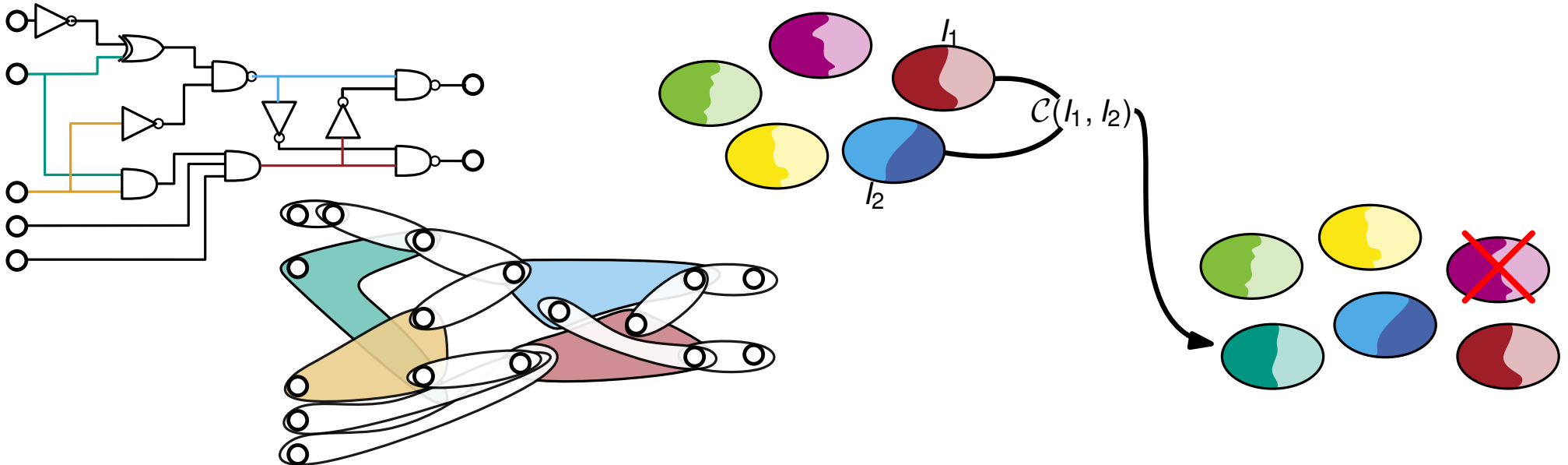


# Memetic Multilevel Hypergraph Partitioning

GECCO'18 · July 17, 2018

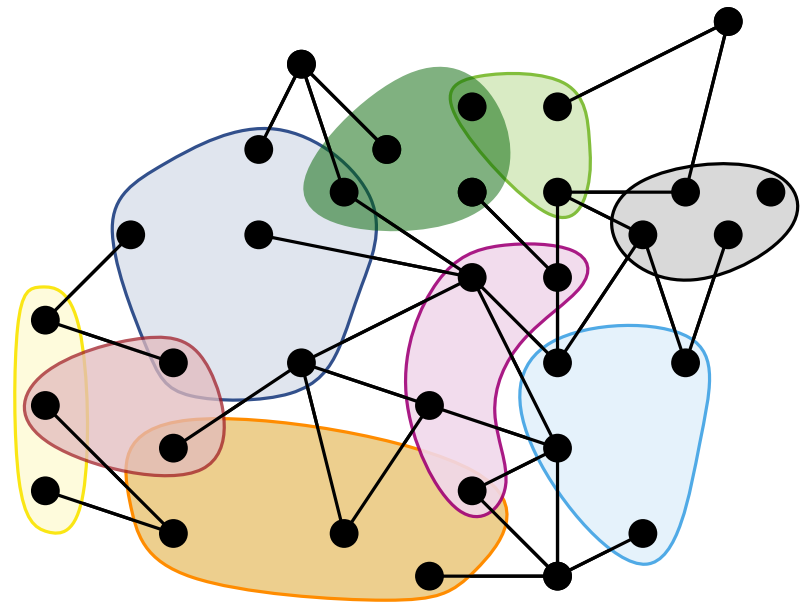
Robin Andre, Christian Schulz, Sebastian Schlag

INSTITUTE OF THEORETICAL INFORMATICS ·



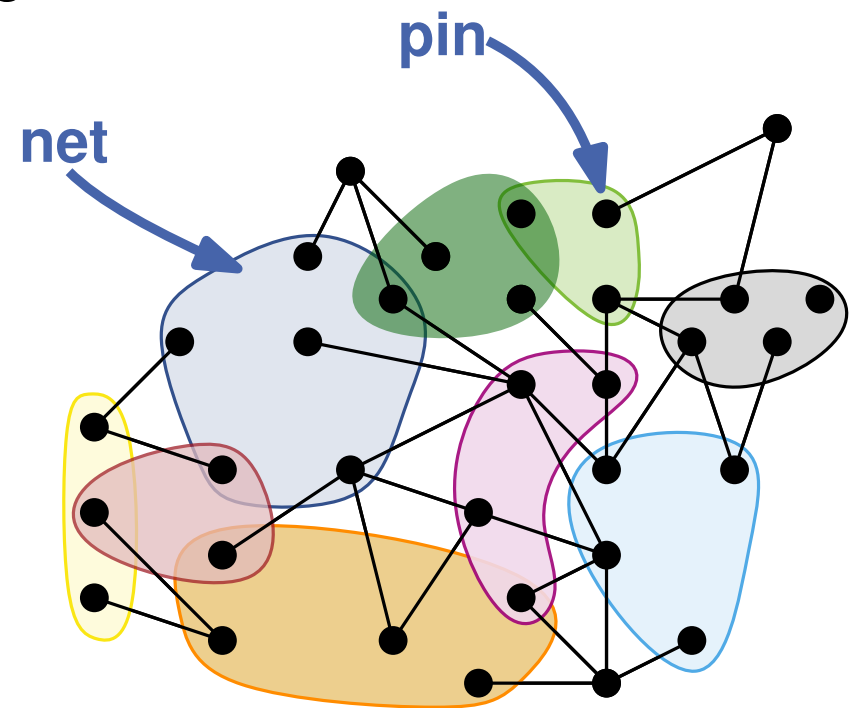
# Hypergraphs

- generalization of graphs  
⇒ hyperedges connect  $\geq 2$  nodes
- graphs  $\Rightarrow$  dyadic (**2-ary**) relationships
- hypergraphs  $\Rightarrow$  (**d-ary**) relationships
- hypergraph  $H = (V, E, c, \omega)$ 
  - vertex set  $V = \{1, \dots, n\}$
  - edge set  $E \subseteq \mathcal{P}(V) \setminus \emptyset$
  - node weights  $c : V \rightarrow \mathbb{R}_{\geq 1}$
  - edge weights  $\omega : E \rightarrow \mathbb{R}_{\geq 1}$



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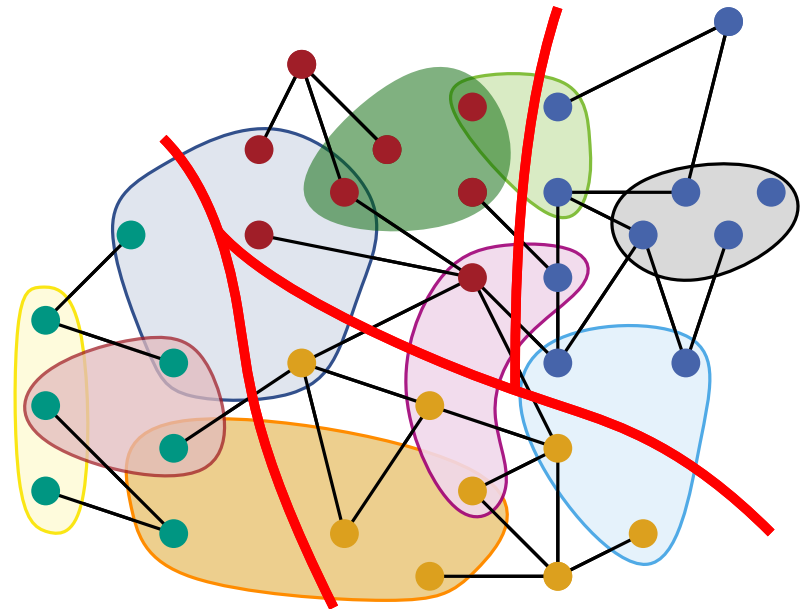


# $\varepsilon$ -Balanced Hypergraph Partitioning Problem

Partition hypergraph  $H = (V, E, c, \omega)$  into  $k$  disjoint blocks  $\Pi = \{V_1, \dots, V_k\}$  such that:

- blocks  $V_i$  are **roughly equal-sized**:

$$c(V_i) \leq (1 + \varepsilon) \left\lceil \frac{c(V)}{k} \right\rceil$$



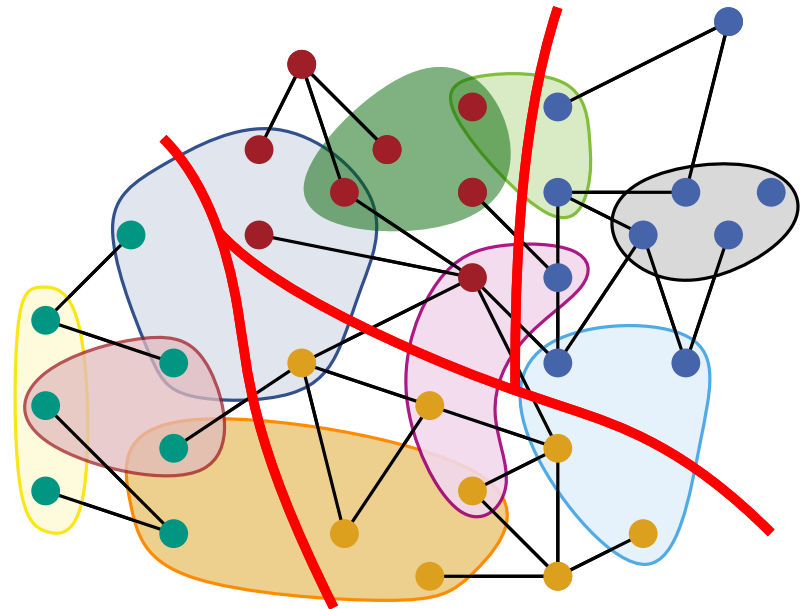
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imbalance  
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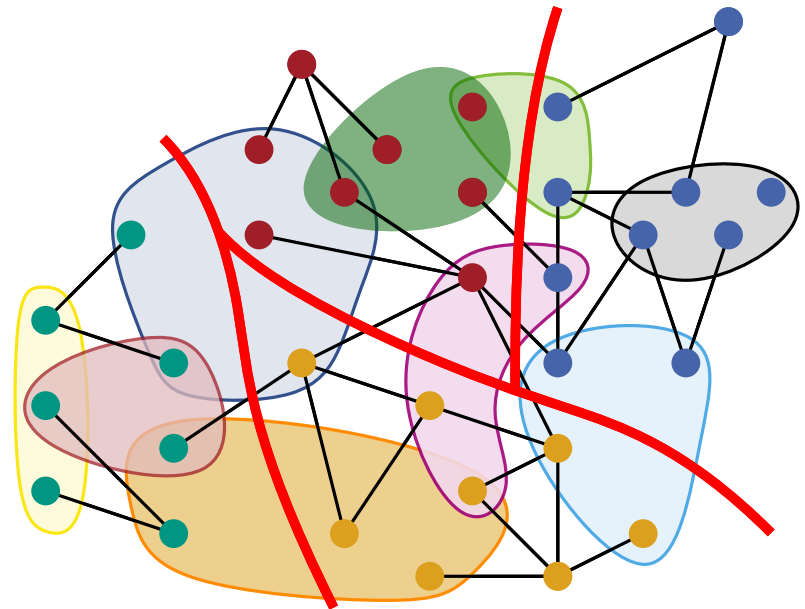
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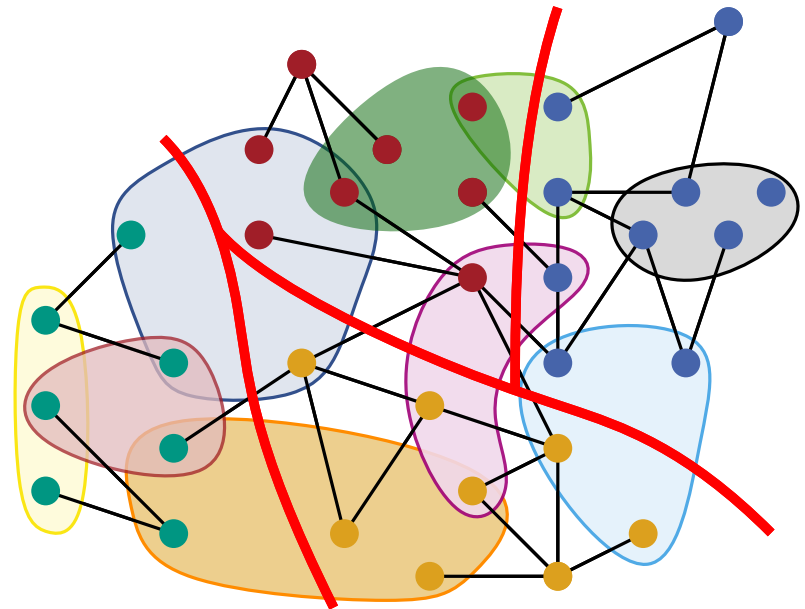
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$$\sum_{e \in \text{cut}} (\lambda - 1) \omega(e)$$

connectivity:  
# **blocks** connected by net  $e$



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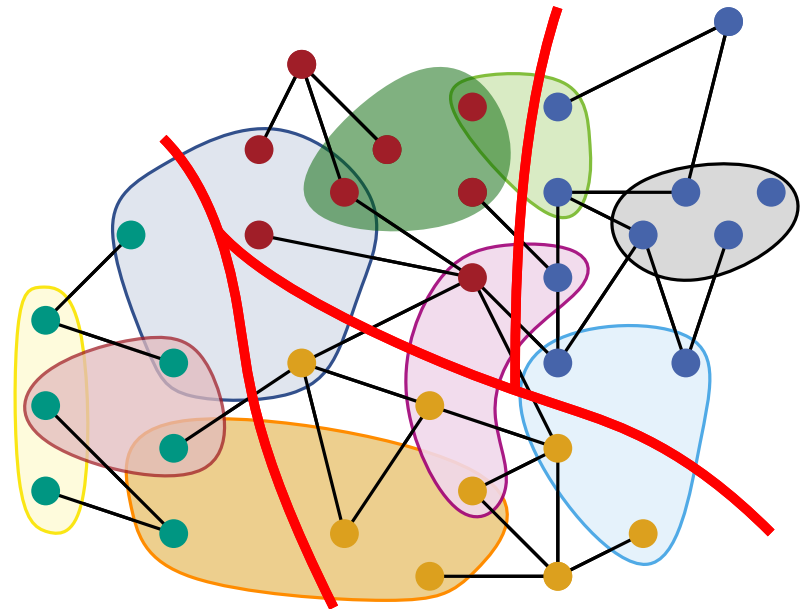
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imbalance  
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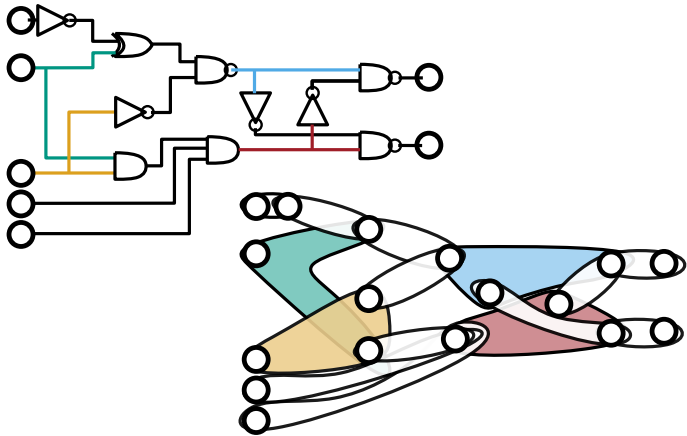
$$\sum_{e \in \text{cut}} (\lambda - 1) \omega(e) = 12$$

connectivity:  
# **blocks** connected by net  $e$





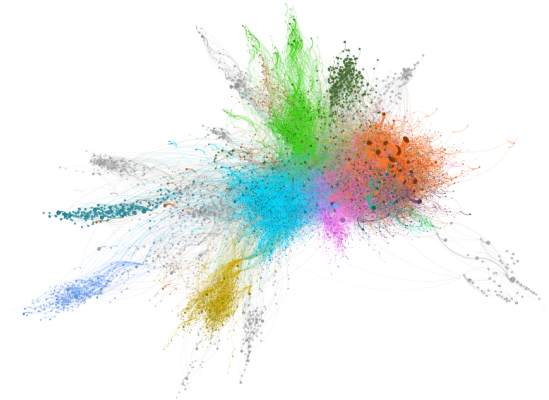
# Applications



VLSI Design



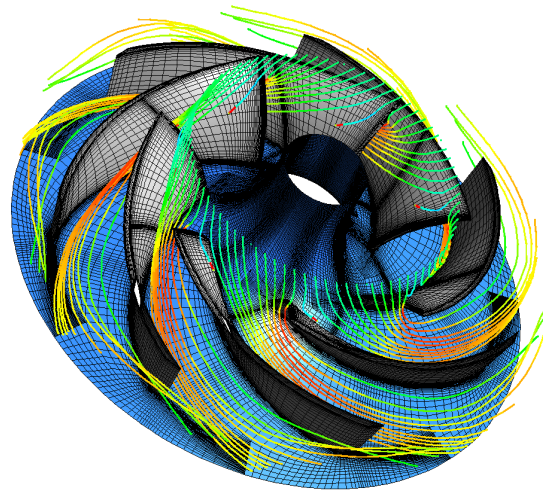
Warehouse Optimization



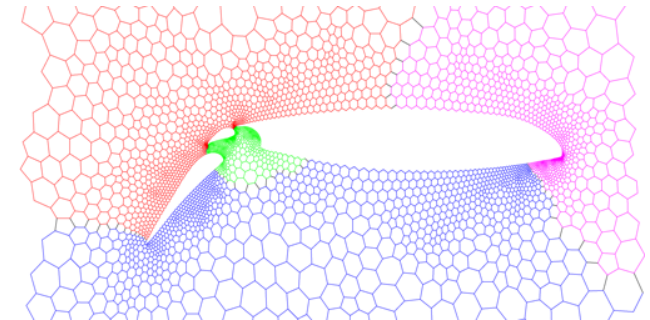
Complex Networks



Route Planning

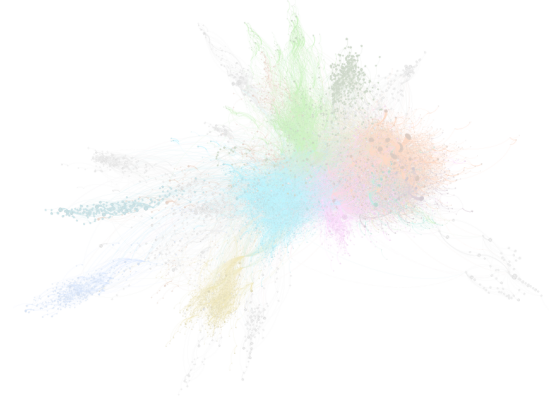
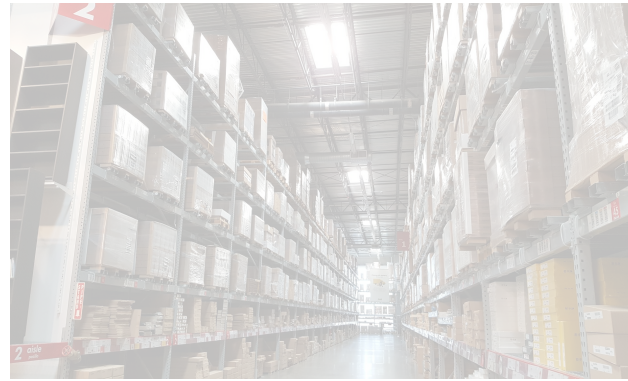
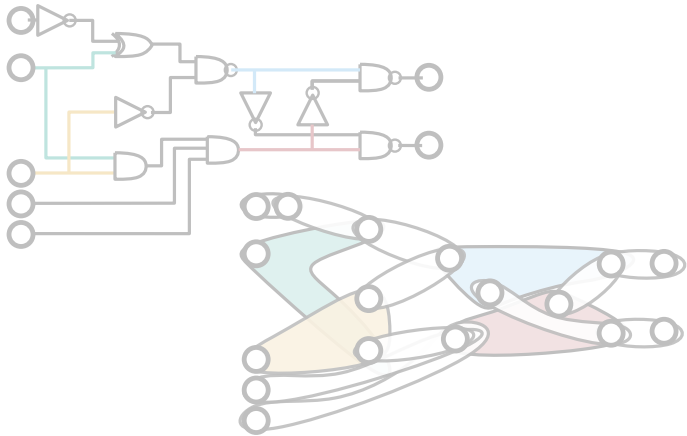


