More Hierarchy in Route Planning Using Edge Hierarchies

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INSTITUTE OF THEORETICAL INFORMATICS · ALGORITHMICS GROUP


Motivation

- Dijkstra's algorithm is too slow
- Last decade: split up into preprocessing and query phases
- Hierachical Route Planning
  - Roads in the middle of a path are more important than on the ends
- Contraction Hierarchies have one level per node (crossing)

- Today: One level per edge (road)

Problem Definition

- Vertices = Crossings
- Edges = Roads
- Edge weights = metric to optimize

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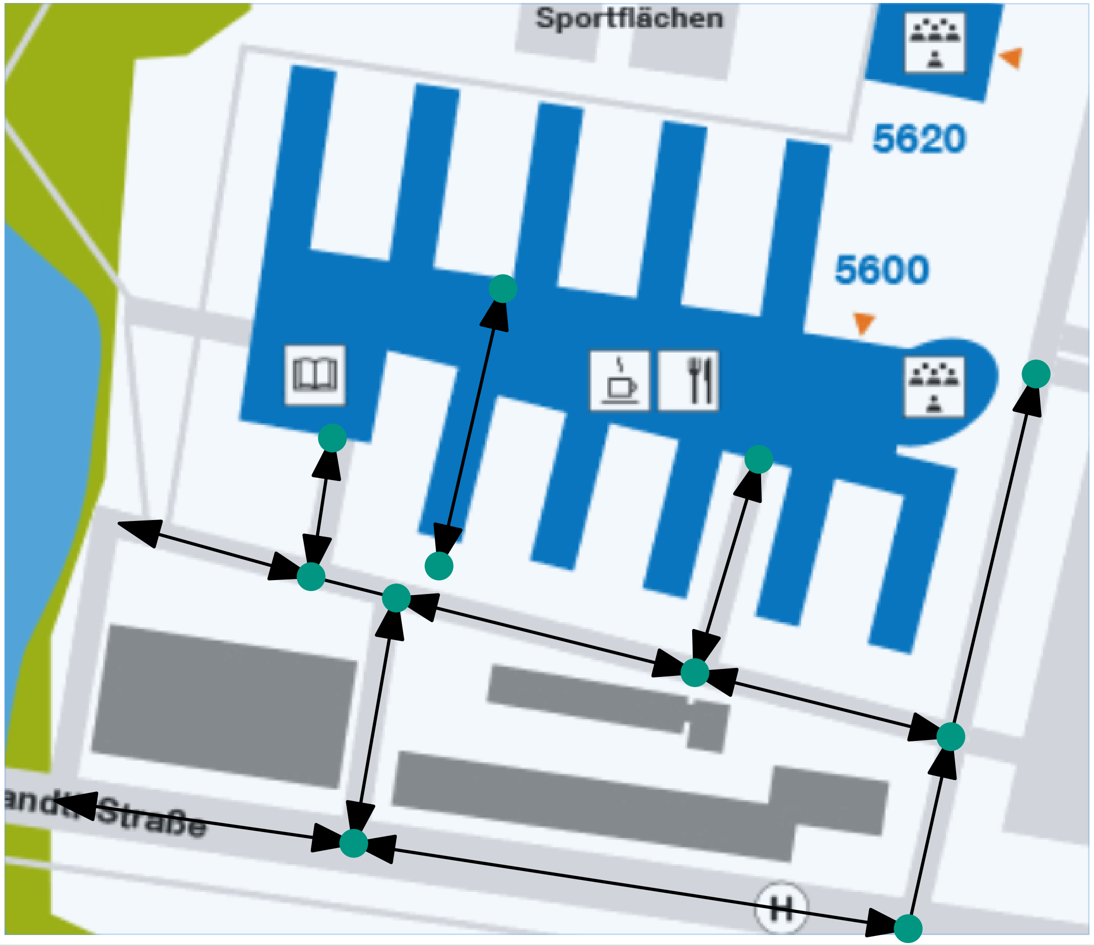

