls		1.678 ± 0.024s	3.606 ± 0.752

(initialization with Runnalls' algorithm; $N = 200$, $K = 10$)

- a good initial solution is mandatory
- local search primarily improves variance
- refinement has single-most impact on approximation quality

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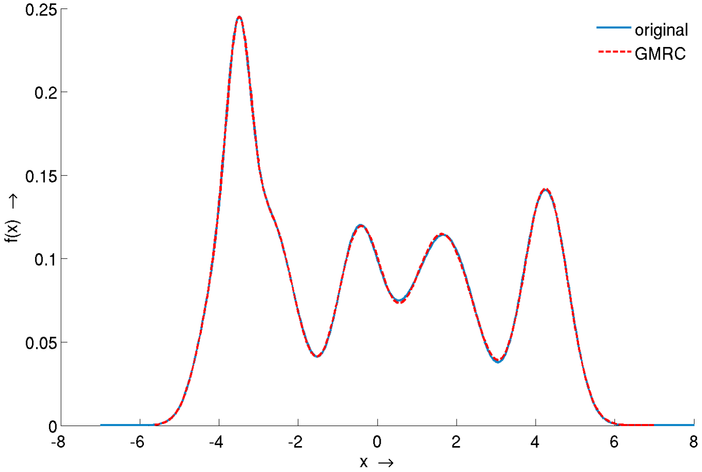

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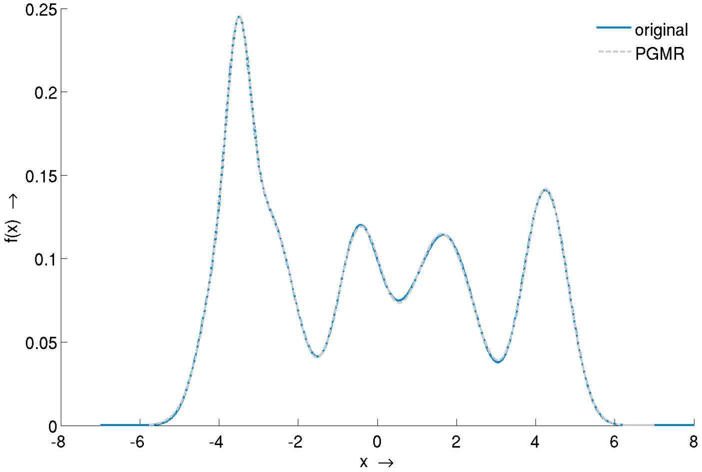

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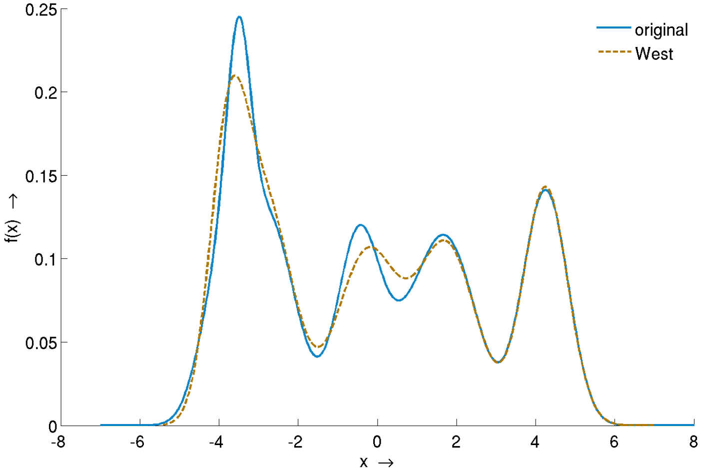

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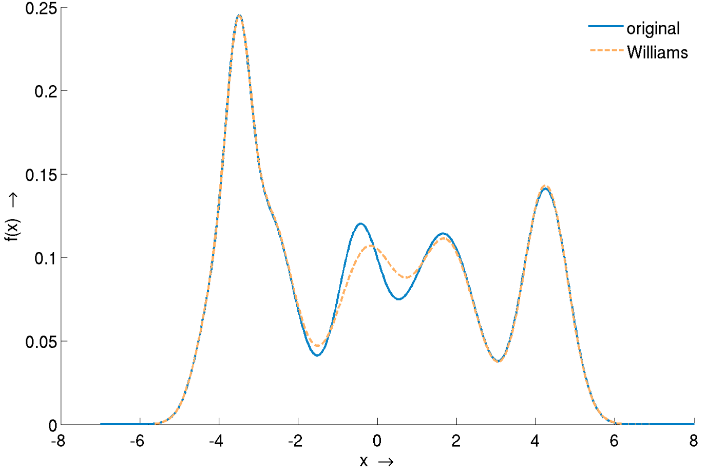

Conclusion

Summary

- novel mixture reduction algorithm
  - top-down approach, using a global deviation measure
  - based on k-means clustering method
  - combines discrete and continuous optimization methods
- compared to the current state-of-the-art PGMR:
  - faster computation
  - similar approximation quality

Outlook

- extension to multivariate Gaussian mixtures
- refine empirical choice of
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  - k-means as clustering approach
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Thank you for your attention!

time for questions

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