Simulation



$\mathbb{R}^{n \times n} \ni Ax = b \in \mathbb{R}^n$   
Scientific Computing

# Applications

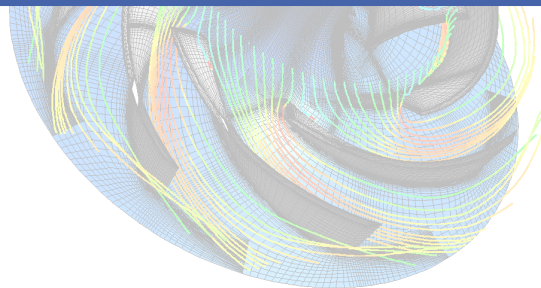


VLSI Design      Warehouse Optimization      Complex Networks

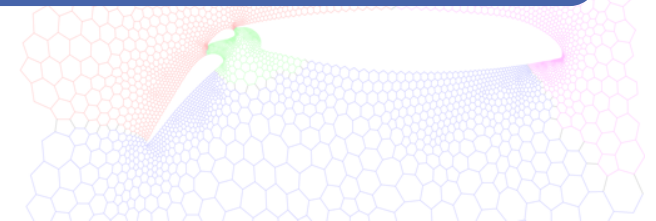
- hypergraph partitioning is **NP-hard**
- even finding **good approximate** solutions for graphs is NP-hard  
⇒ our focus: **solution quality**



Route Planning



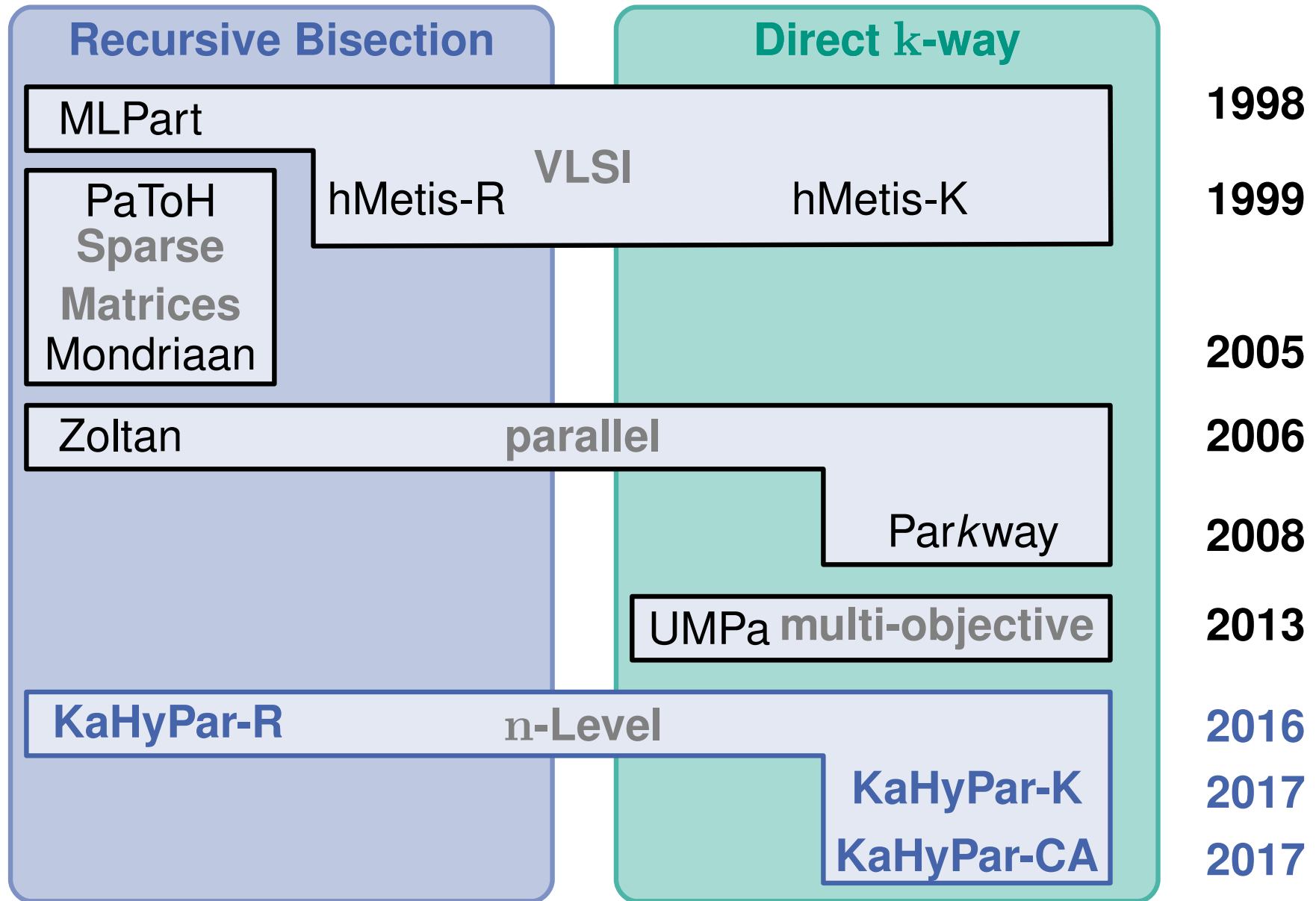
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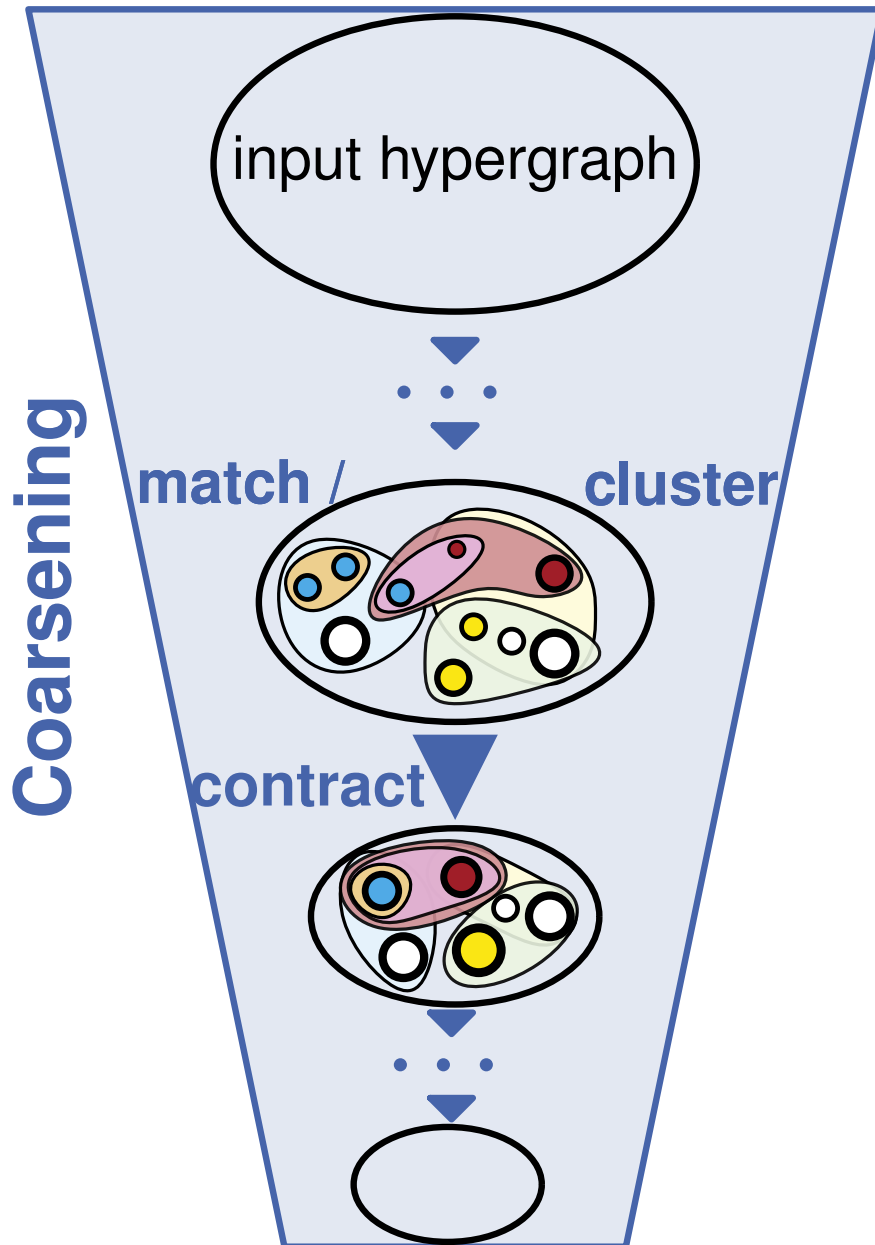
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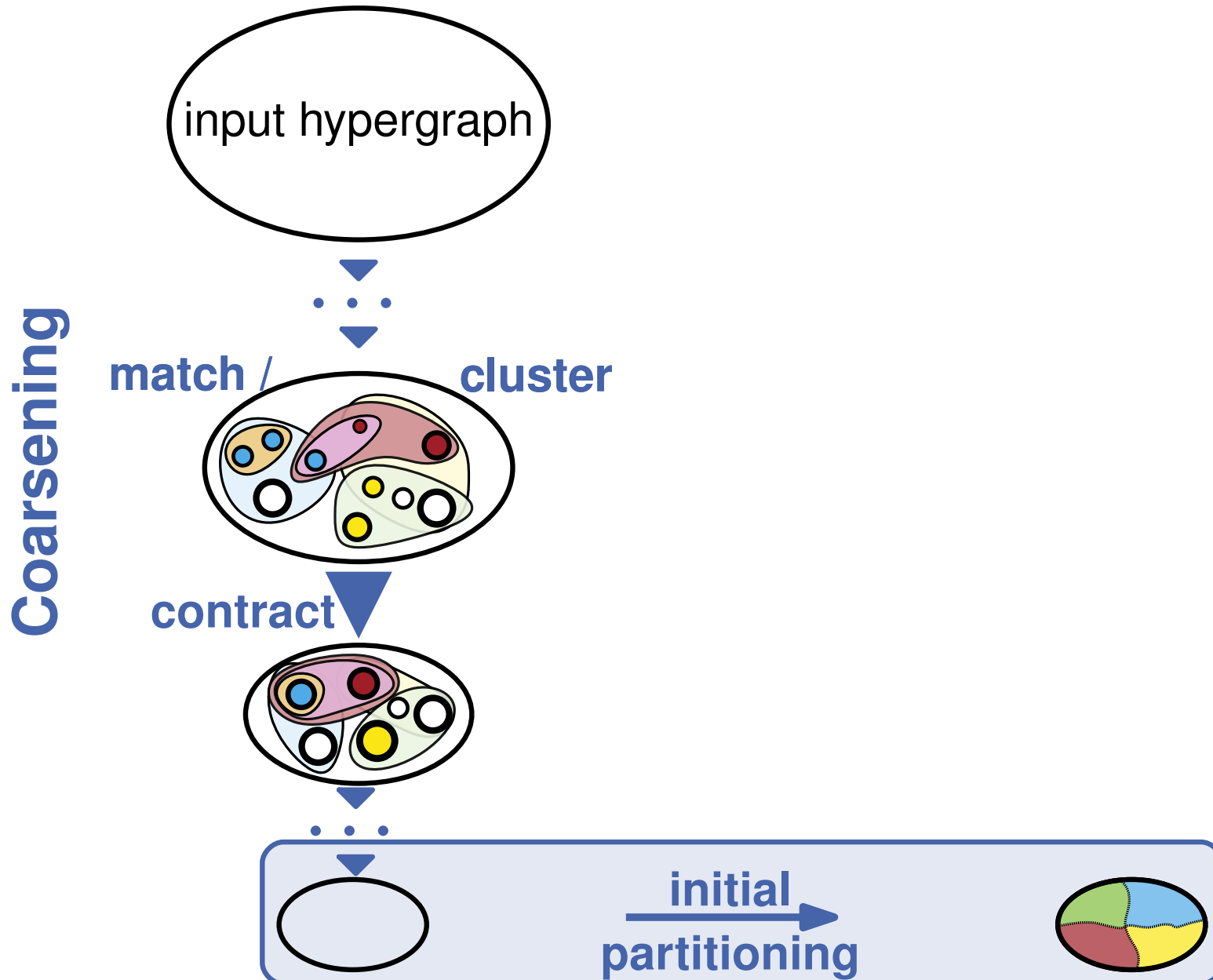
# Taxonomy of Hypergraph Partitioning Tools