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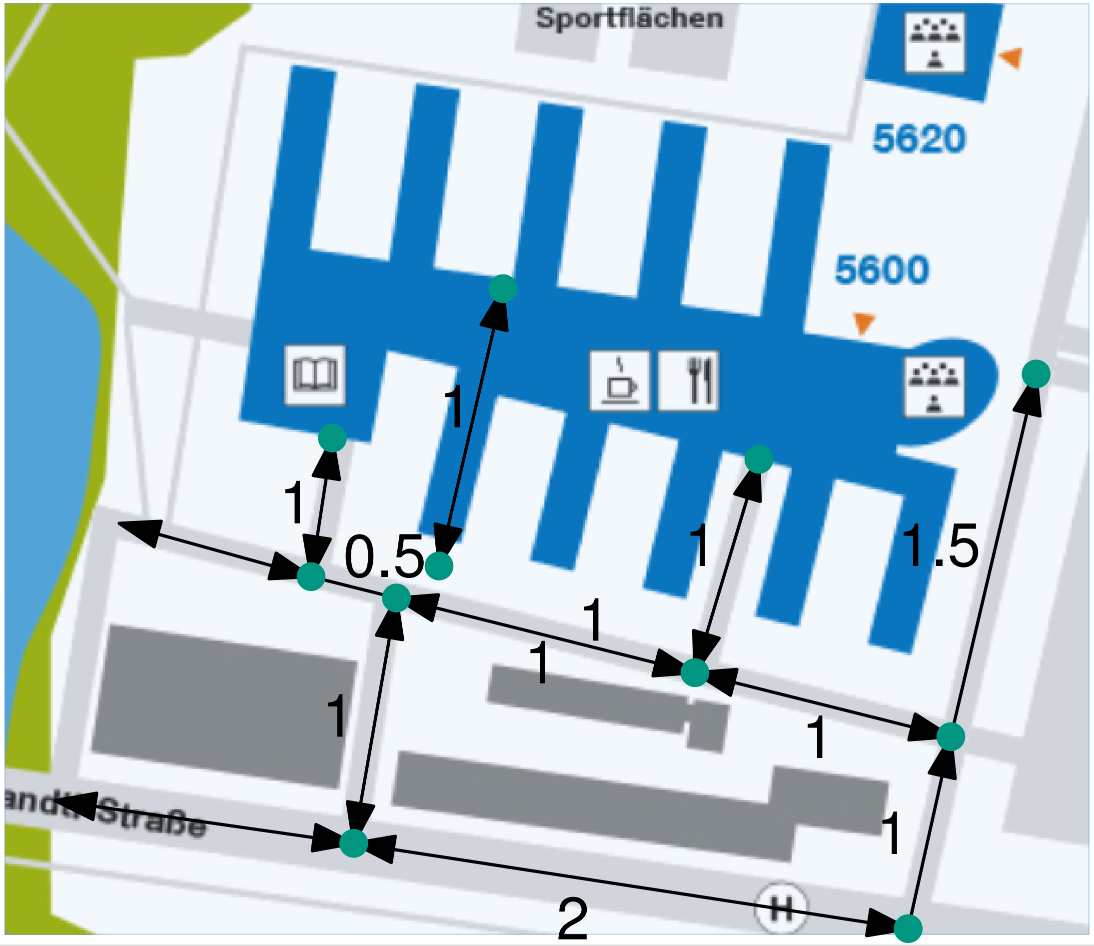

