

### **Evolution and Evaluation of the Penalty Method for Alternative Graphs**

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#### shortest paths

 multitude of speed-up techniques (AF, CH, HL, ...; sub-microsecond queries)

### alternative routes

### non-optimal routes

(heuristiccs; quality  $\iff$  speed)

speed-up techniques do not work well (algorithms do not use or relax them)



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### **Motivation**

what are alternative graphs?

### alternative graph (AG)

- extension to alternative routes
  - encode multiple (good) alternative routes (between one source and one target)
  - $\Rightarrow$  interaction between alternative routes
  - compact representation of options
- intermediate data structure
  - $\Rightarrow$  sparse, directed graph

Kobitzsch, Radermacher, Schieferdecker:

2

 $\Rightarrow$  usable with expensive algorithms





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what has been done before? (1)



[ABRAHAM ET AL. 10]

- via-node approach (plateau method)
  - $ightarrow\,$  concatenation of two shortest paths
  - → variants: X-BDV (sec.), X-CHV (millisec.)





### quality measures

- not too much longer
- sufficiently different
- reasonable

(stretch) (sharing) al optimality)

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#### "Alternative Routes in Road Networks"

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#### 4 Kobitzsch, Radermacher, Schieferdecker: Evolution and Evaluation of the Penalty Method for Alternative Graphs

### **Related Work**

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### "Alternative Route Graphs in Road Networks"

[BADER ET AL. 11]

- penalty method (classical method)
  - ightarrow putting penalties on arc costs
  - $\rightarrow$  based on Dijkstra (sec.)
- no alternative route extraction

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c(a)

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### **Optimization Potential**

what can we do better?

#### current state-of-the-art

- fast "one-hop" alternative routes
- slow alternative graphs



### goal

fast alternatives with diverse structure

### approach

- focus on penalty method
  - ⇒ improve for interactive use (speed-up techniques)
  - $\Rightarrow$  extract alternative routes (quality criteria similar to via-node approach)
  - $\Rightarrow$  analyze guality & structure of results





#### generic workflow

```
while {termination condition false} do
1
2
      {compute shortest path}
3
4
      {add penalties to graph}
5
6
  end
7
8
  {select shortest paths + combine to alternative graph}
9
10
  fextract alternative routes }
11
```

path selectionpenalization (not covered today)

(modified from previous work)

- fast computation
- alternative route extraction

```
(not in previous work)
```



### basic approach (classical method)

- perform "enough" iterations
  - $\Rightarrow$  generate set of shortest paths
- select good subset for AG

(implementation details left open)

- exploit quality measures
  - ⇒ terminate as paths become too long (allow maximum stretch w.r.t. original metric)
  - ⇒ select path for AG after each iteration (w.r.t. stretch, sharing measures)

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computing shortest paths (1)

### challenges

- compute many shortest paths
  - ⇒ Dijkstra's algorithm [DIJKSTRA 59] → used by previous work

(takes seconds on random queries)

 $\Rightarrow \ \ \, \text{static speed-up techniques} \\ (AF, CH, HL, \ldots \rightarrow \text{costly preprocessing})$ 





#### arc costs change

⇒ Customizable Route Planning (CRP) [DELLING ET AL. 13] → used by our approach (takes milliseconds on random queries)

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 $\Rightarrow$  but still requires some kind of preprocessing

computing shortest paths (2)

### **CRP** preprocessing

- structural preprocessing
  - takes minutes to hours (required once, can be done offline)
  - multi-level partitioning
  - adding shortcut arcs

(boundary nodes of each cell become cliques)

#### metric customization

compute shortcut costs

(required when arc costs change)

 takes seconds<sup>1</sup> / tenths of a second<sup>2</sup> (using multiple cores and vector units)





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[DELLING ET AL. 13]<sup>1</sup>

[Delling&Werneck 13]<sup>2</sup>



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#### computing shortest paths (3)

#### adaptive customization

- only update cells with changes
- only update k levels
  - $\Rightarrow$  restrict query algorithm to k levels
  - $\Rightarrow$  preprocessing times  $\iff$  query times
  - ⇒ beneficial for short queries (less overhead)

### dynamic level selection

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(depending on hop count, optimized on different query set)





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### $\Rightarrow$ tens of milliseconds for query and customization





#### alternative route extraction (1)



- take constituting routes?
  - $\Rightarrow$  misses synergy effects
- take any route?
  - $\Rightarrow$  arbitrarily bad
- our approach:  $CRP-\pi$ 
  - two-step procedure (via-node & penalty methods)
    extraction on small graph
    - (expensive methods become feasible)





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### **procedure** 1. via-node approach

- based on X-BDV
  - $\rightarrow$  local optimality disabled (not applicable in AG)
- exhaustively compute routes
- keep all routes
- 2. penalty method
  - based on classical penalty method → initialize with X-BDV routes
  - fixed number of iterations (stretch no good termination criterion)
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## Penalty Method

alternative route extraction (2)

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## Experimental Setup

#### hardware/software

- 4 Intel Xeon E5-4640 @ 2.4 GHz, 512 GiB RAM (32 cores, total)
- Ubuntu 12.04, gcc 4.6.1 (full optimizations)

#### data

- road network of Western Europe (provided by PTV AG)
  - directed, weighted graph, single SCC
  - 18 million vertices, 24 million arcs (degree two vertices removed)
  - travel-time metric
- 1 000 queries at random / data point (of random Dijkstra rank / of fixed Dijkstra rank)









iterations of penalty method



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#### engineering impact



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#### engineering impact



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#### engineering impact

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engineering impact - multi-cores