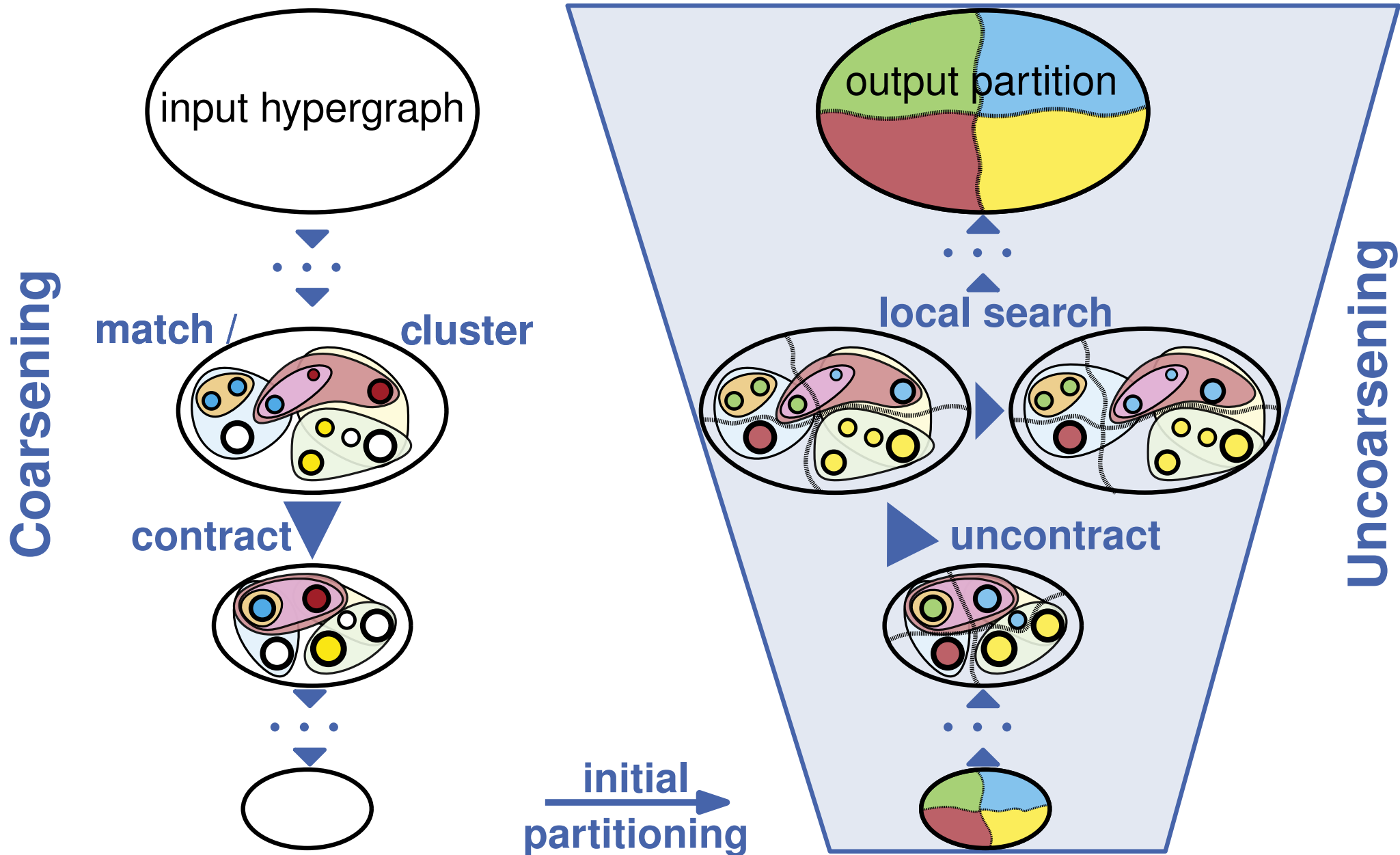
# The Multilevel (ML) Framework



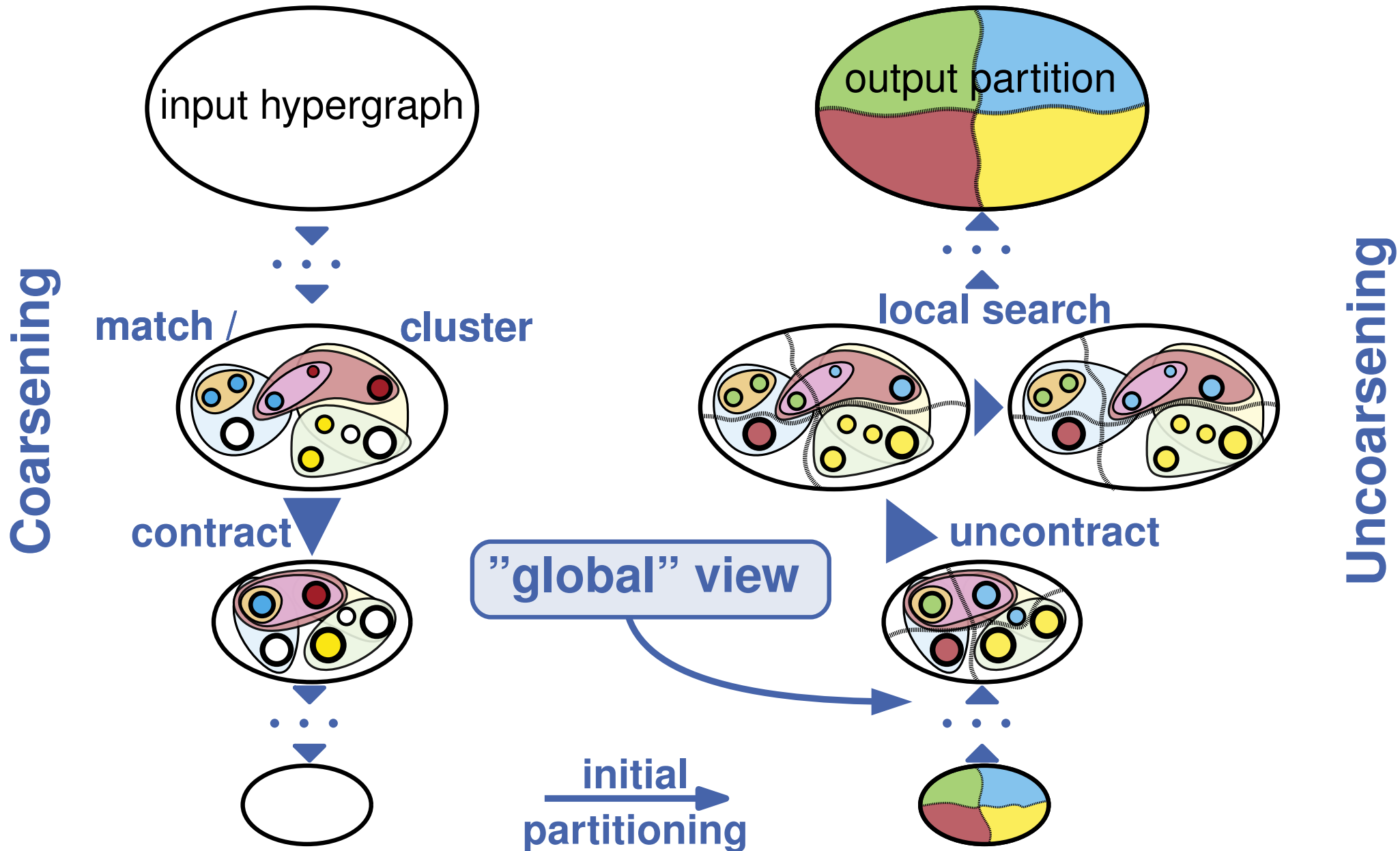
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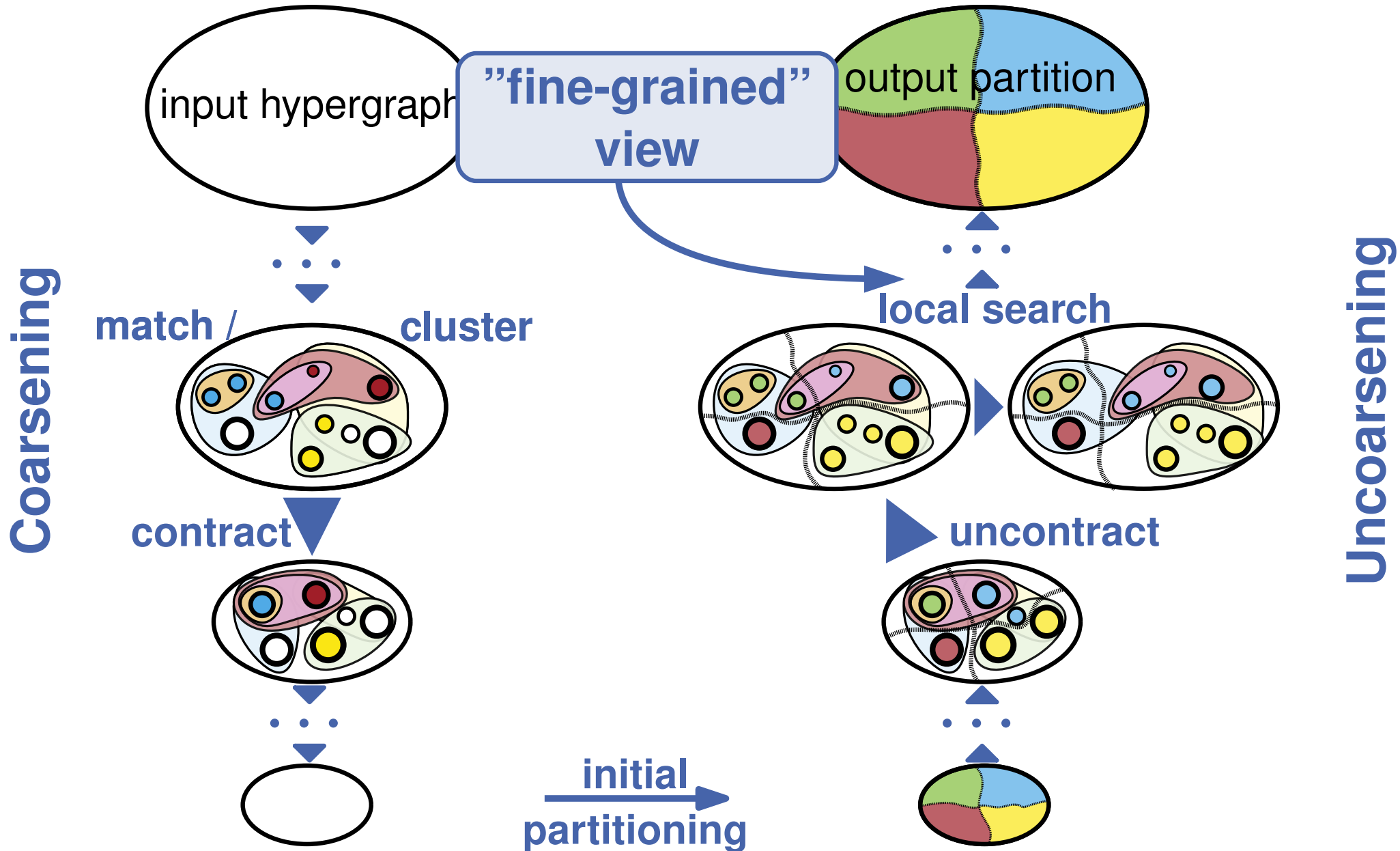
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# The Multilevel (ML) Framework





# Evolutionary Algorithm – Outline

---

## Algorithm 1: Steady-State EA

---

create initial population  $\mathcal{P}$   
**while** *stopping criterion not fulfilled* **do**  
    select parents  $l_1, l_2$  from  $\mathcal{P}$   
    recombine  $l_1$  with  $l_2$  to offspring  $o$   
    mutate offspring  $o$   
    evict individual from  $\mathcal{P}$  using  $o$   
**return** *fittest individual*

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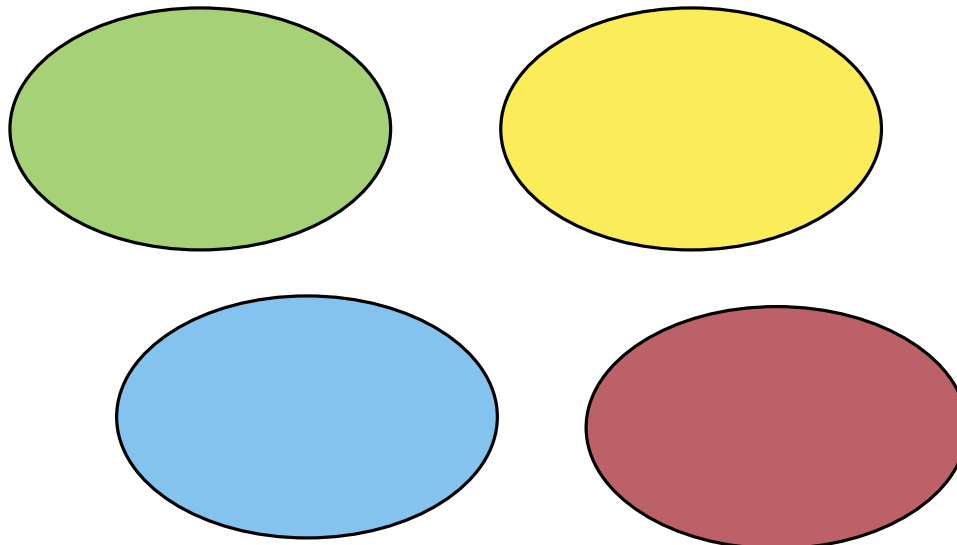
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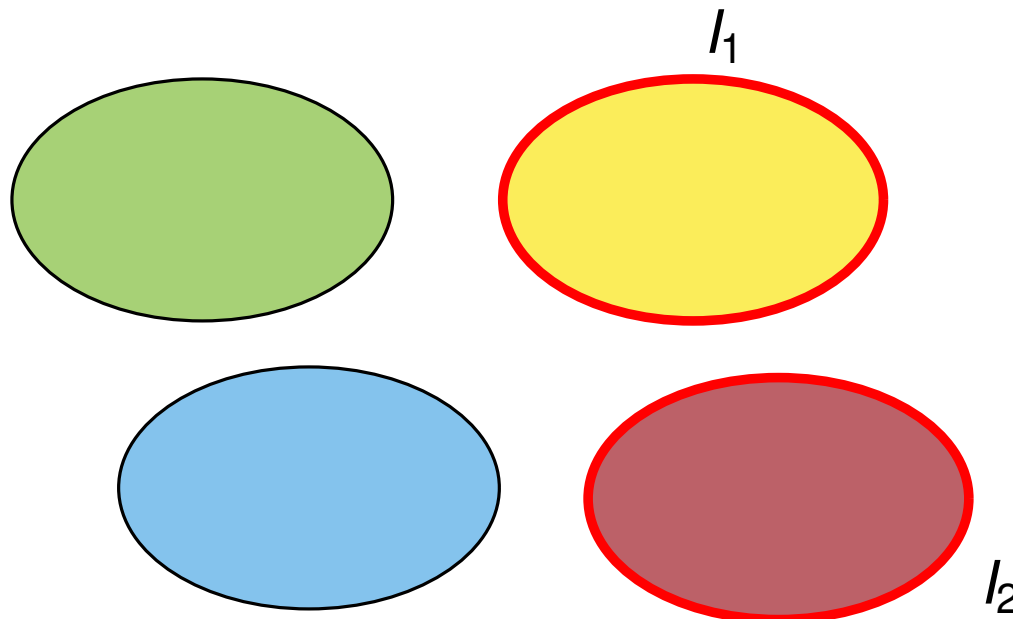
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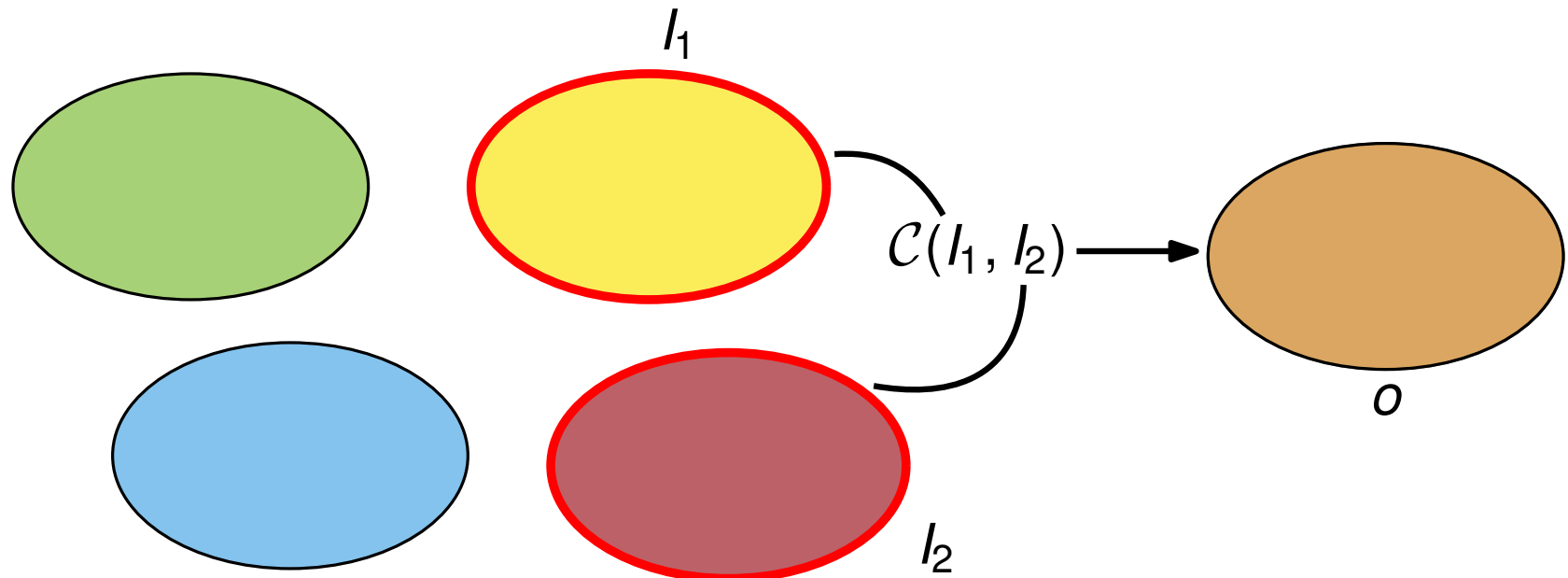
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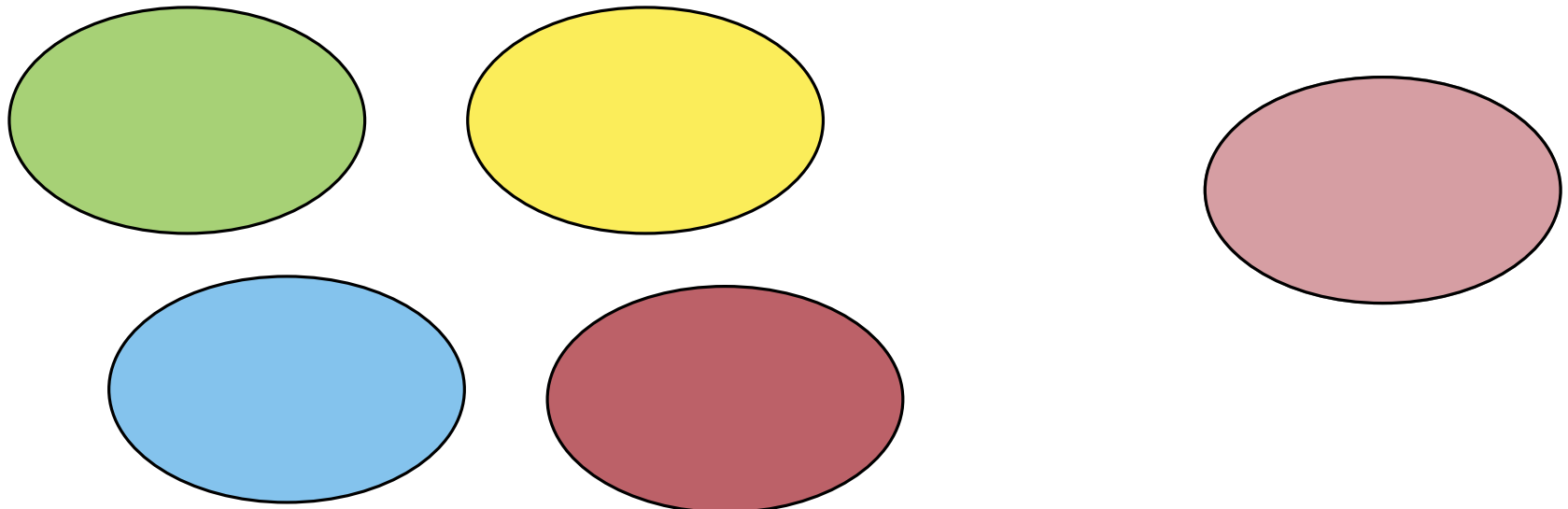
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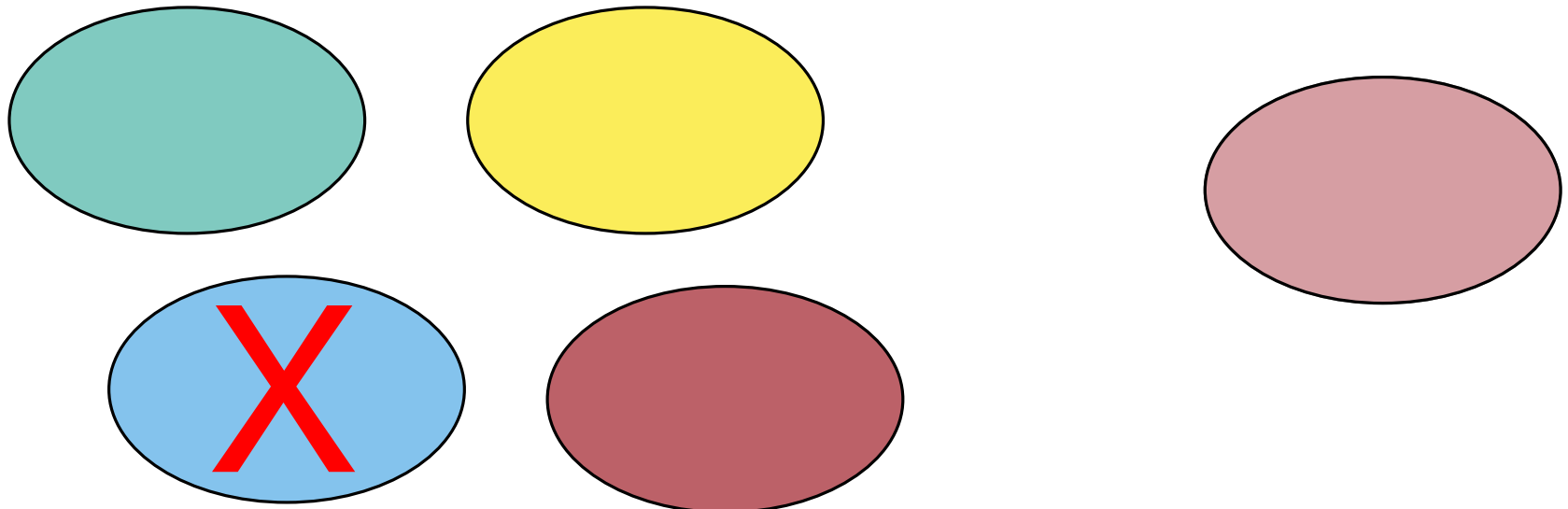
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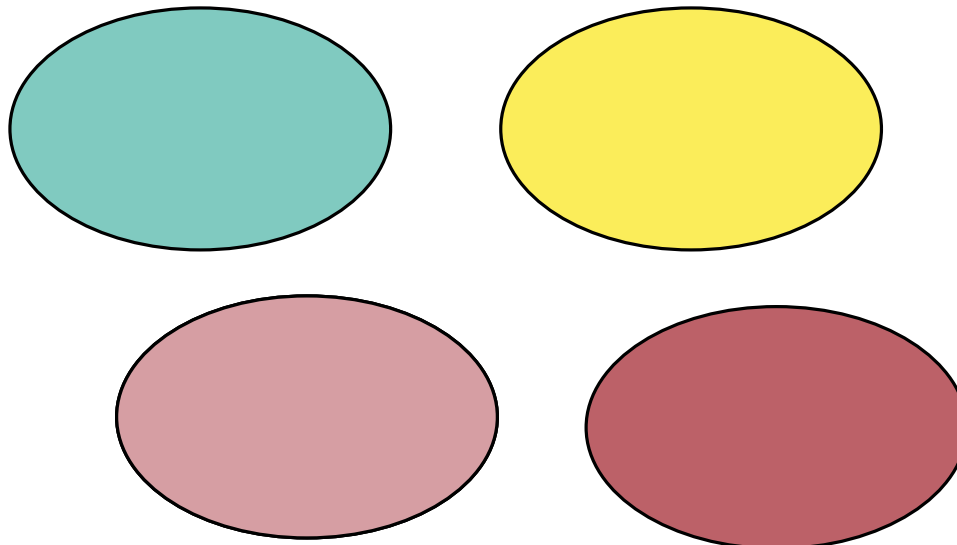
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**evict** individual from  $\mathcal{P}$  using  $o$

**return** *fittest individual*

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# Evolutionary Algorithm – Outline