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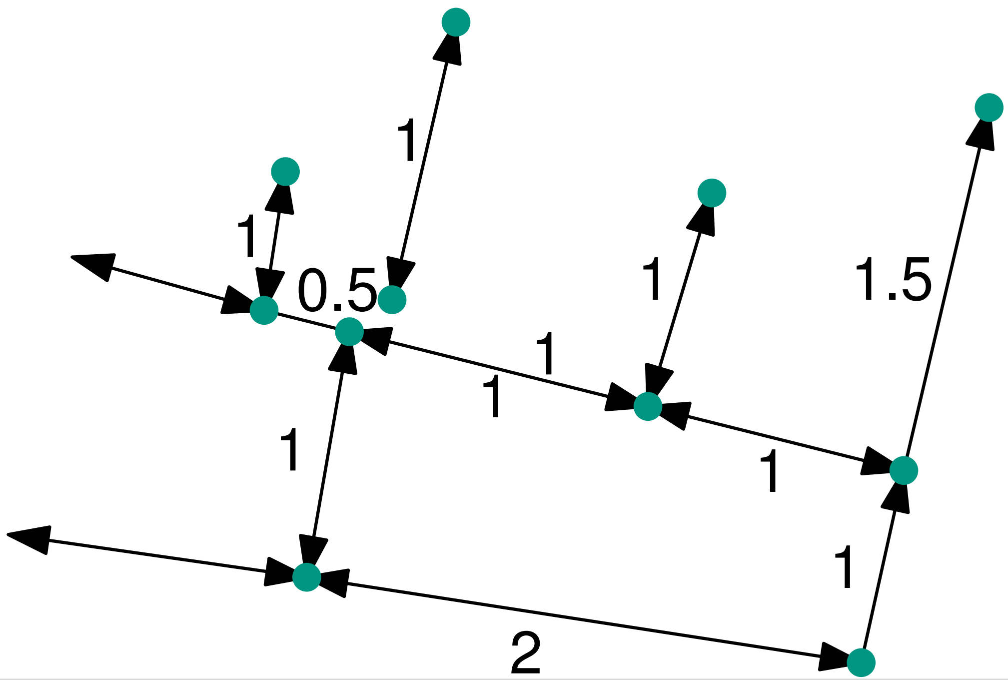
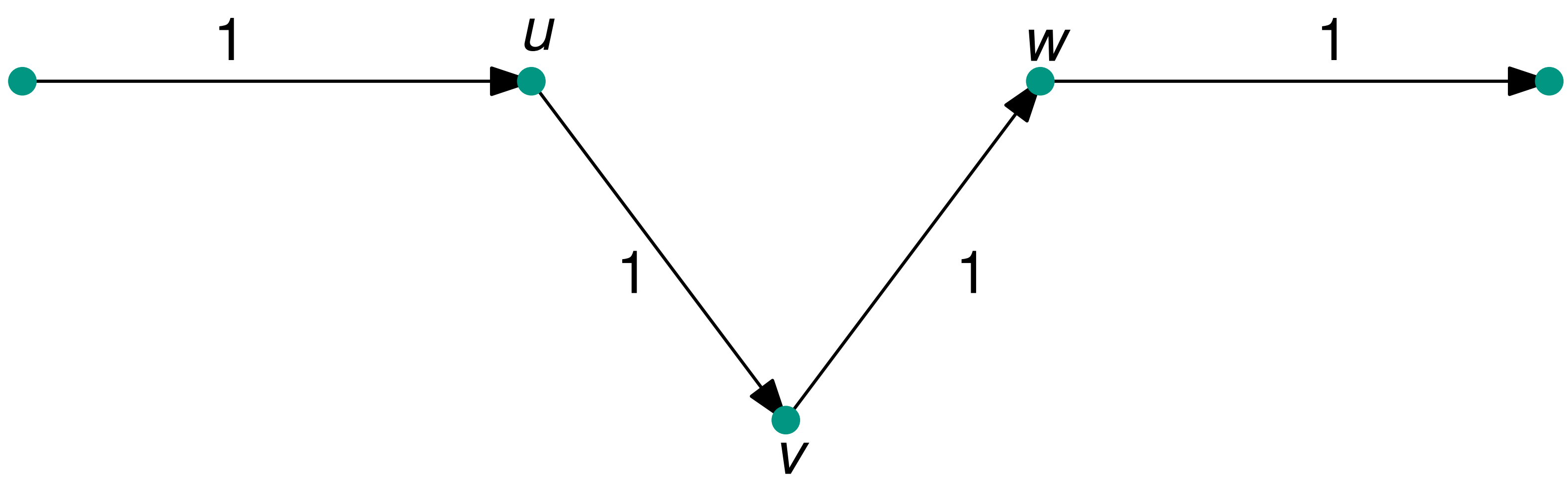


Shortcuts



- Add new edge skipping over one vertex
- Distances unchanged
- Unpack by storing midway vertex