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alternative graph quality

algorithm	rating	queries	decision arcs
[BADER ET AL. 11]	3.21	100	≤ 10.0
[RADERMACHER 12]	2.89	1 000	7.0
$CRP ext{-}\pi$	3.32	1 000	(unfiltered) 17.4
$CRP\text{-}\pi$	2.89	1 000	(filtered) 9.5

comparable results to previous penalty methods

- best previous work only considered tiny test set
- filtering to reduce decision arcs (≈100µs)
  - only for comparison to previous penalty methods
  - reduces potential for extracting multiple alternative routes (not applied in subsequent alternative route analysis)



#### alternative route quality

#	algorithm	SUCCESS [%]	stretch [%]	sharing [%]	optimality [%]
1st	X-BDV	96.0	10.0	41.8	75.4
	X-CHV	89.6	80.4	40.6	68.1
	$CRP\text{-}\pi$	96.3	42.9	31.9	26.9
2nd	X-BDV	87.6	13.8	59.5	65.1
	X-CHV	72.5	269.0	57.6	57.2
	$CRP\text{-}\pi$	84.0	47.6	45.9	22.1
3rd	X-BDV	75.5	17.2	65.6	54.6
	X-CHV	51.4	214.0	63.6	46.8
	$CRP\text{-}\pi$	62.9	67.4	51.8	15.9

- comparable success rates (corresponds to runtimes)
- limited similarity to via-node alternativs (77.9% | 72.7% | 65.5%)



#### alternative route quality

#	algorithm	SUCCESS [%]	stretch [%]	sharing [%]	optimality [%]
1st	X-BDV	96.0	10.0	41.8	75.4
	X-CHV	89.6	80.4	40.6	68.1
	$CRP\text{-}\pi$	96.3	42.9	31.9	26.9
2nd	X-BDV	87.6	13.8	59.5	65.1
	X-CHV	72.5	269.0	57.6	57.2
	$CRP\text{-}\pi$	84.0	47.6	45.9	22.1
3rd	X-BDV	75.5	17.2	65.6	54.6
	X-CHV	51.4	214.0	63.6	46.8
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### Conclusion



#### Summary

- first fast implemenation of penalty method, suited for interactive use (utilizing vector units and multi-core capabilities of modern CPUs)
- extracted routes of high quality, and distinct from via-node approach (first quantitative analysis of extracted routes from alternative graphs)

#### **Open Problems**

- general classification scheme for good alternatives (set of criteria, not tailored to a specific approach)
- improve runtime to compete with via-node approach (combination with [PARASKEVOPOULOS&ZAROLIAGIS 13] seems promising)

### Thank you for your attention!





# Time for questions!

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### References



[DIJKSTRA 59] A Note on Two Problems in Connexion with Graphs [ABRAHAM ET AL. 10] Alternative Routes in Road Networks [BADER ET AL. 11] Alternative Route Graphs in Road Networks [Delling et al. 13] Customizable Route Planning Faster Customization of Road Networks Improved Alternative Route Planning Schnelle Berechnung von Alternativgraphen





## backup slides

why consider alternative routes?

#### business perspective

#### provide options

(users have varied preferences)

#### overcome flaws in model and data (shortest paths need not be best in reality)

#### research perspective

- building blocks (traffic simulation, stochastic routing)
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peering into the distant past...



### simple approaches to find distinct routes

### k-shortest path

(meaningful alternatives only for large k)

#### multi-criteria optimization

 $(distance \iff difference)$ 

#### time-dependent routes



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(alternatives not guaranteed, limited data)

### insufficient solutions to the problem



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### Penalty Method penalization (1)

#### requirements

- enable discovery of diverse routes
  - ⇒ penalties on arcs of current route (discourage previous routes)
  - ⇒ penalties on adjoined arcs (discourage meandering)
- quick discovery of diverse routes
  - $\Rightarrow$  generate lots of new information in each iteration
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#### $\Rightarrow$ how to choose penalty values?



# additive penalties (classical method)

Penalty Method

penalization (2)

- add fraction of arc/path costs (w.r.t. original metric)
  - $\Rightarrow$  shortest paths only change slowly

#### geometrically growing penalties (our approach)

### multiply arc costs on current route by small factor

- $\Rightarrow$  often used paths quickly become undesirable
- add  $\frac{1}{2}\sqrt{(\text{current route cost})}$  to adjoined arc costs

 $\Rightarrow$  discourages short detours on long routes







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restriction of maximum crp level



Evolution and Evaluation of the Penalty Method for Alternative Graphs



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restriction of maximum crp level



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restriction of maximum crp level



Evolution and Evaluation of the Penalty Method for Alternative Graphs

26



optimal selection of maximum crp level



Evolution and Evaluation of the Penalty Method for Alternative Graphs

alternative route structure



#### differences between penalty & via-node alternatives

### procedure

- consider all CRP-π routes
- for each, find via-node alternative with most overlap

(test each vertex on CRP- $\pi$  route as via-node)

#### average maximum overlap

- **77.9% | 72.7% | 65.5%** (1st 3rd alternative)
- higher order alternatives increasingly distinct

(first routes likely extracted by via-node approach)

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## **Quality Analysis**

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alternative route structure

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#### $\Rightarrow$ CRP- $\pi$ provides distinct routes a via-node approach cannot find

### **Quality Analysis**

alternative route structure