---

## Algorithm 1: Steady-State MA

---

create initial population  $\mathcal{P}$  + local search

**while** *stopping criterion not fulfilled* **do**

    select parents  $l_1, l_2$  from  $\mathcal{P}$

    recombine  $l_1$  with  $l_2$  to offspring  $o$

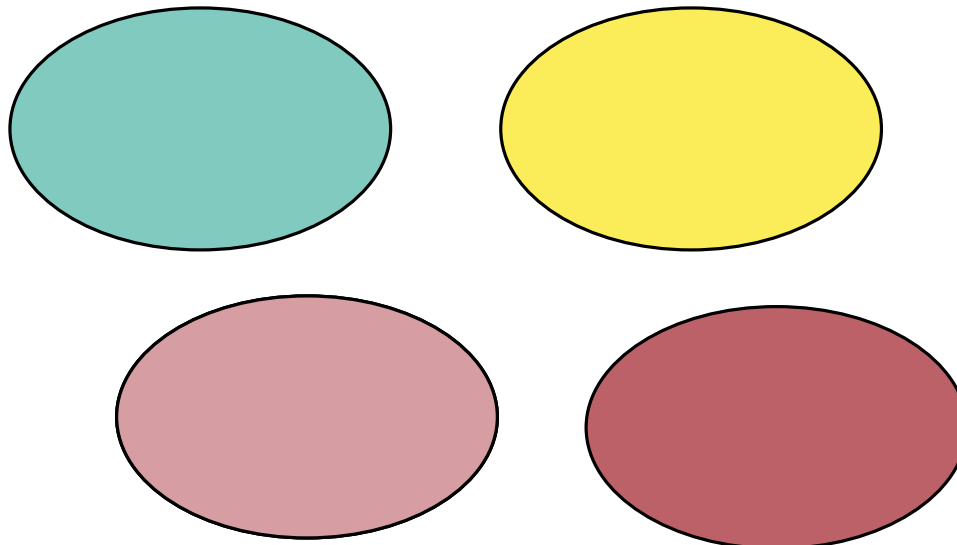
    mutate offspring  $o$  + local search

    evict individual from  $\mathcal{P}$  using  $o$

**return** *fittest individual*

---

memetic  
algorithm





# Evolutionary Algorithms for HGP

|                      | # | Population  | Recombination | Mutation     | LS  | ML |
|----------------------|---|-------------|---------------|--------------|-----|----|
| Saab & Rao '89       | k | bin packing | –             | rand. greedy | –   | –  |
| Hulin '91            | 2 | rand.       | 2-point       | rand.        | –   | –  |
| Bui & Moon '94       | 2 | rand.       | 5-point       | rand.        | FM  | –  |
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Individuals/Population

Partition 01011101010101101

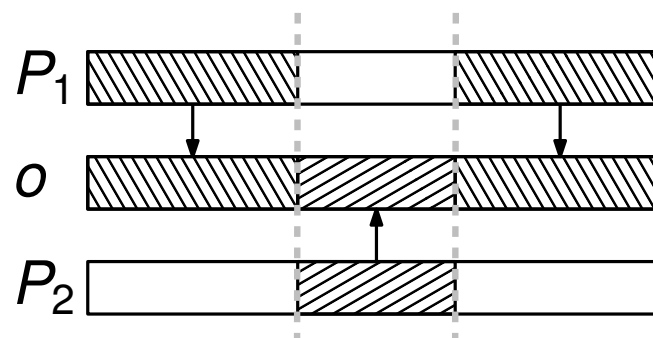
|-----|

|V|

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## 2-point crossover



# Evolutionary Algorithms for HGP

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random mutation

01011101010101101

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11010101000101001

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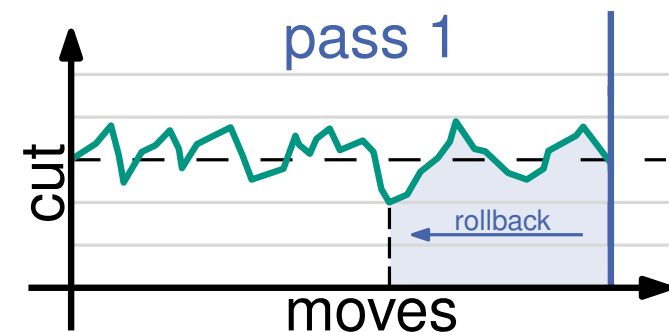
## Algorithm 2: FM Local Search

```

while improvement found do
    while  $\neg$  done do
        find best move
        perform best move
        rollback to best solution
    
```

pass

can worsen solution



# Evolutionary Algorithms for HGP

|                      | # | Population  | Recombination | Mutation     | LS  | ML |
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- **no** usage of multilevel paradigm
- considered **not** competitive with state-of-the-art tools [Cohoon et al. '03]
- benchmarked on **small & outdated** hypergraphs

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| Armstrong         |   |             |               |              | kFM | —  |

## Our Contribution:

first **memetic multilevel** HGP algorithm

⇒ **problem-specific** recombine & mutation operators

⇒ **extensive** experiments on **large** benchmark set

- **no** usage of multilevel paradigm
- considered **not** competitive with state-of-the-art tools [Cohoon et al. '03]
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# Memetic Multilevel HGP – Initial Population

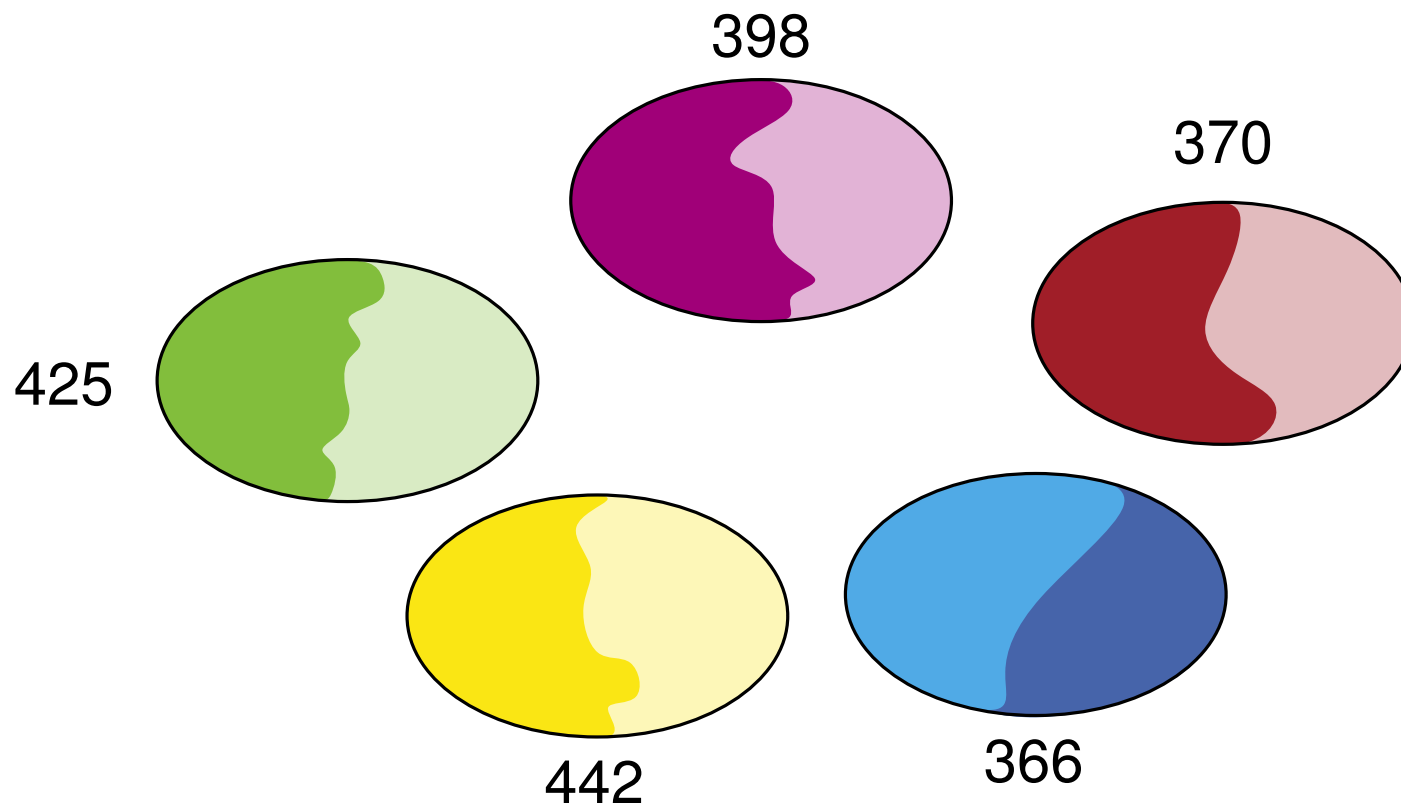
generate initial population  $\mathcal{P}$

■ individuals  $\rightsquigarrow$  **high-quality** partitions of KaHyPar-CA

■ **dynamic** population size  $|\mathcal{P}| := \max(3, \min(50, \delta \cdot \frac{t}{t_l}))$

■ **fitness**:  $(\lambda - 1)$  objective

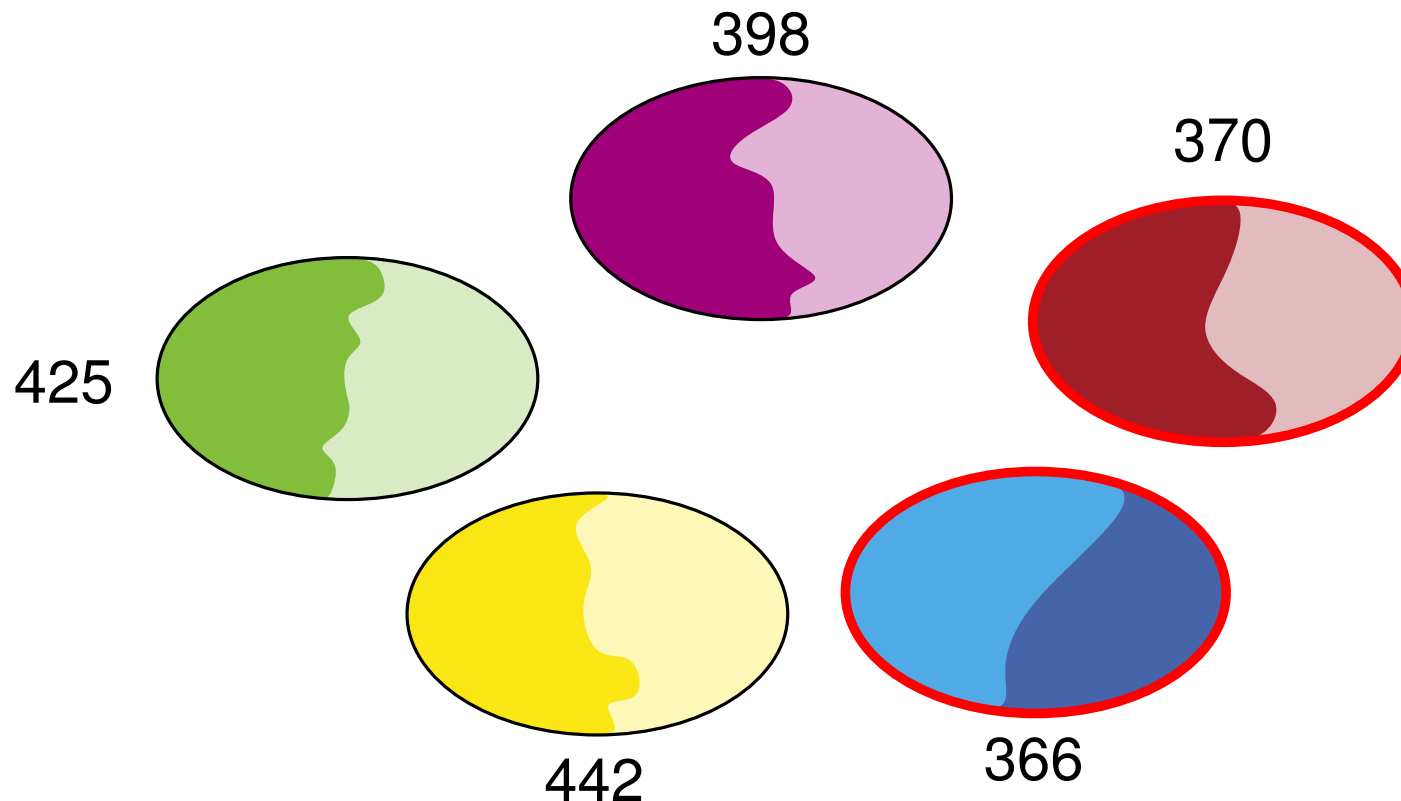
total time  $t$   
time for 1 partition  $t_l$   
tuning param.  $\delta$



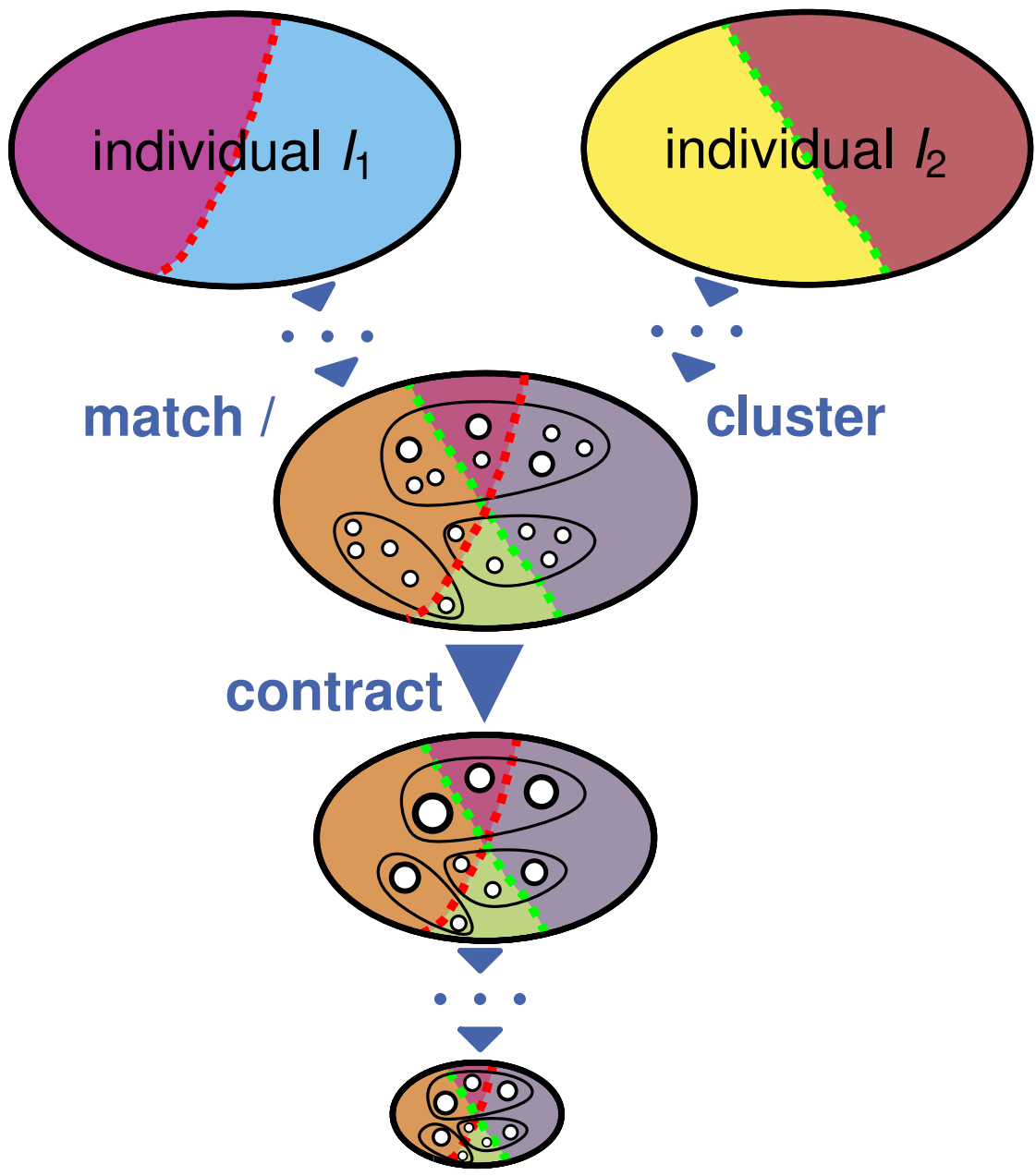
# Memetic Multilevel HGP – Parent Selection

**select** individuals for recombination

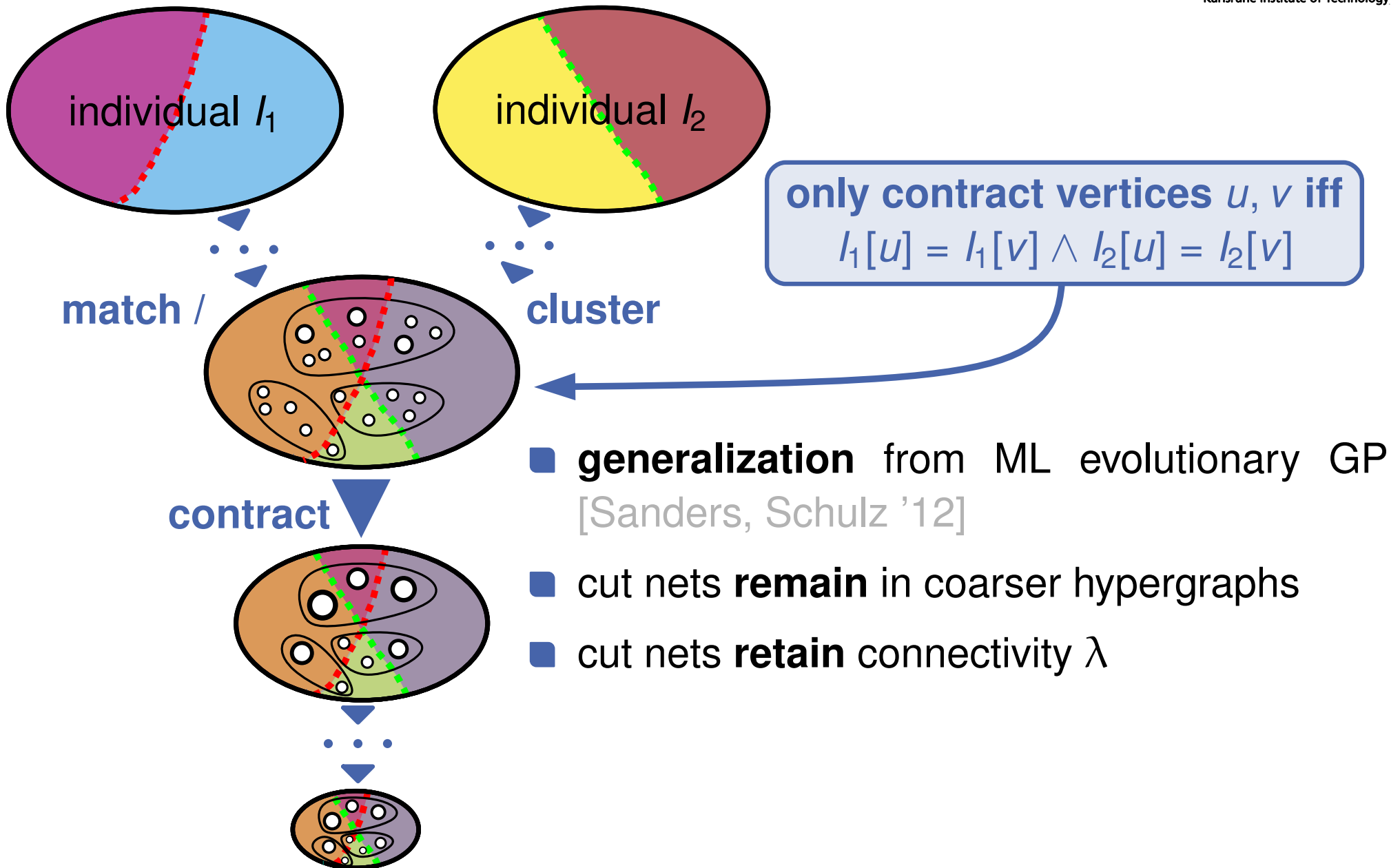
- **Two-Point Recombine**  $\Rightarrow$  **binary** tournament selection
- **Edge-Frequency Multi-Recombine**  $\Rightarrow$  use  $\lceil \sqrt{|\mathcal{P}|} \rceil$  **best**