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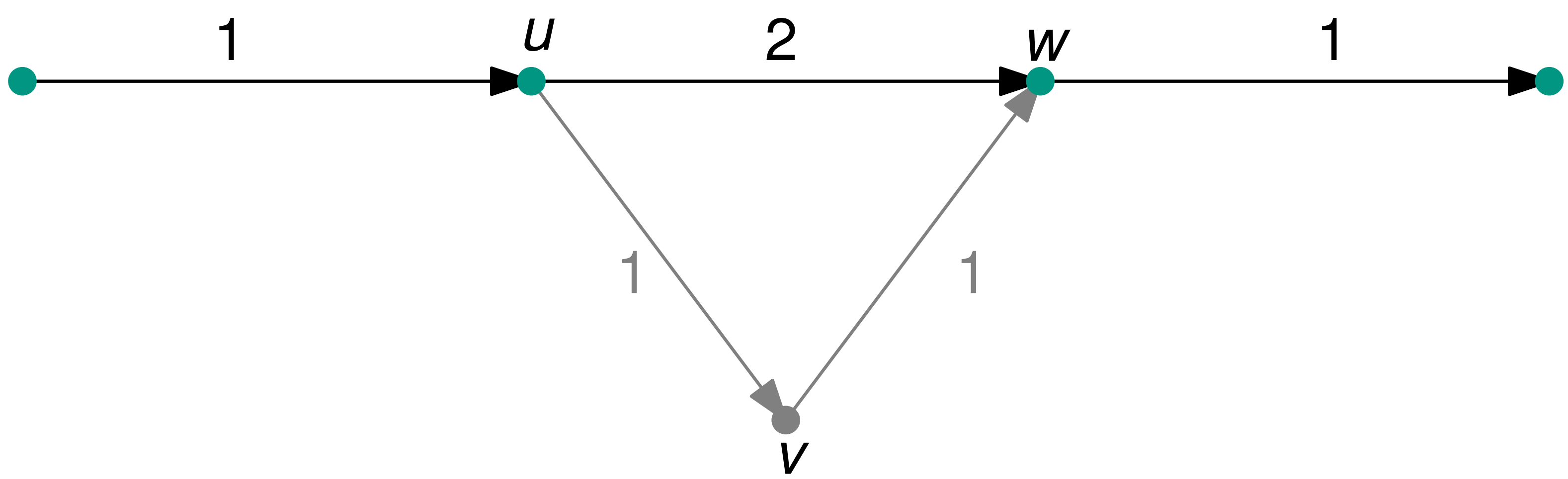

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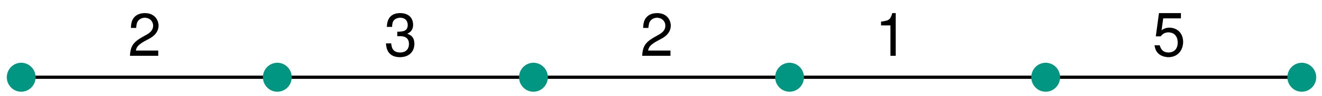

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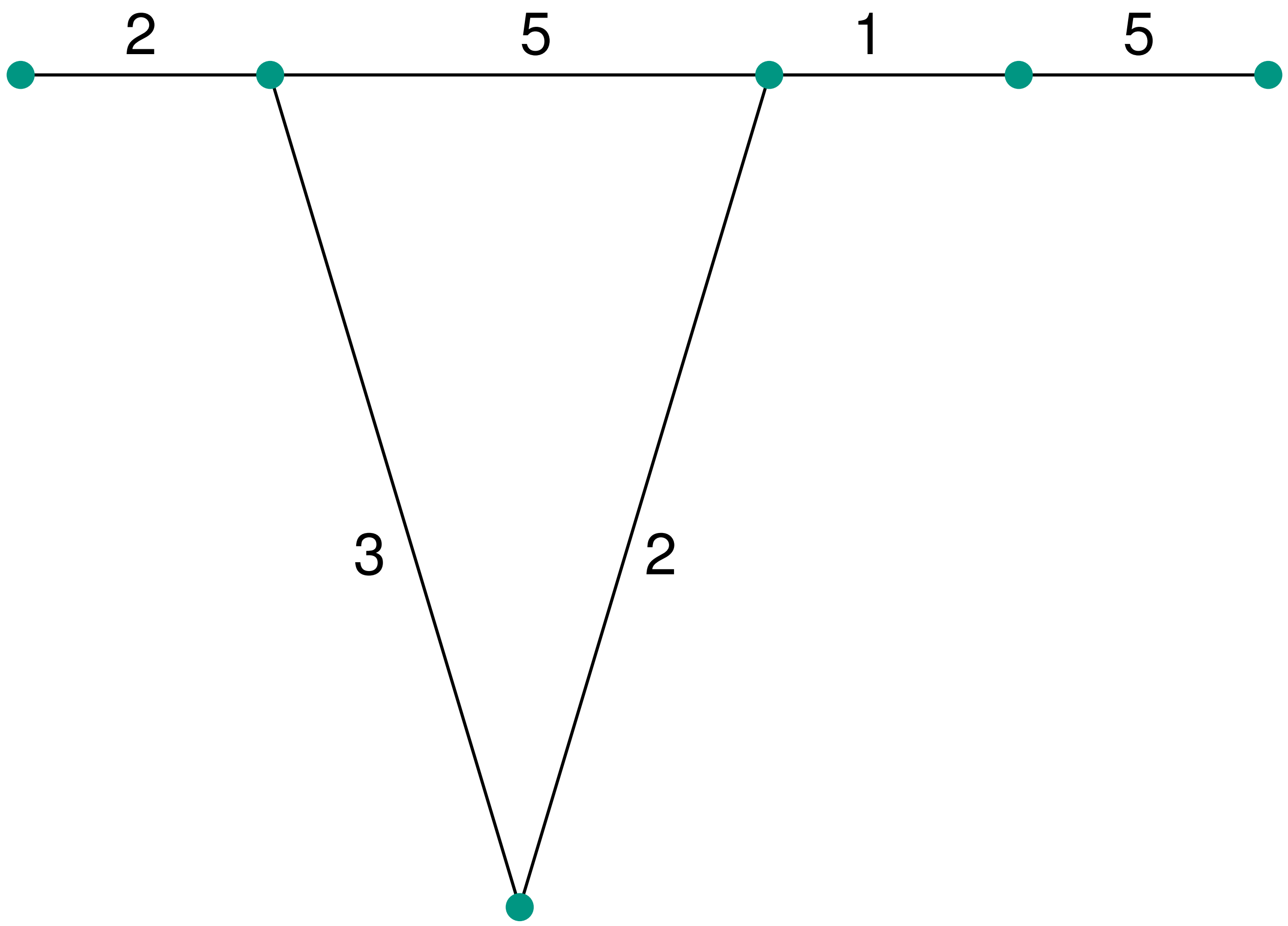