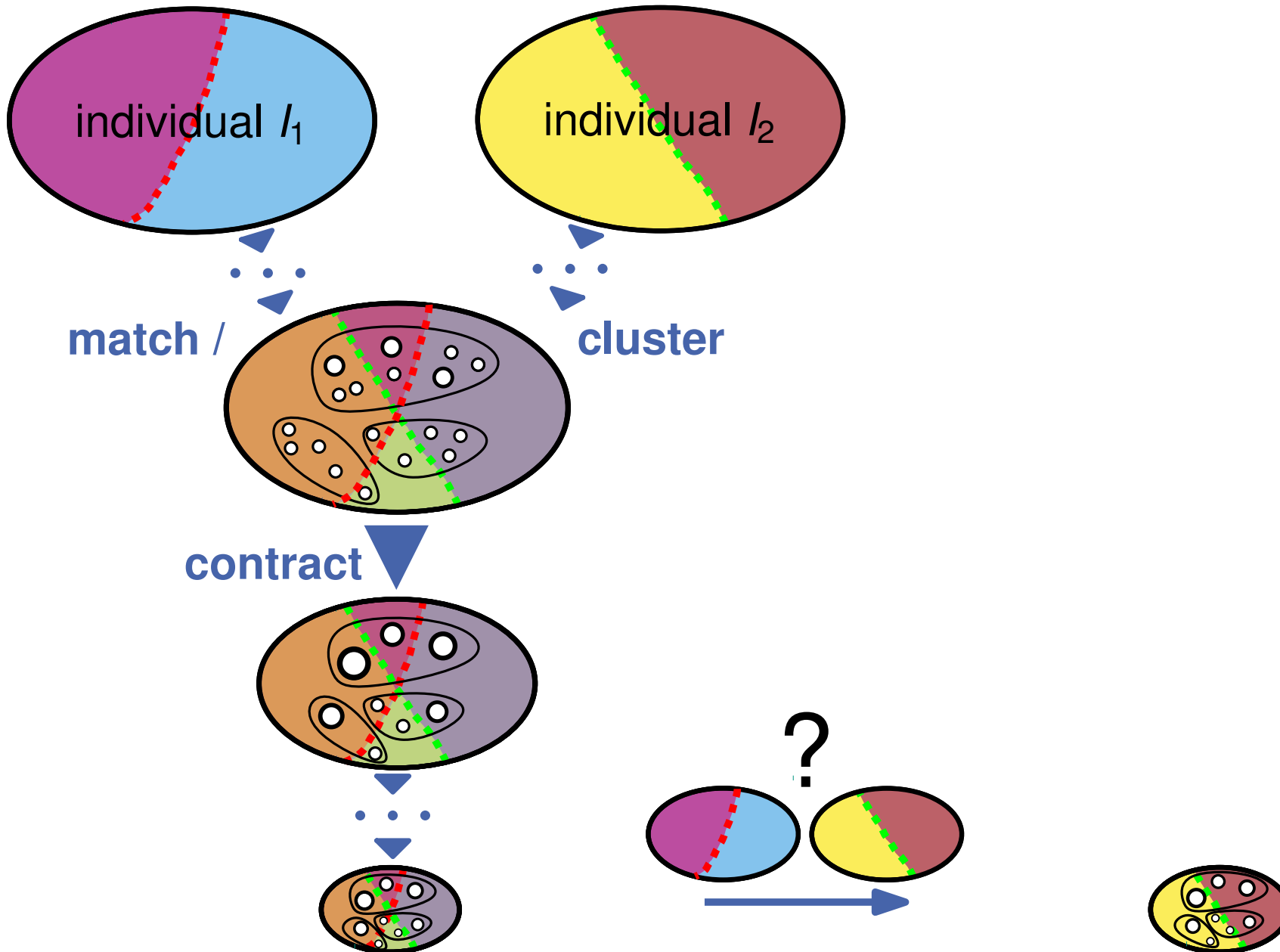
# Memetic Multilevel HGP – 2-Point Recombine (+C)



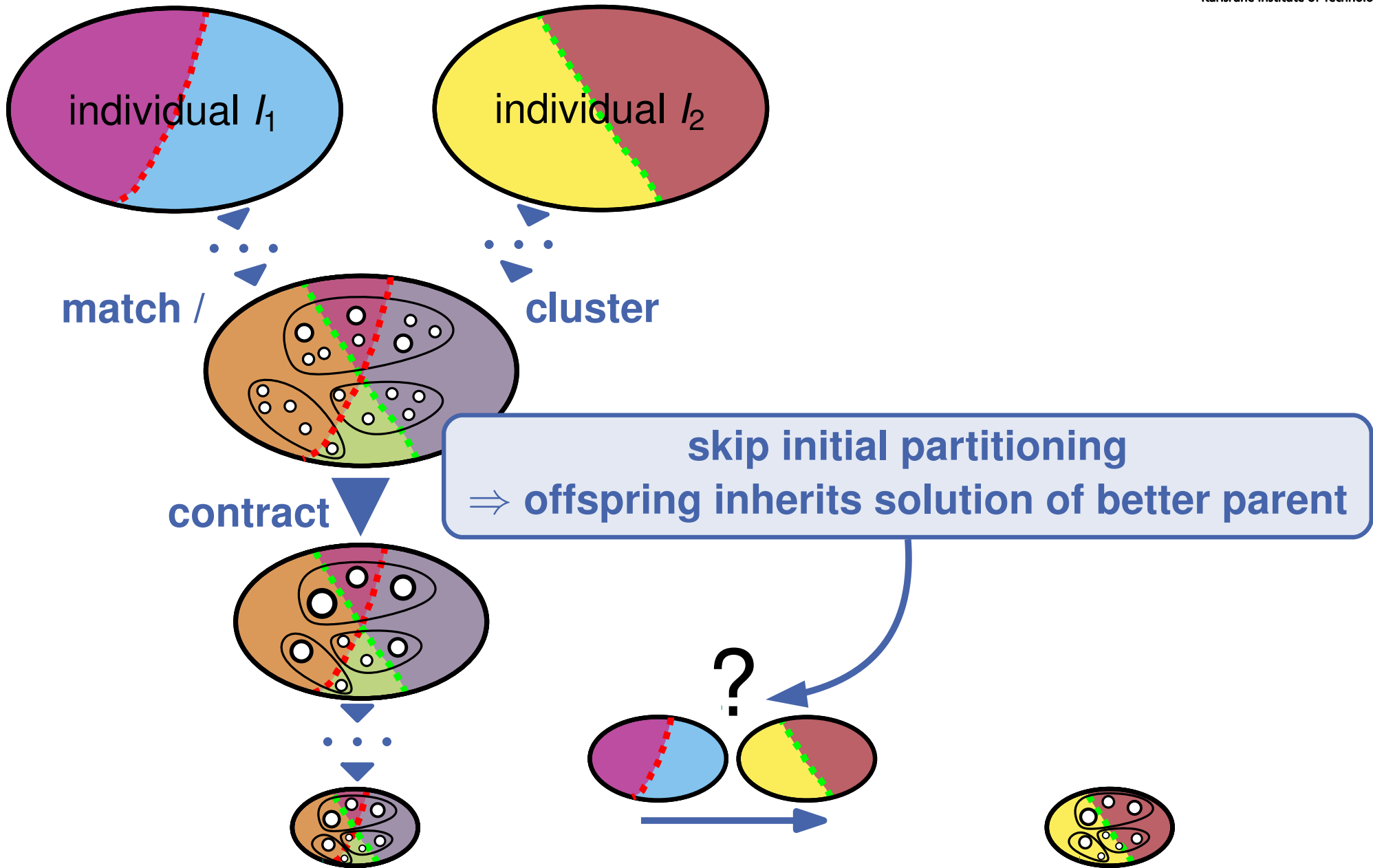
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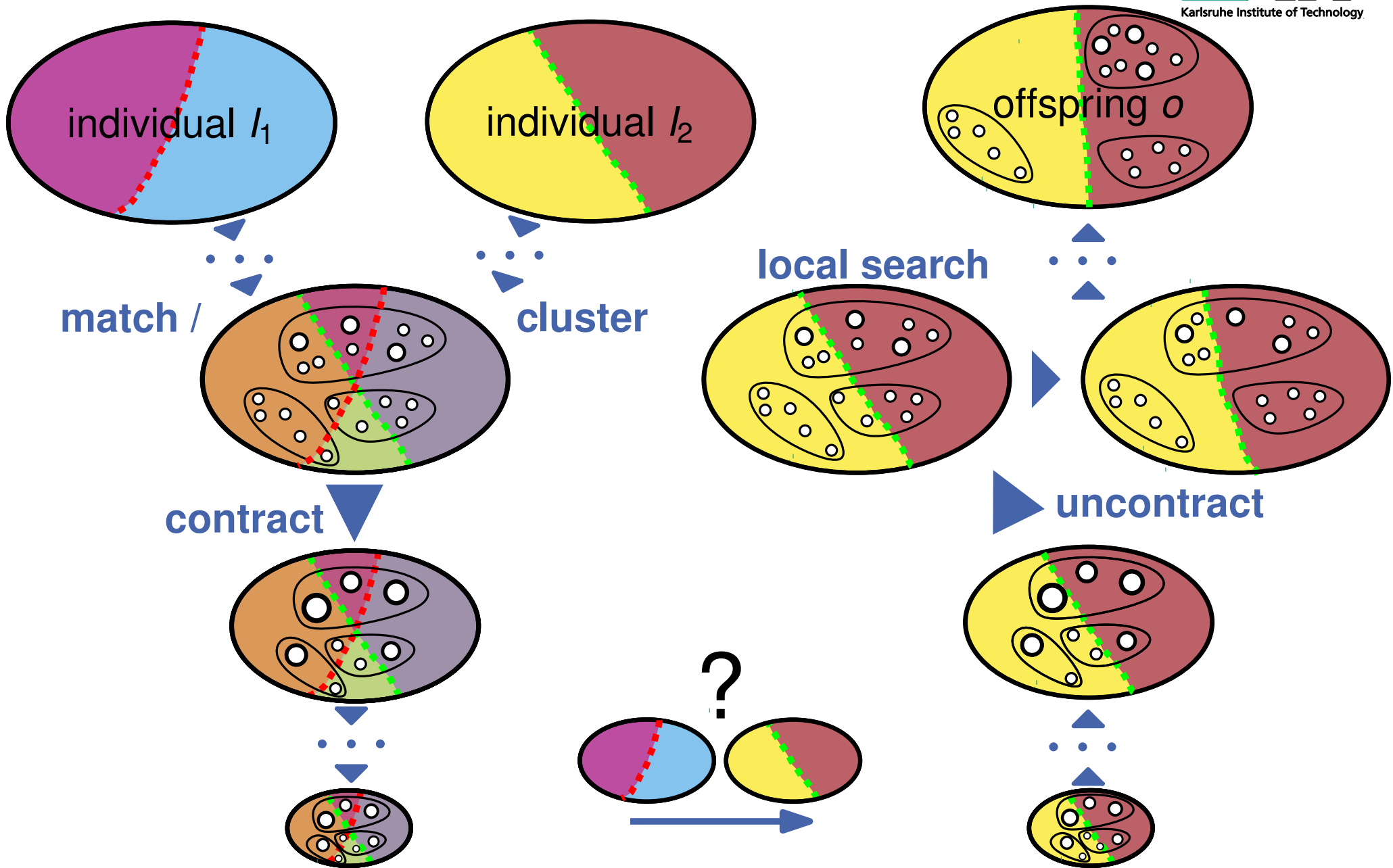
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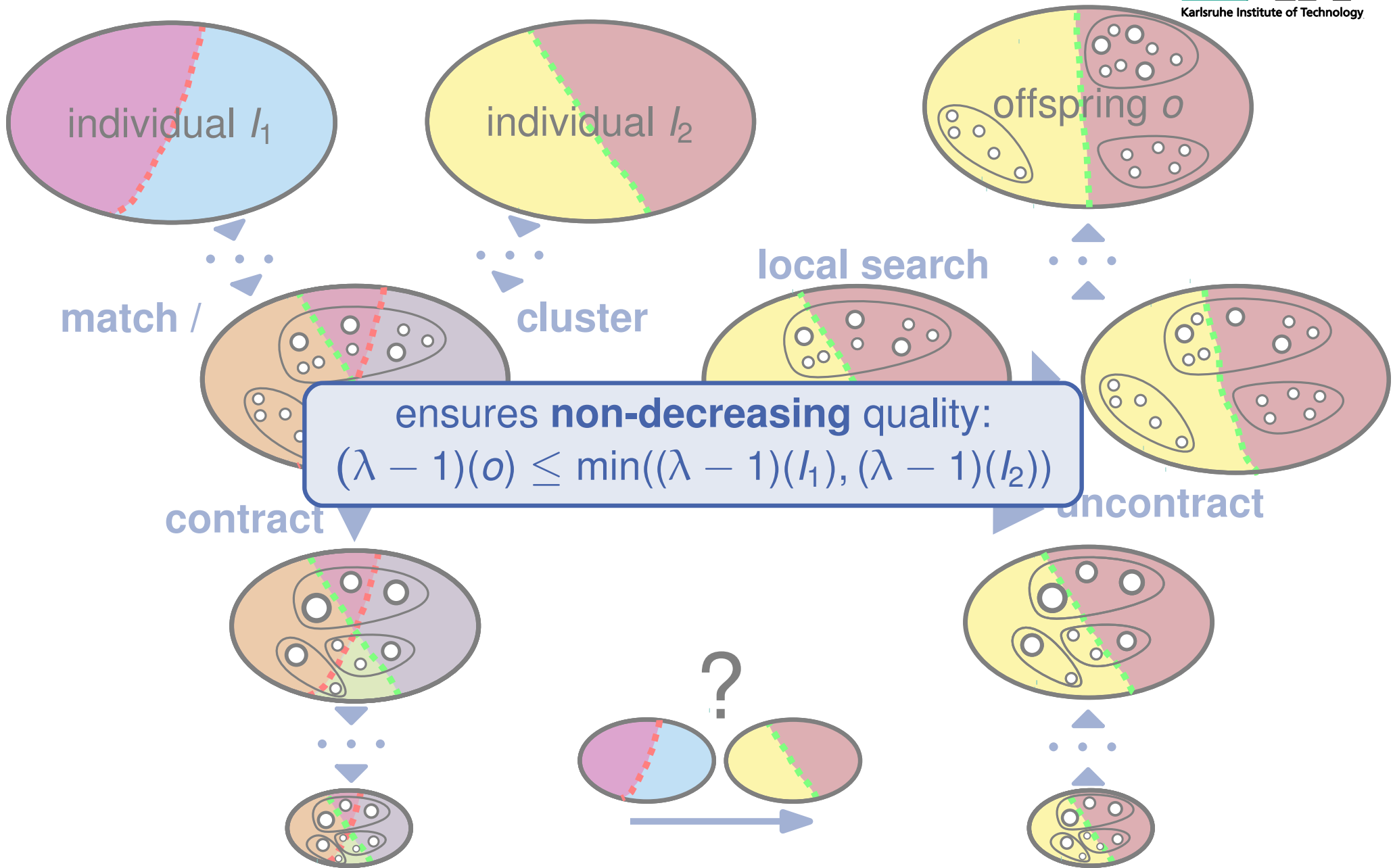
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**Idea:** frequently occurring cut nets likely part of high quality solutions

- look at  $t = \lceil \sqrt{|\mathcal{P}|} \rceil$  **best** individuals
- compute **edge frequency**  $f(e) := |\{I \in t \mid \lambda(e) > 1\}|$
- prefer contractions of vertices incident to **low frequency** nets:

$$r(u, v) := \frac{1}{c(v) \cdot c(u)} \sum_{e \in \{I(v) \cap I(u)\}} \frac{\exp(-\gamma f(e))}{|e|} \quad [\text{Wichlund, Aas '98}]$$

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large number ...

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[Wichlund, Aas '98]

... with small size

... of **low-frequency** nets ...

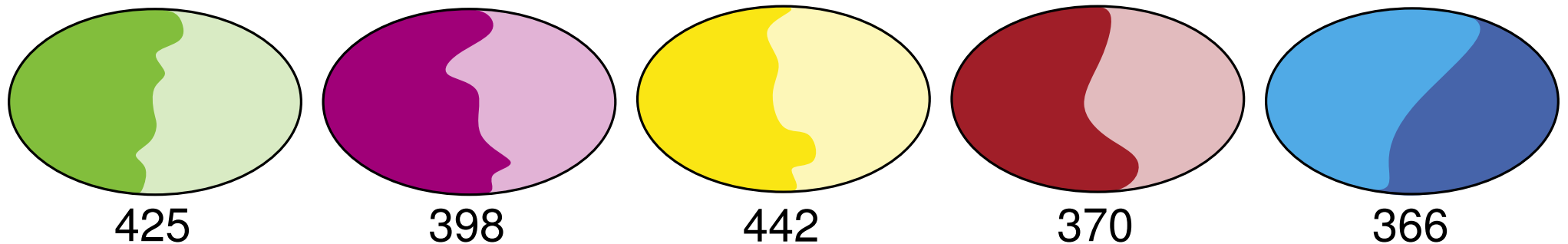
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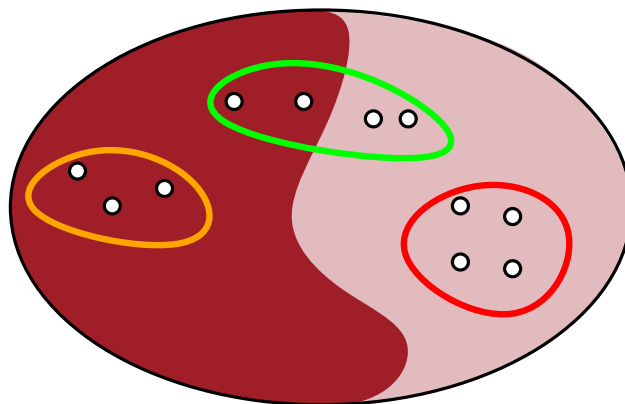
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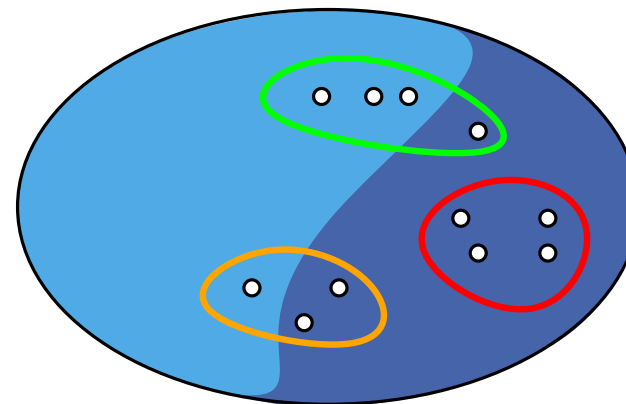
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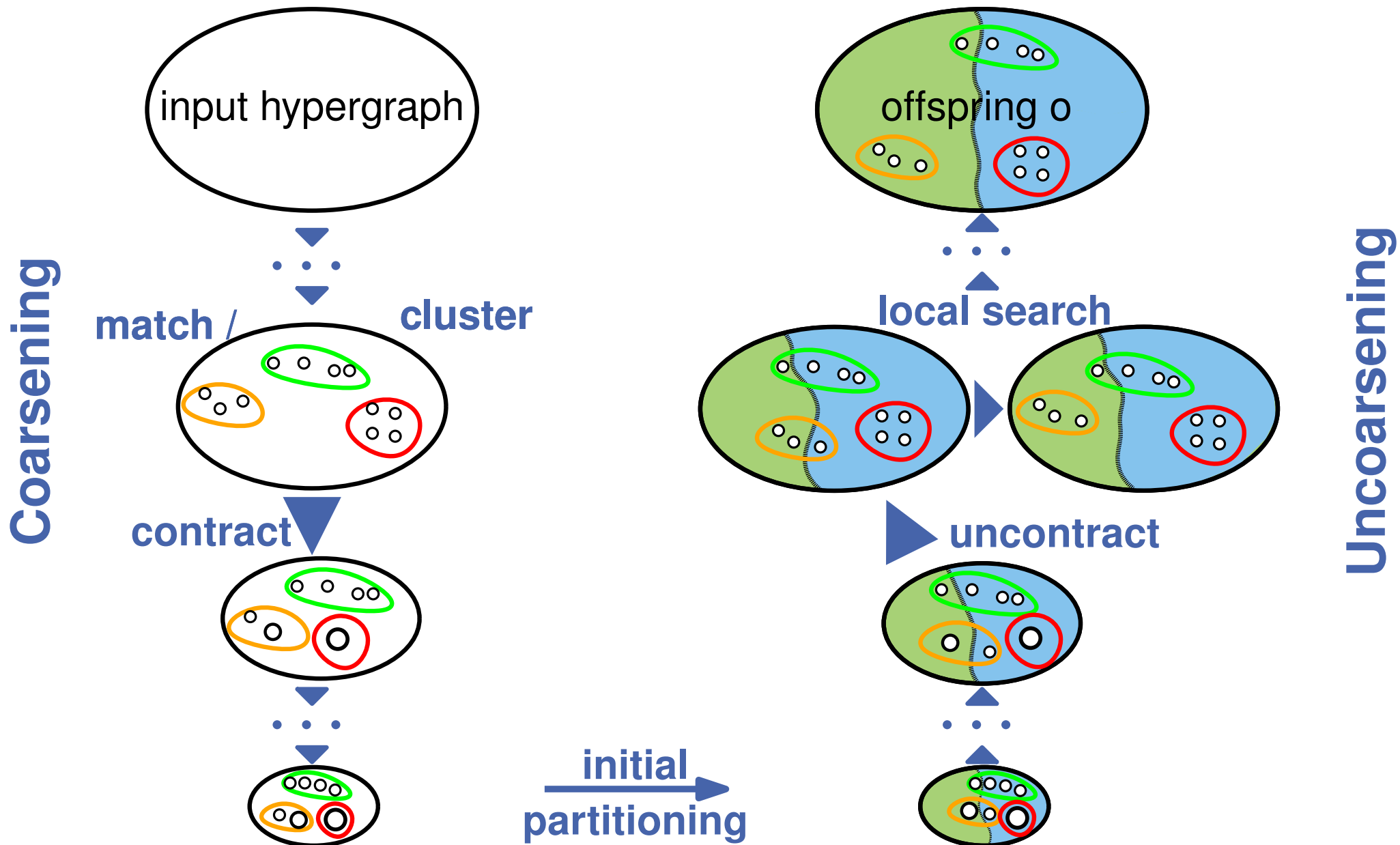