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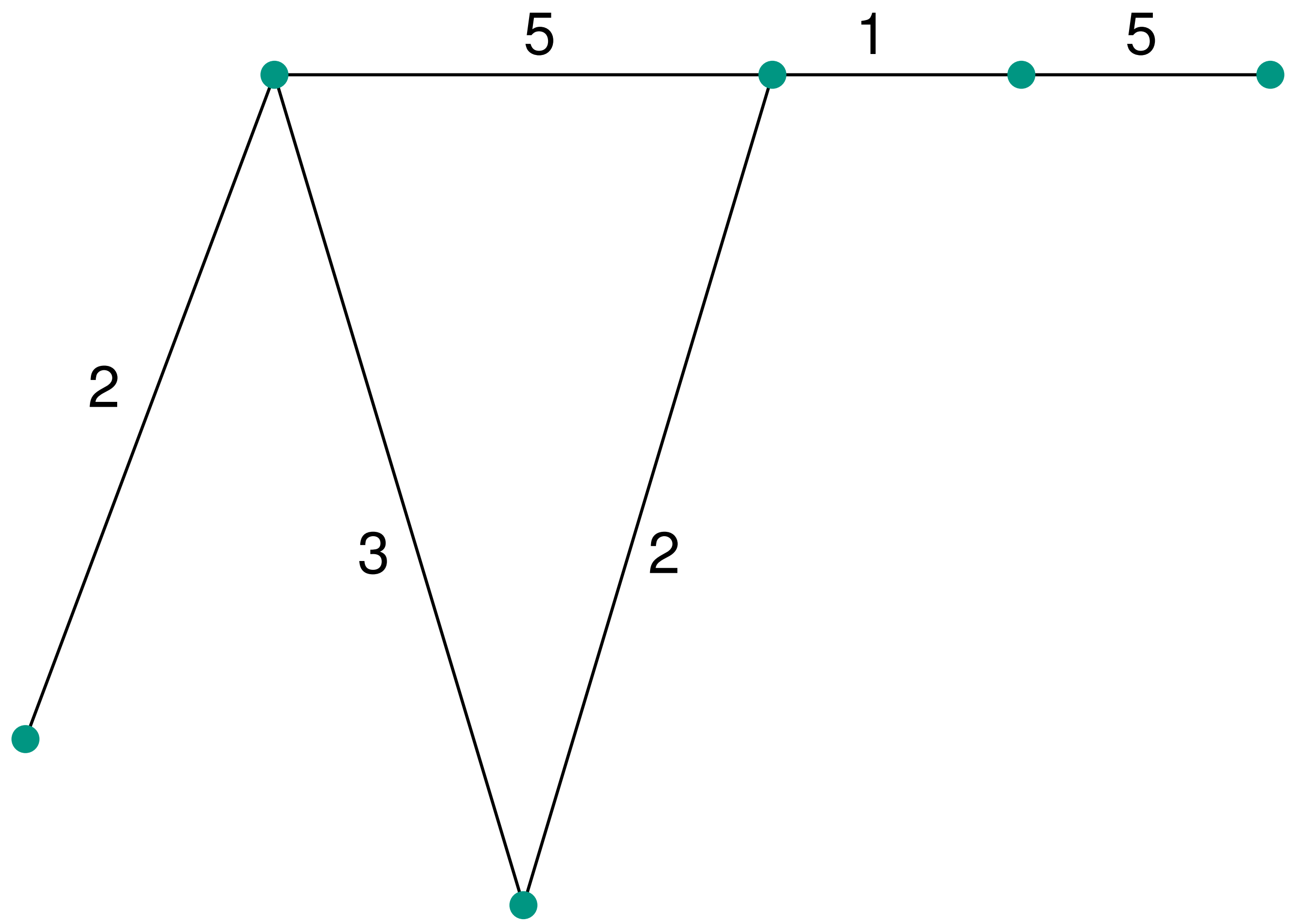

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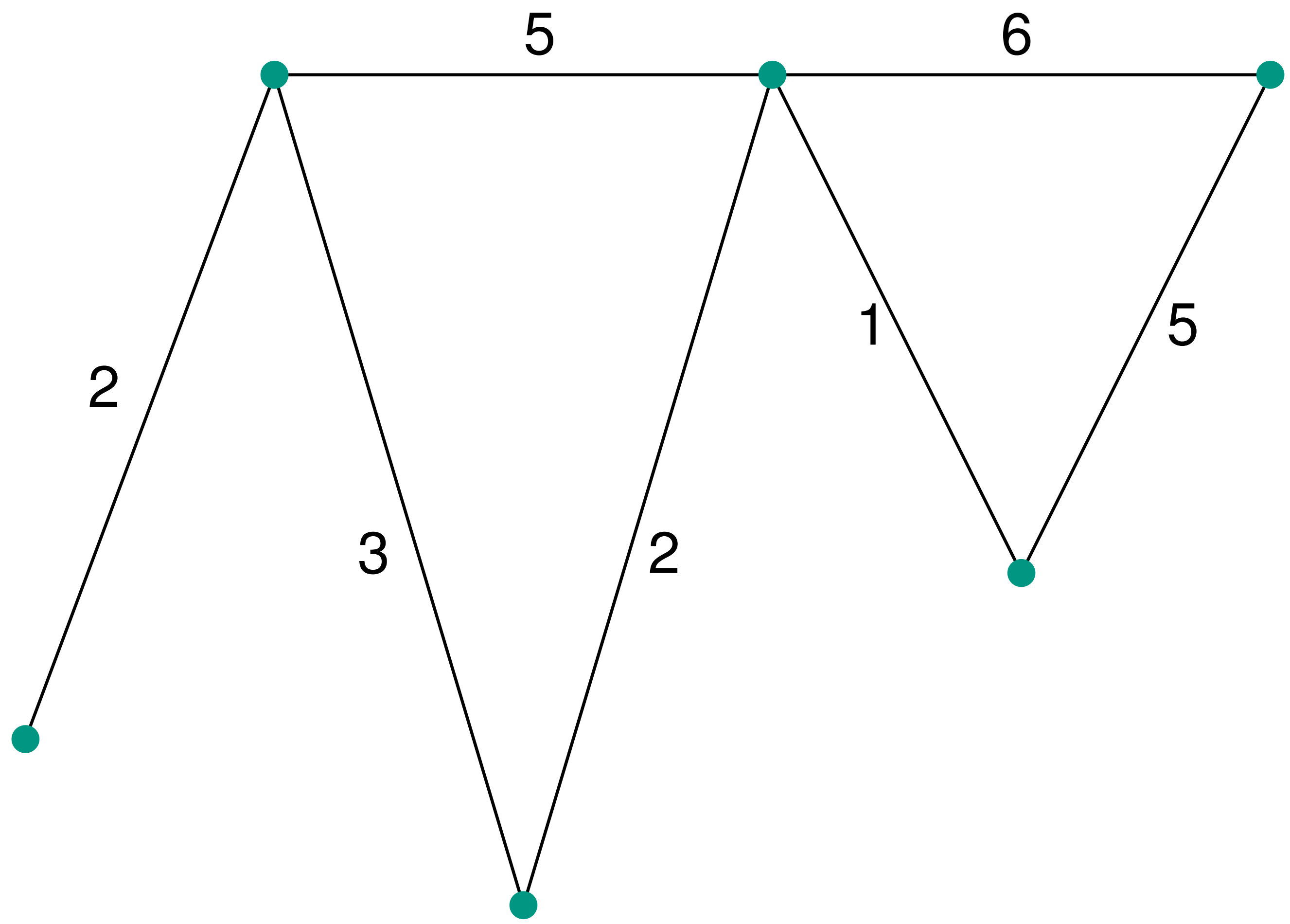