370

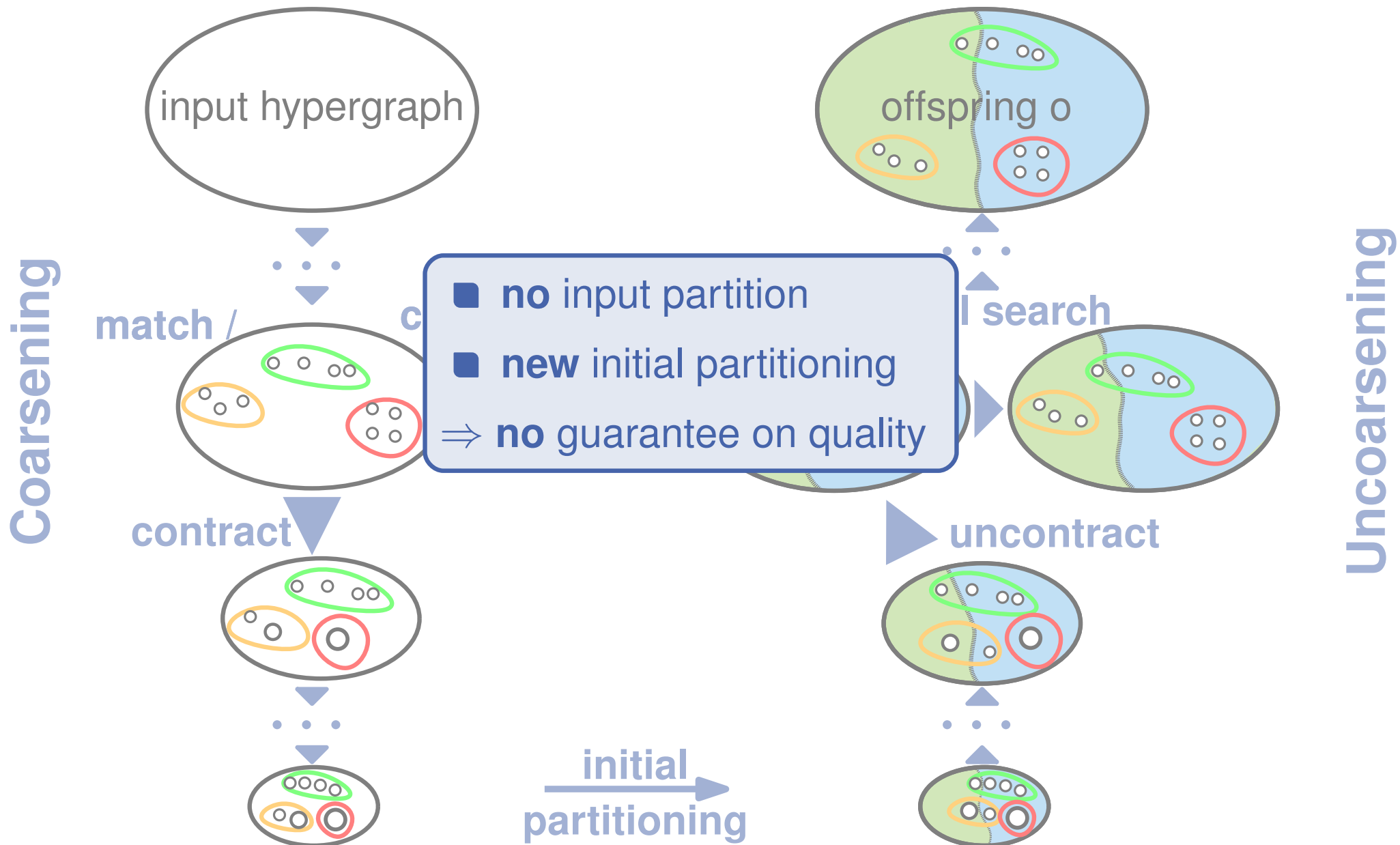


366

# Memetic Multilevel HGP – Multi-Recombine (+ER)

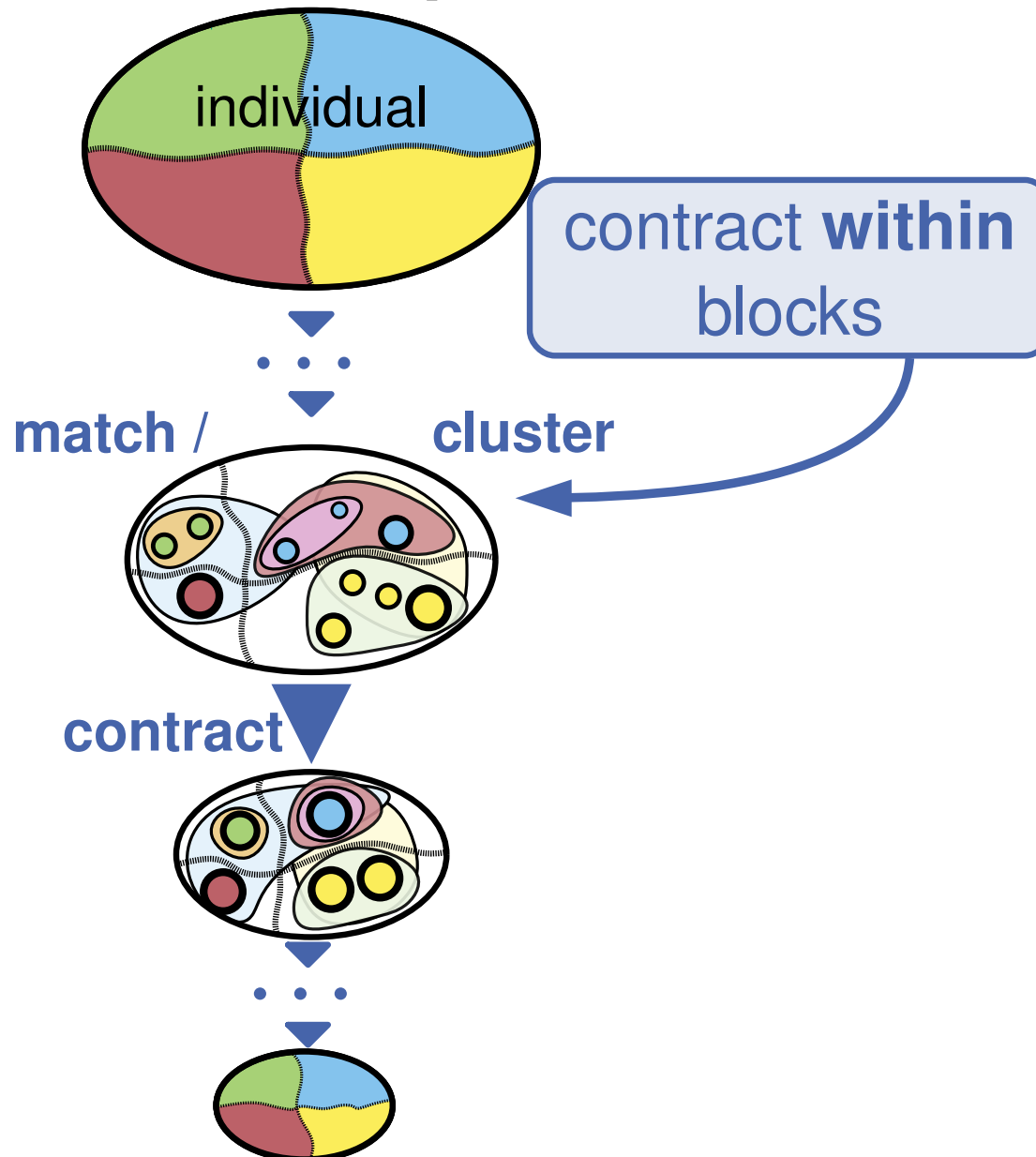


# Memetic Multilevel HGP – Multi-Recombine (+ER)



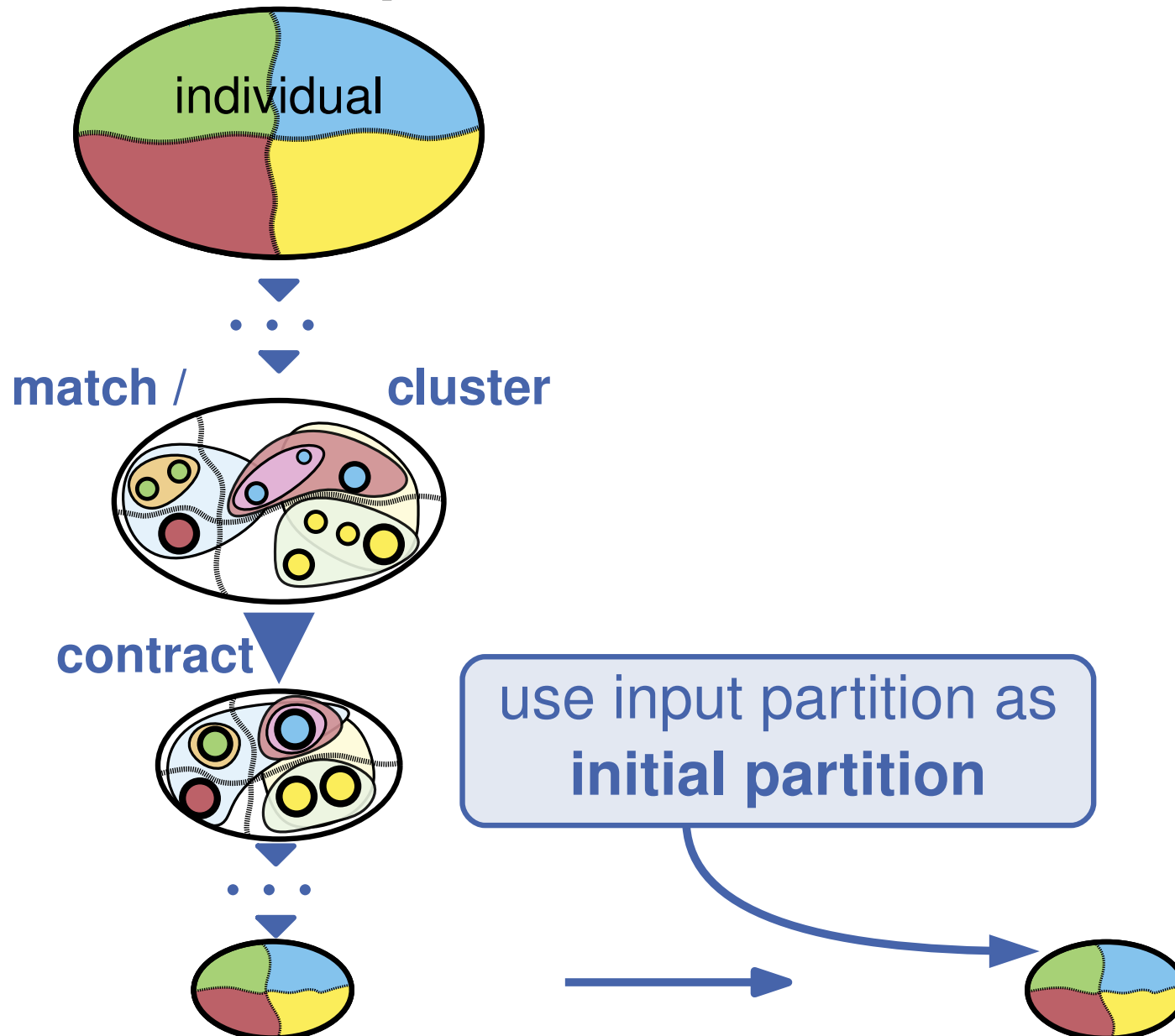
# Memetic Multilevel HGP – V-Cycle Mutation (+M)

[Sanders, Schulz '12]



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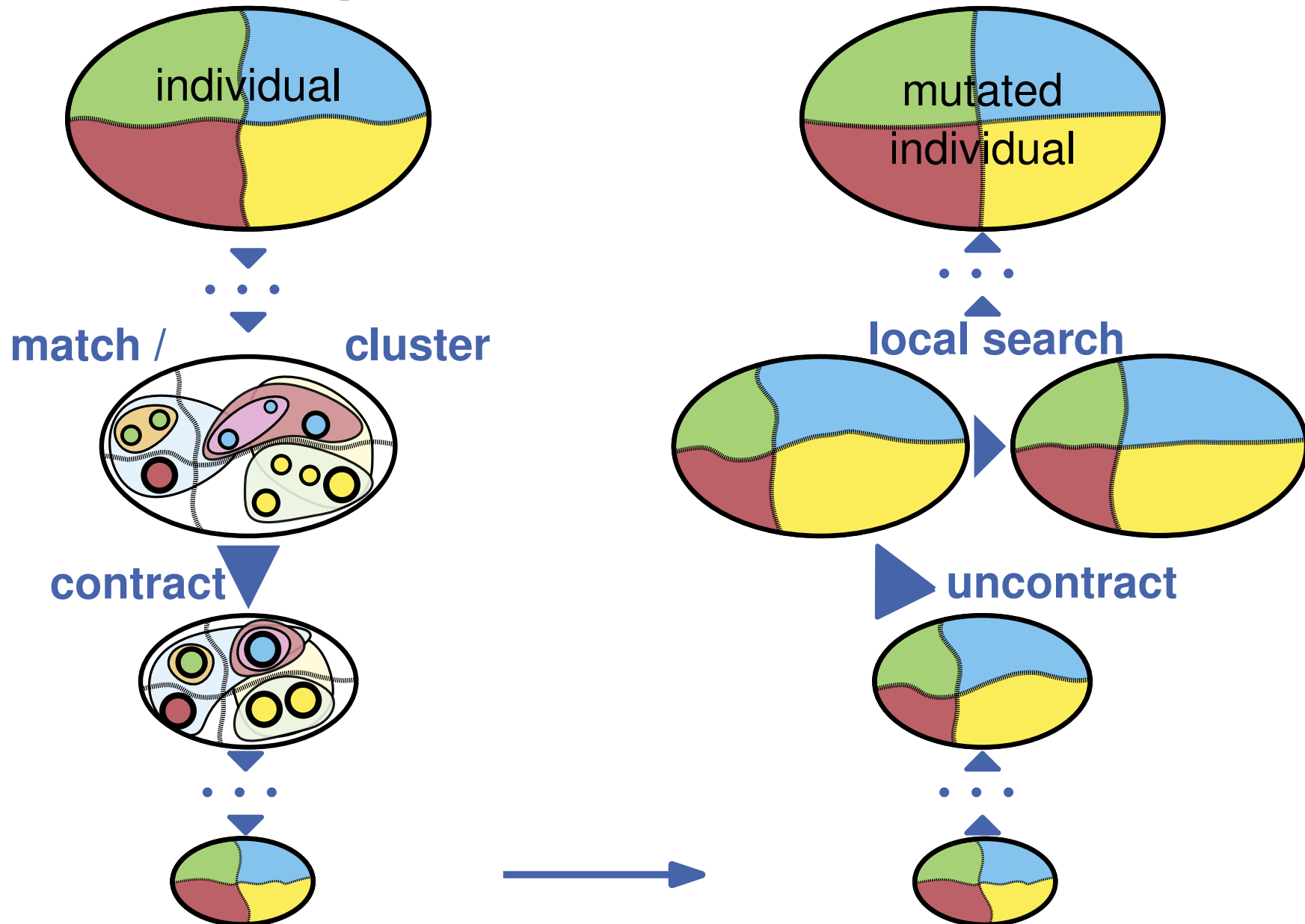
[Sanders, Schulz '12]





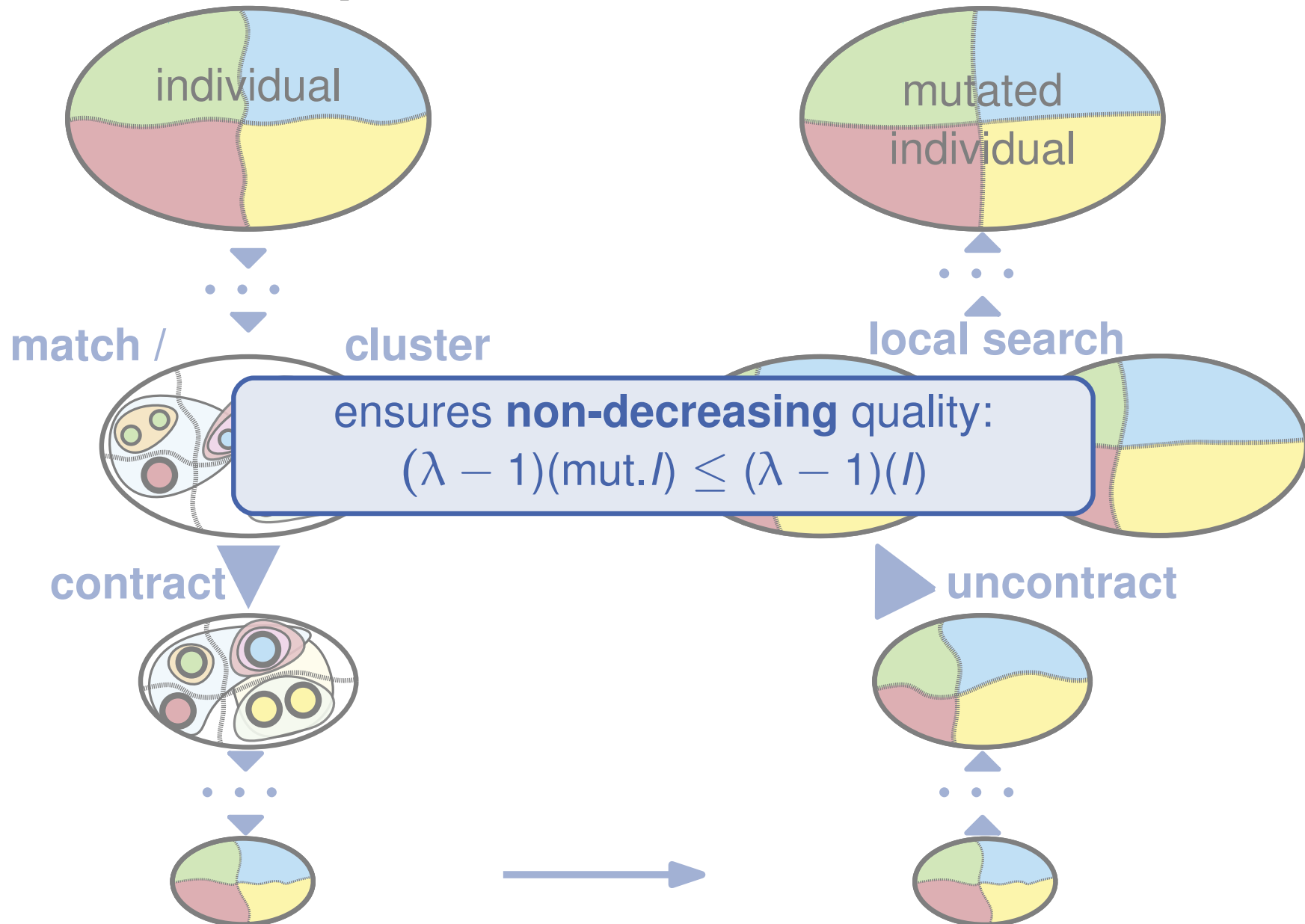
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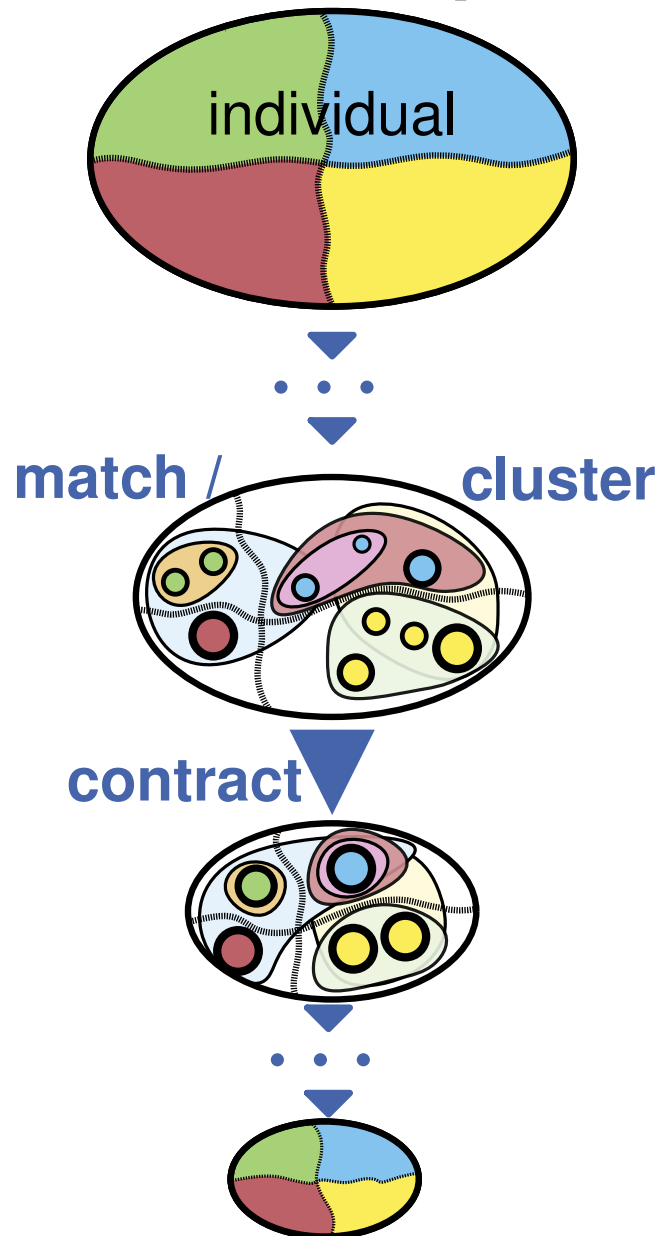
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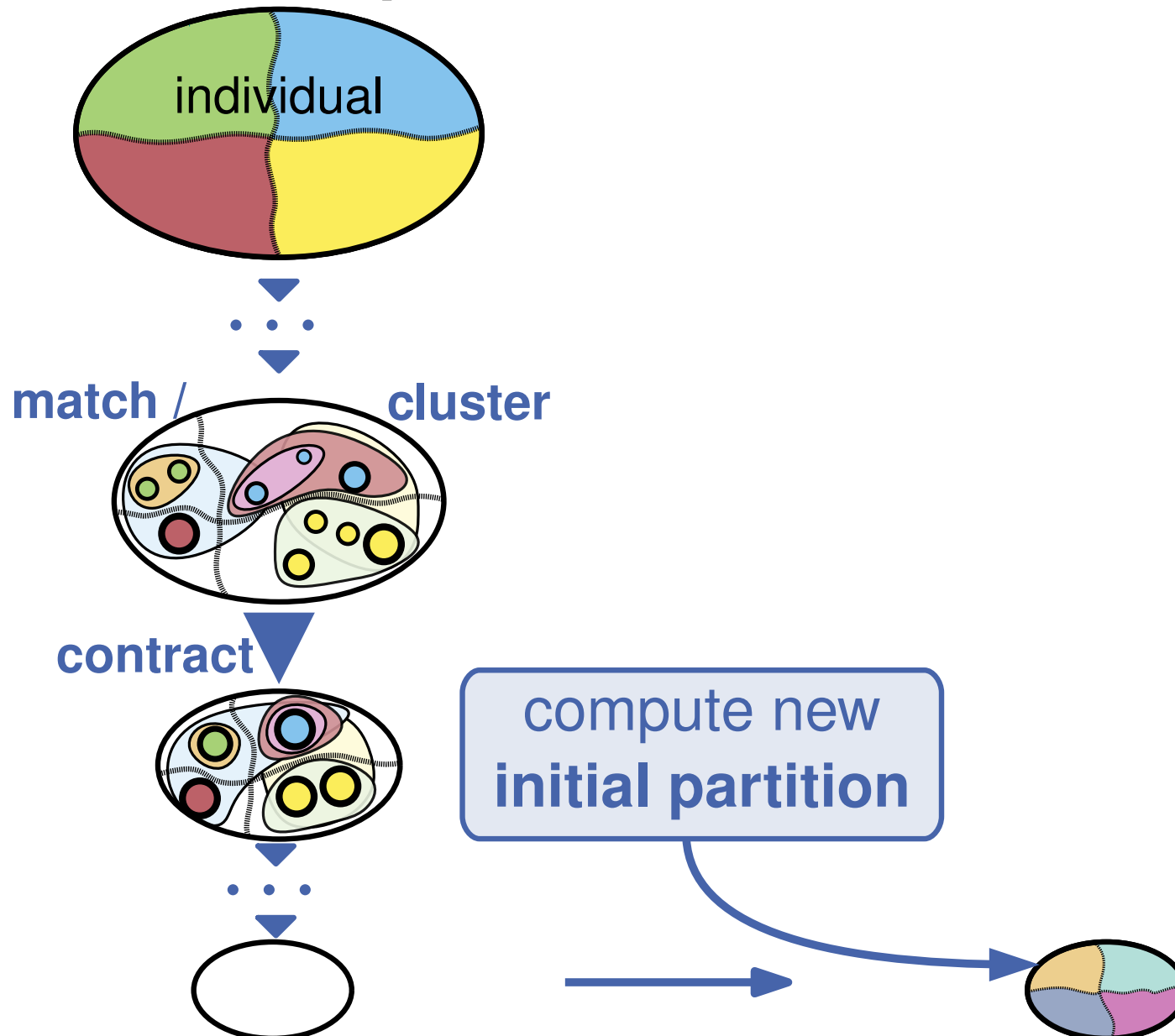
[Sanders, Schulz '12]

## New Initial Partitioning



# Memetic Multilevel HGP – V-Cycle Mutation (+M) New Initial Partitioning

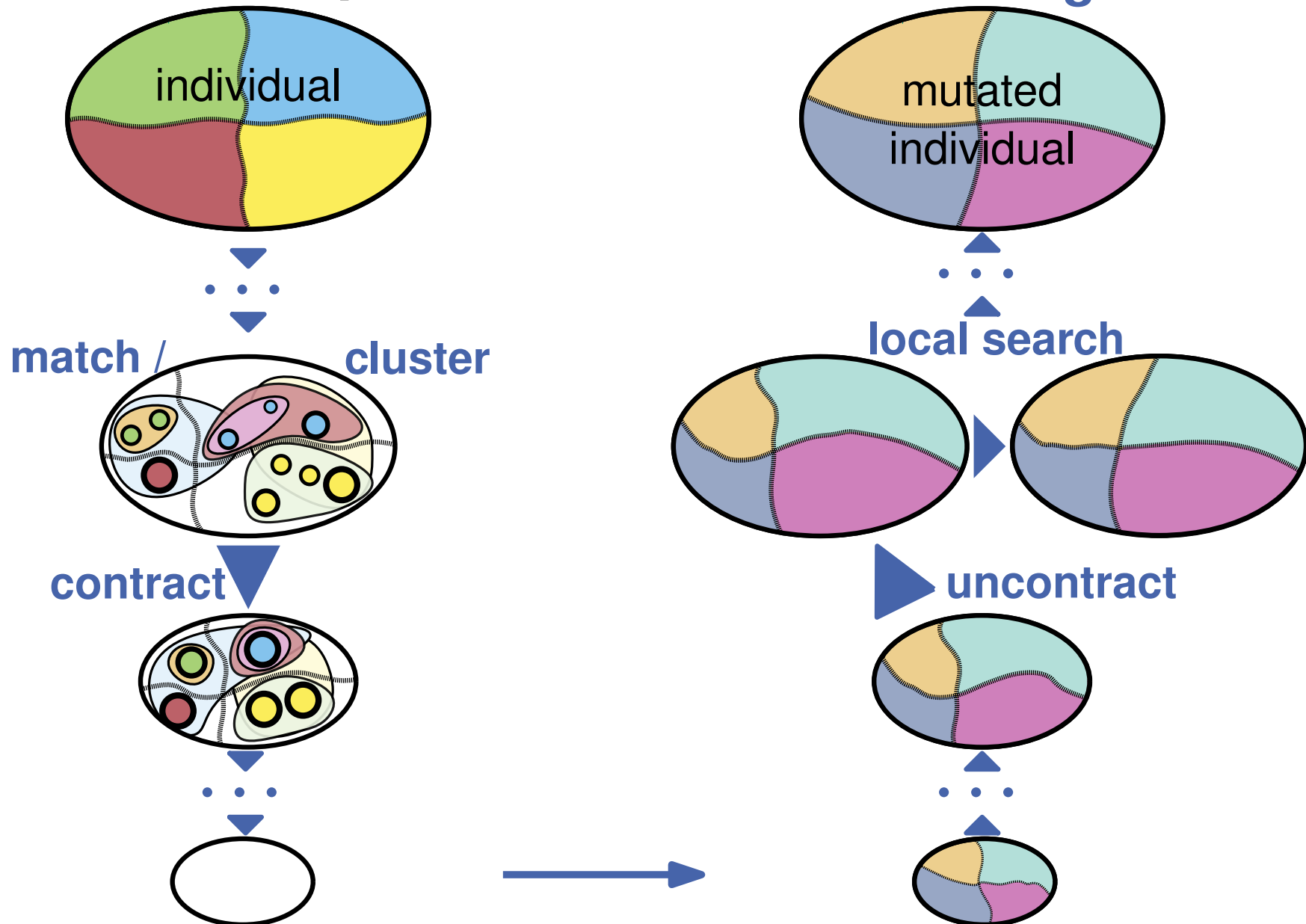
[Sanders, Schulz '12]



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[Sanders, Schulz '12]

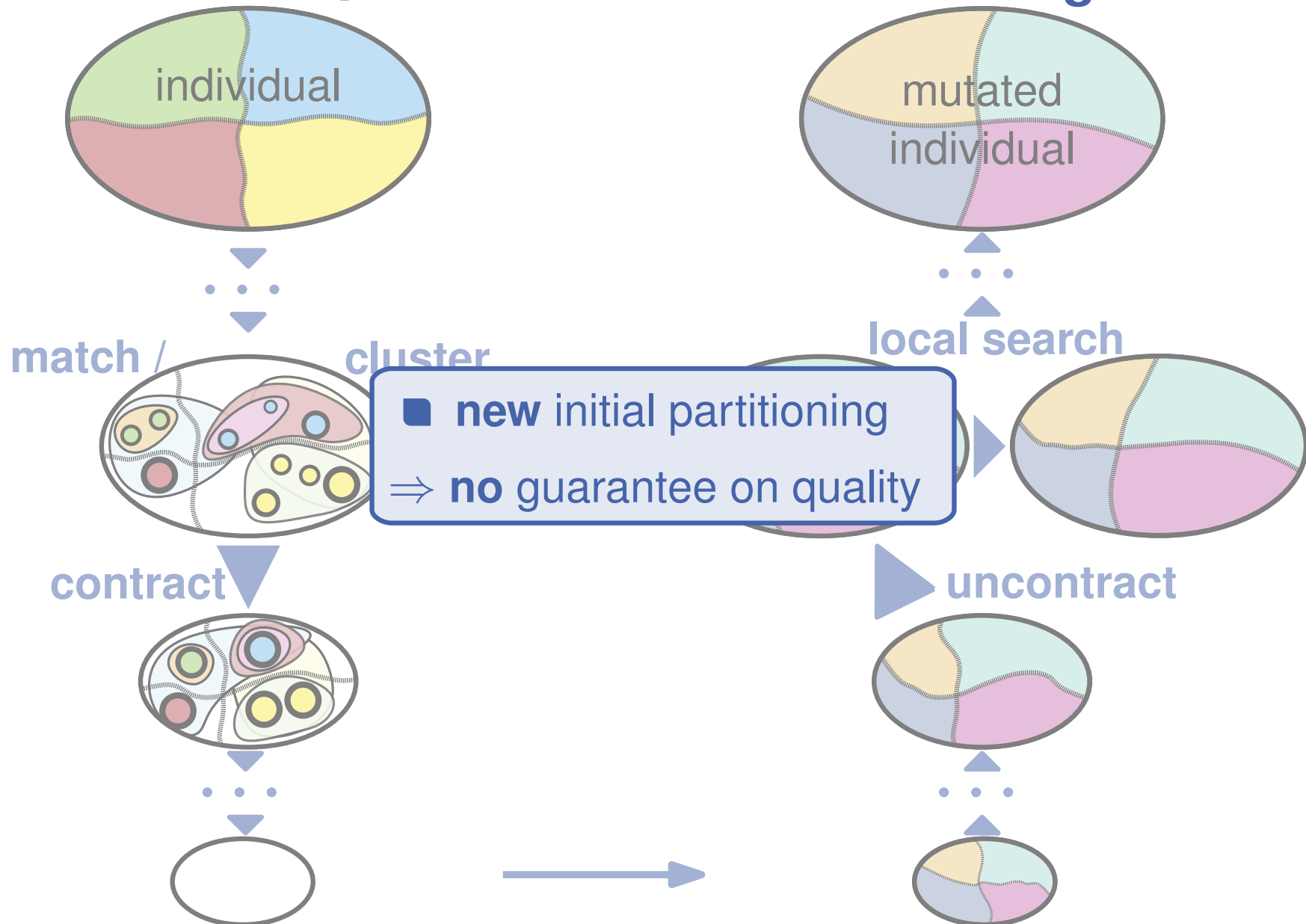
## New Initial Partitioning



# Memetic Multilevel HGP – V-Cycle Mutation (+M)

## New Initial Partitioning

[Sanders, Schulz '12]



# Memetic Multilevel HGP – Replacement Rule

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similarity measure:  $d(l, o) := |D_l \ominus D_o|$

$$D_x := \{(e, m(e)) : e \in E_x\}$$

$$m(e) := (\lambda(e) - 1)$$

symmetric  
difference

multiset of cut nets



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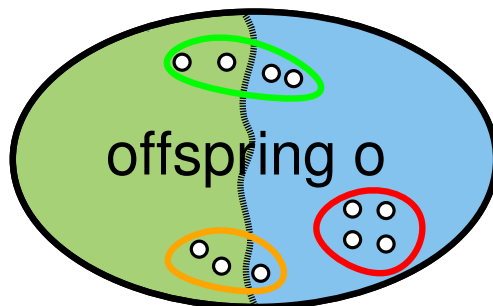
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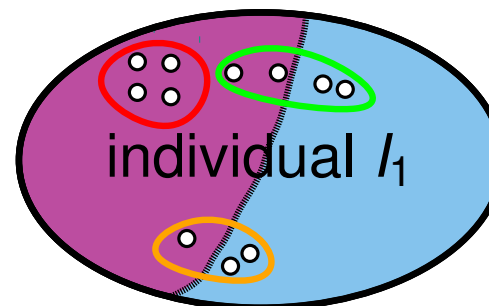
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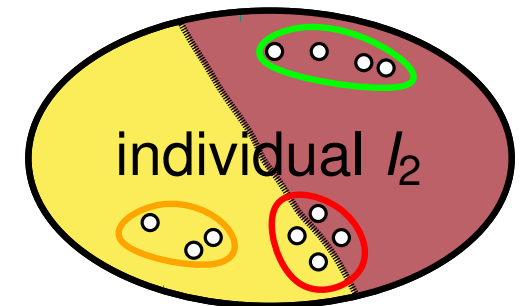
398