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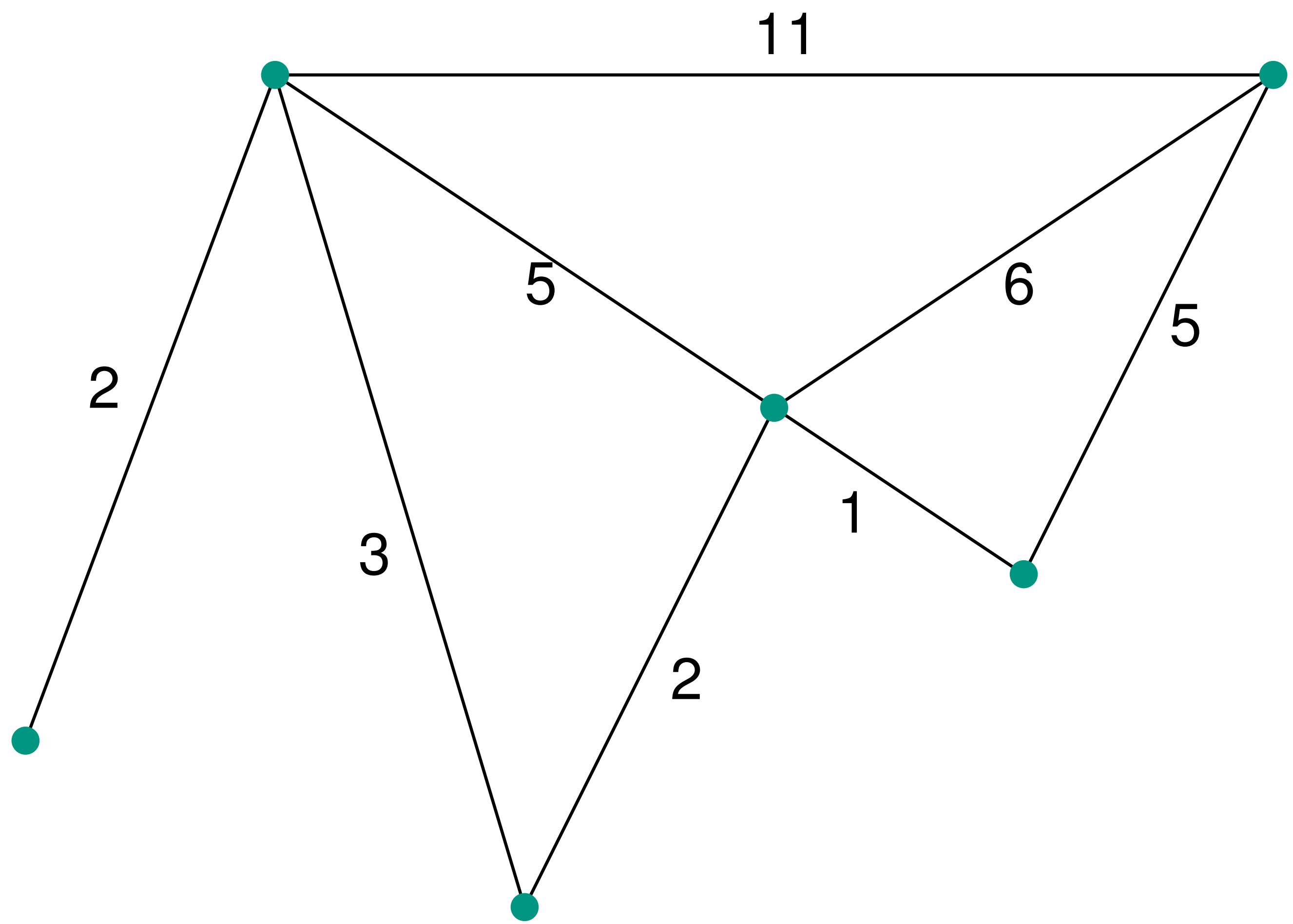