$$D_o = \{e, e\}$$



398

$$D_{l_1} = \{e, e\}$$



442

$$D_{l_2} = \{e\}$$

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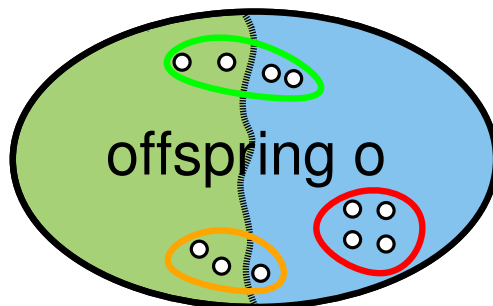
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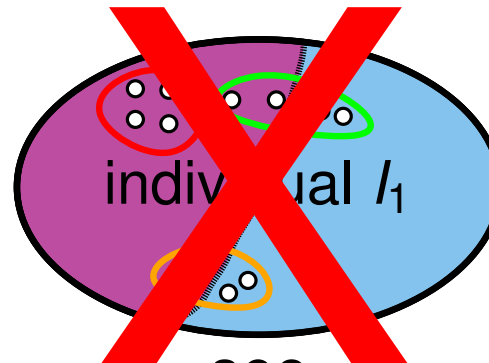
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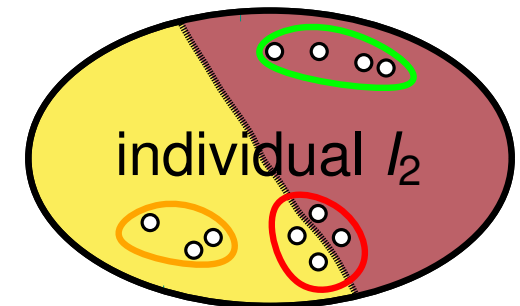
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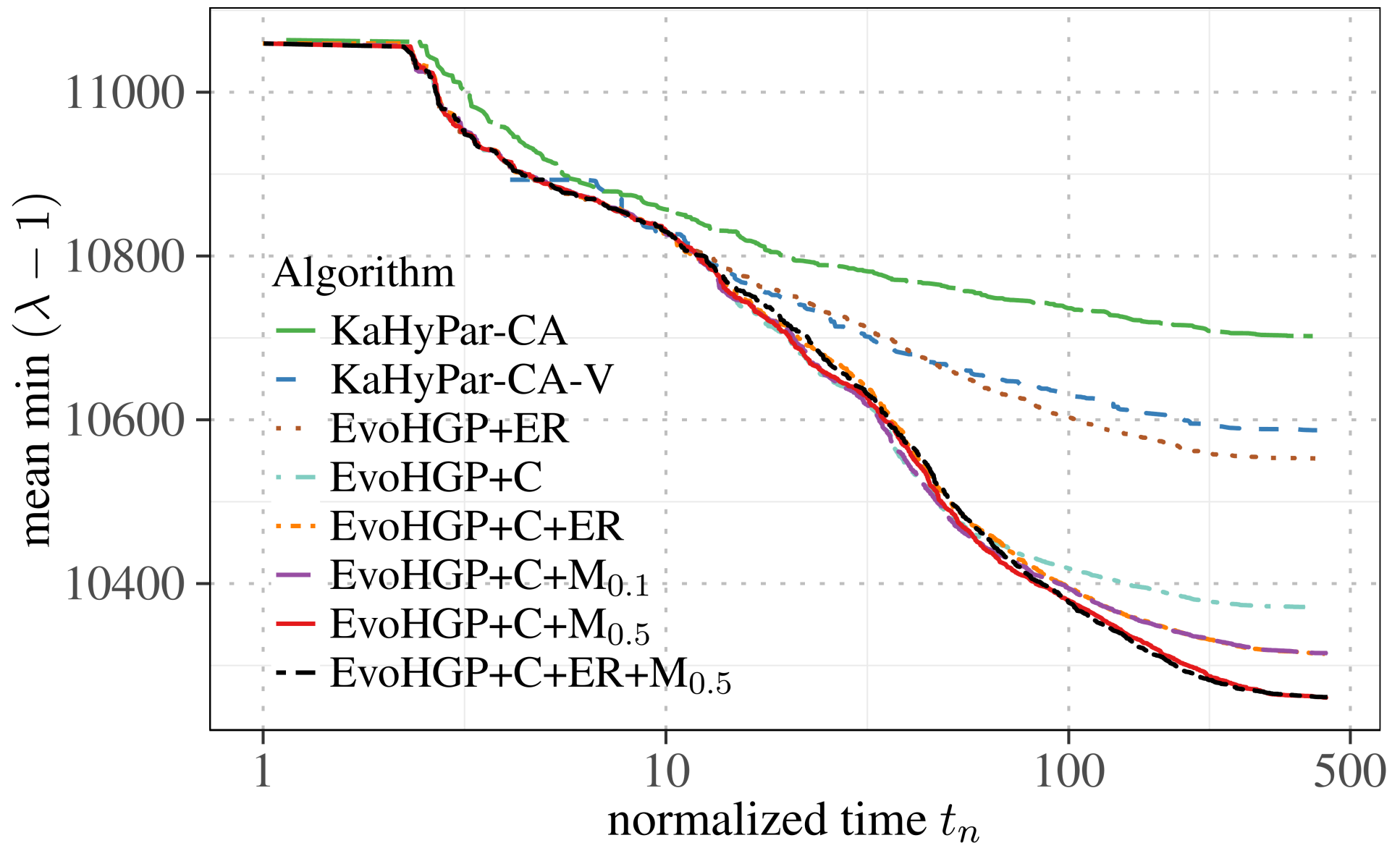
$$D_{l_2} = \{e\}$$

# Experiments – Benchmark Setup

- system: Intel Xeon E5-2670 @ 2.6 Ghz, 64 GB RAM
  
- # hypergraphs: [publicly available] total    subset
  - SuiteSparse Matrix Collection 32    5
  - SAT Competition 2014 (3 representations) 18.3    5.3
  - ISPD98 & DAC2012 VLSI Circuits 14    5
  
- $k \in \{2, 4, 8, 16, 32, 64, 128\}$  with imbalance:  $\varepsilon = 3\%$
- timelimit: 8h / instance
- 5 repetitions / instance
  
- comparing **EvoHGP** (KaHyPar-E) with:
  - KaHyPar-CA
  - KaHyPar-CA + V-Cycles
  - hMetis-R & hMetis-K
  - PaToH-Default & PaToH-Quality

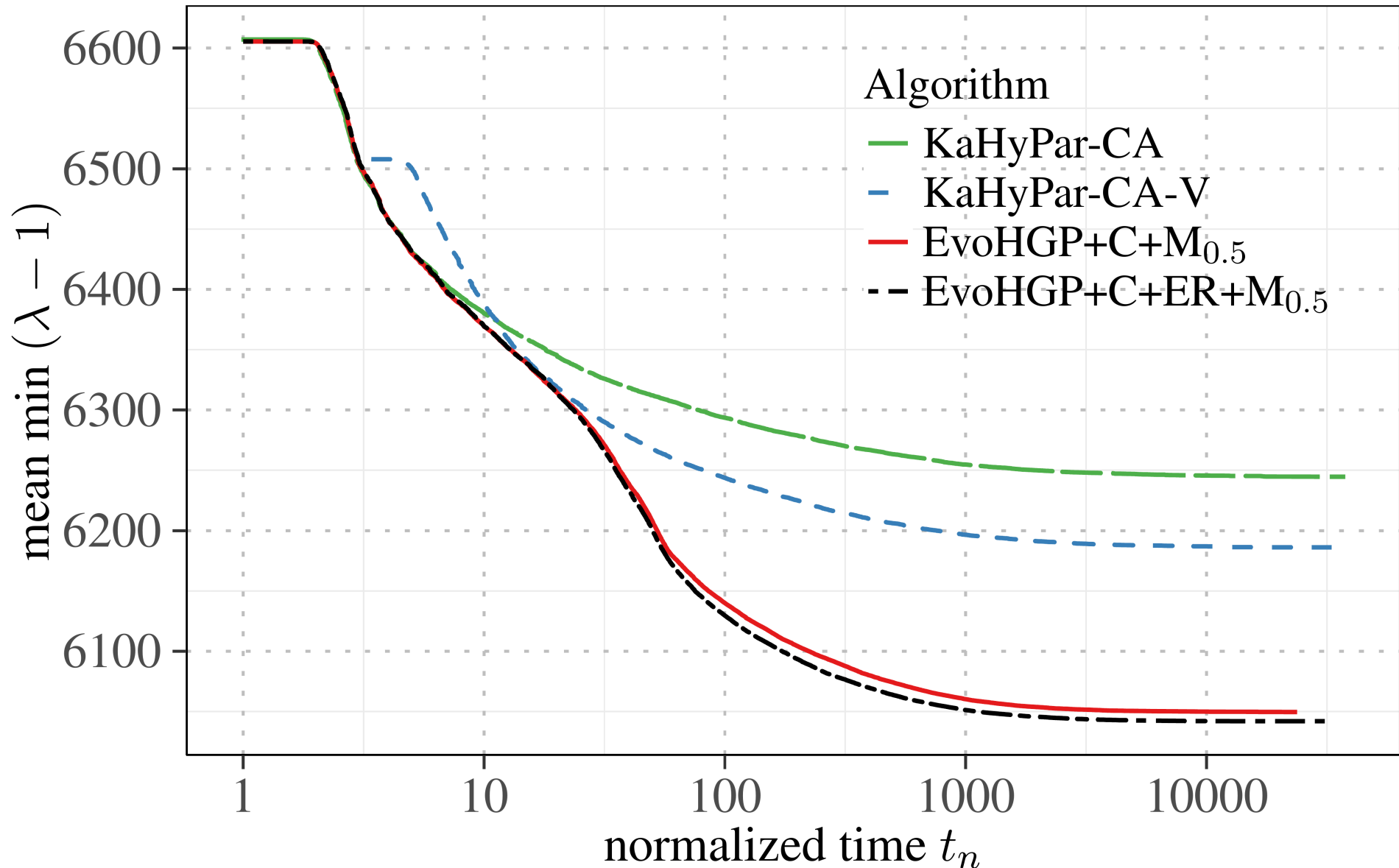
# Influence of Algorithmic Components

Instance Subset



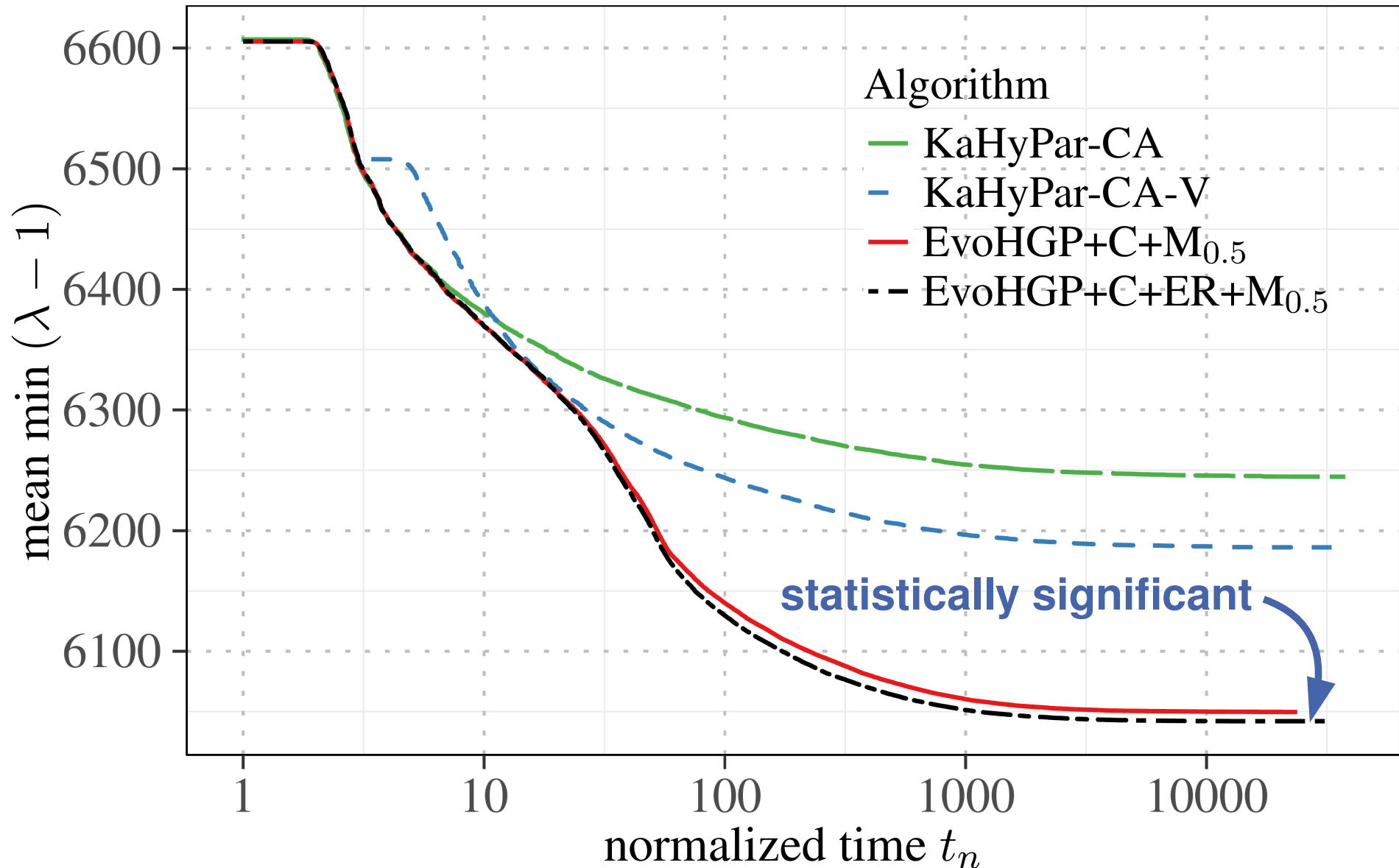
# EvoHGP vs. KaHyPar-CA & KaHyPar-CA-V

All Instances



# EvoHGP vs. KaHyPar-CA & KaHyPar-CA-V

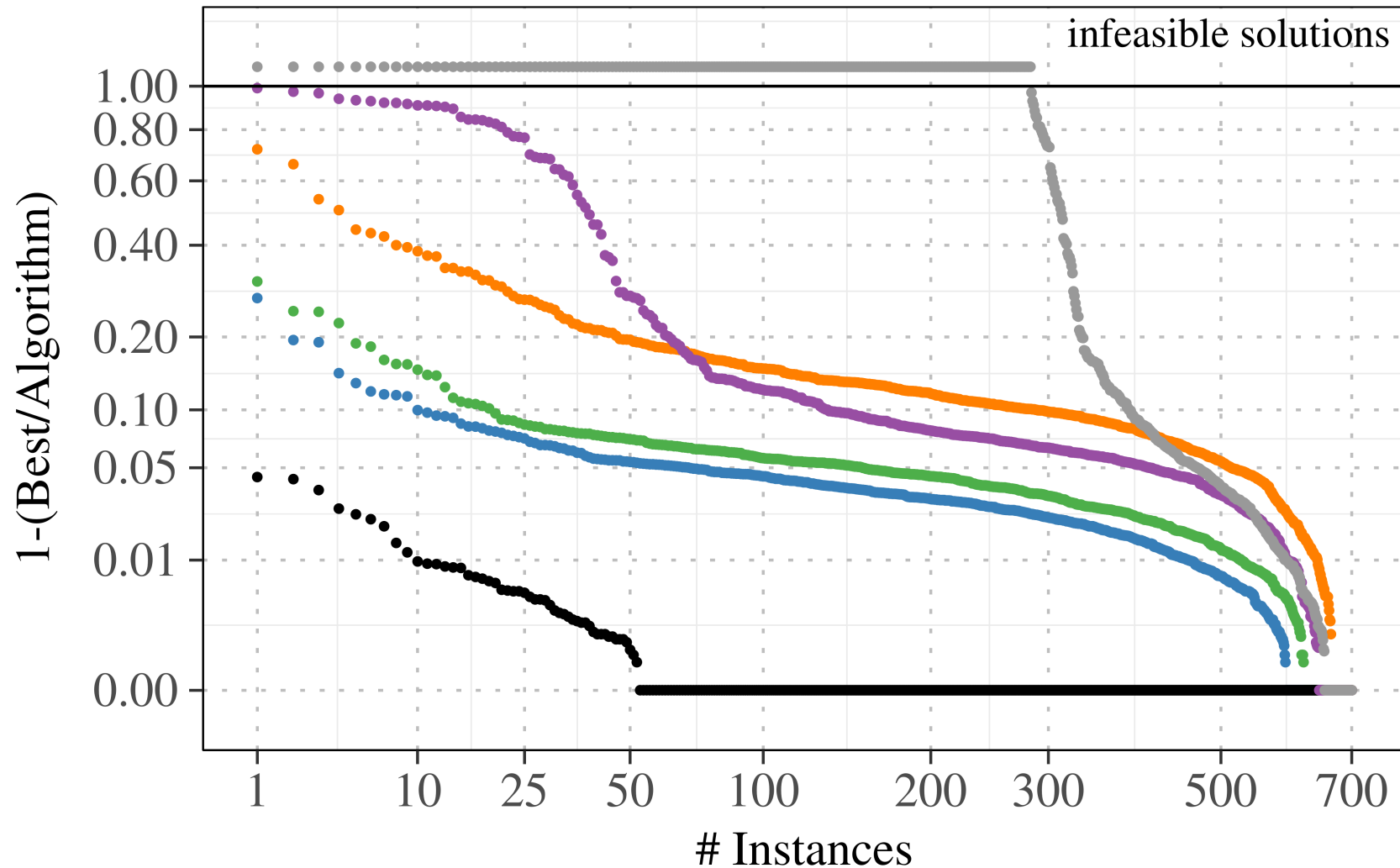
All Instances



# EvoHGP Improvement over KaHyPar-CA & -CA-V

| $k$ | KaHyPar-CA vs. EvoHGP |                        | KaHyPar-CA-V vs. EvoHGP |                        |
|-----|-----------------------|------------------------|-------------------------|------------------------|
|     | +C+M <sub>0.5</sub>   | +C+ER+M <sub>0.5</sub> | +C+M <sub>0.5</sub>     | +C+ER+M <sub>0.5</sub> |
| all | 3.3%                  | 3.4%                   | 2.3%                    | 2.4%                   |
| 2   | 0.9%                  | 0.9%                   | 0.3%                    | 0.4%                   |
| 4   | 1.3%                  | 1.4%                   | 0.8%                    | 1.0%                   |
| 8   | 2.7%                  | 2.9%                   | 1.9%                    | 2.0%                   |
| 16  | 3.5%                  | 3.6%                   | 2.5%                    | 2.6%                   |
| 32  | 4.3%                  | 4.6%                   | 3.2%                    | 3.5%                   |
| 64  | 4.9%                  | 5.0%                   | 3.5%                    | 3.6%                   |
| 128 | 5.4%                  | 5.4%                   | 3.7%                    | 3.7%                   |

# Comparison with Best Non-Evo Algorithms



- |           |            |                                |
|-----------|------------|--------------------------------|
| Algorithm | ● hMetis-K | ● KaHyPar-CA                   |
|           | ● PaToH-D  | ● KaHyPar-CA-V                 |
|           | ● hMetis-R | ● EvoHGP+C+ER+M <sub>0.5</sub> |



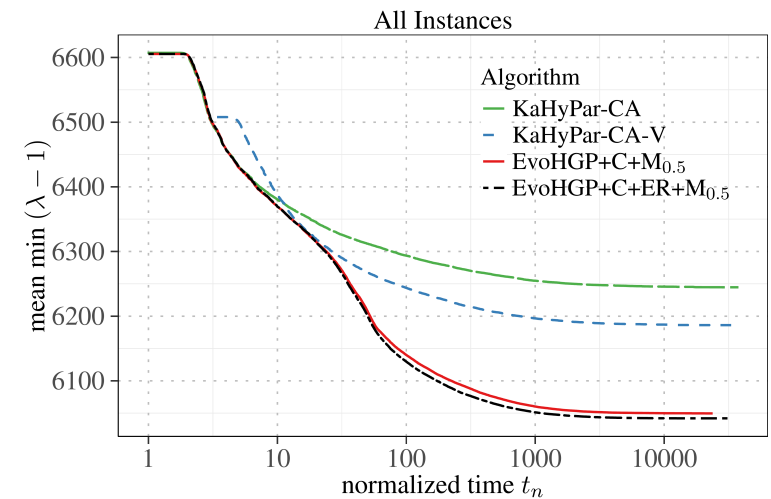
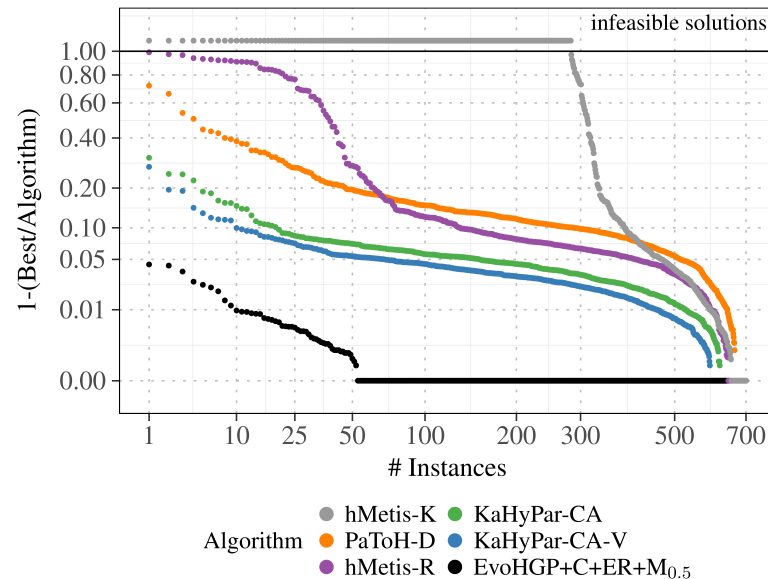
# Conclusion & Discussion

## KaHyPar-E – memetic **multilevel** $k$ -way HGP

- problem-**specific** recombination
- problem-**specific** mutation

## Future Work:

- shared-memory parallelization
- distributed-memory parallelization



**KaHyPar-Framework**  
Open-Source:  
<http://kahypar.org>