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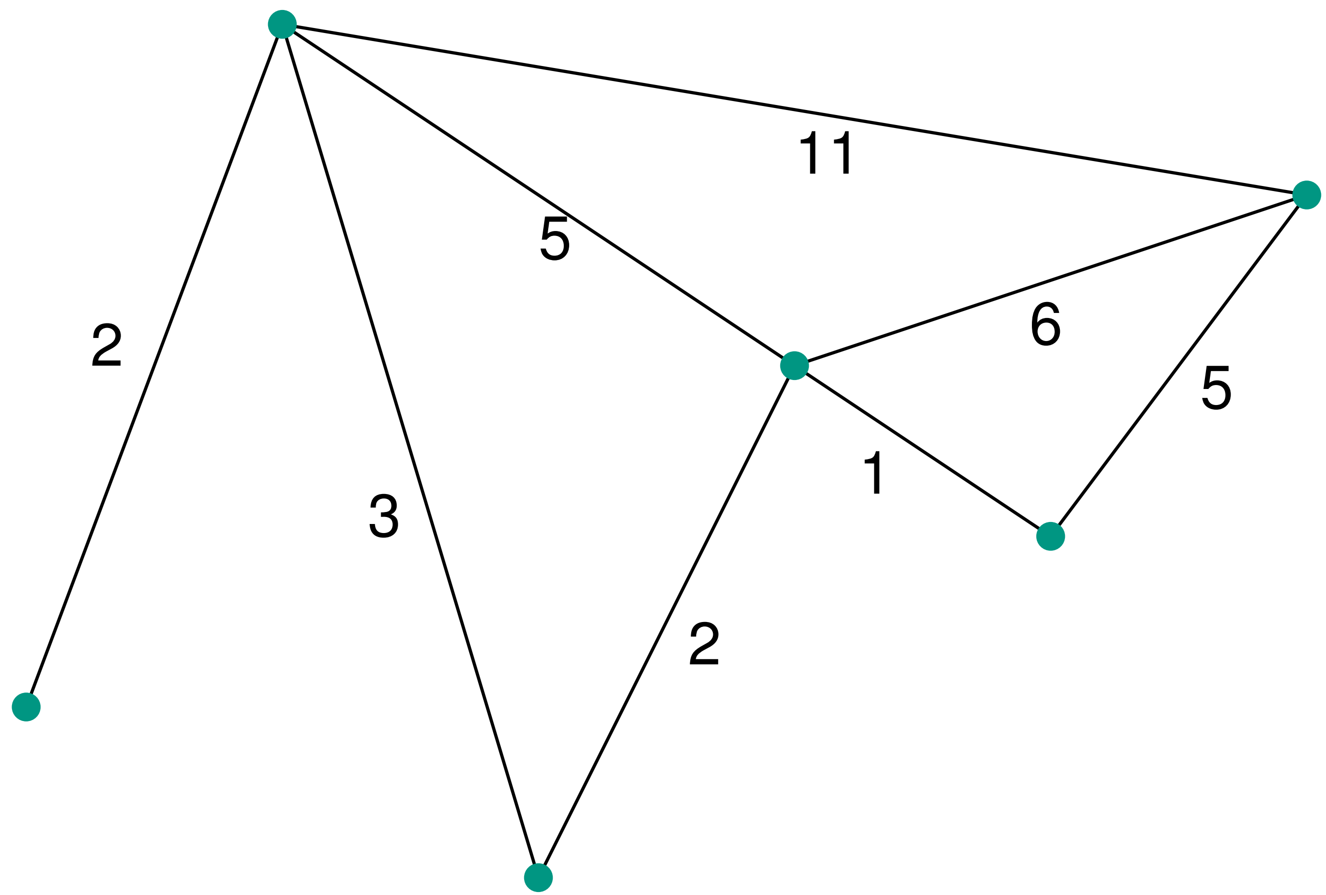

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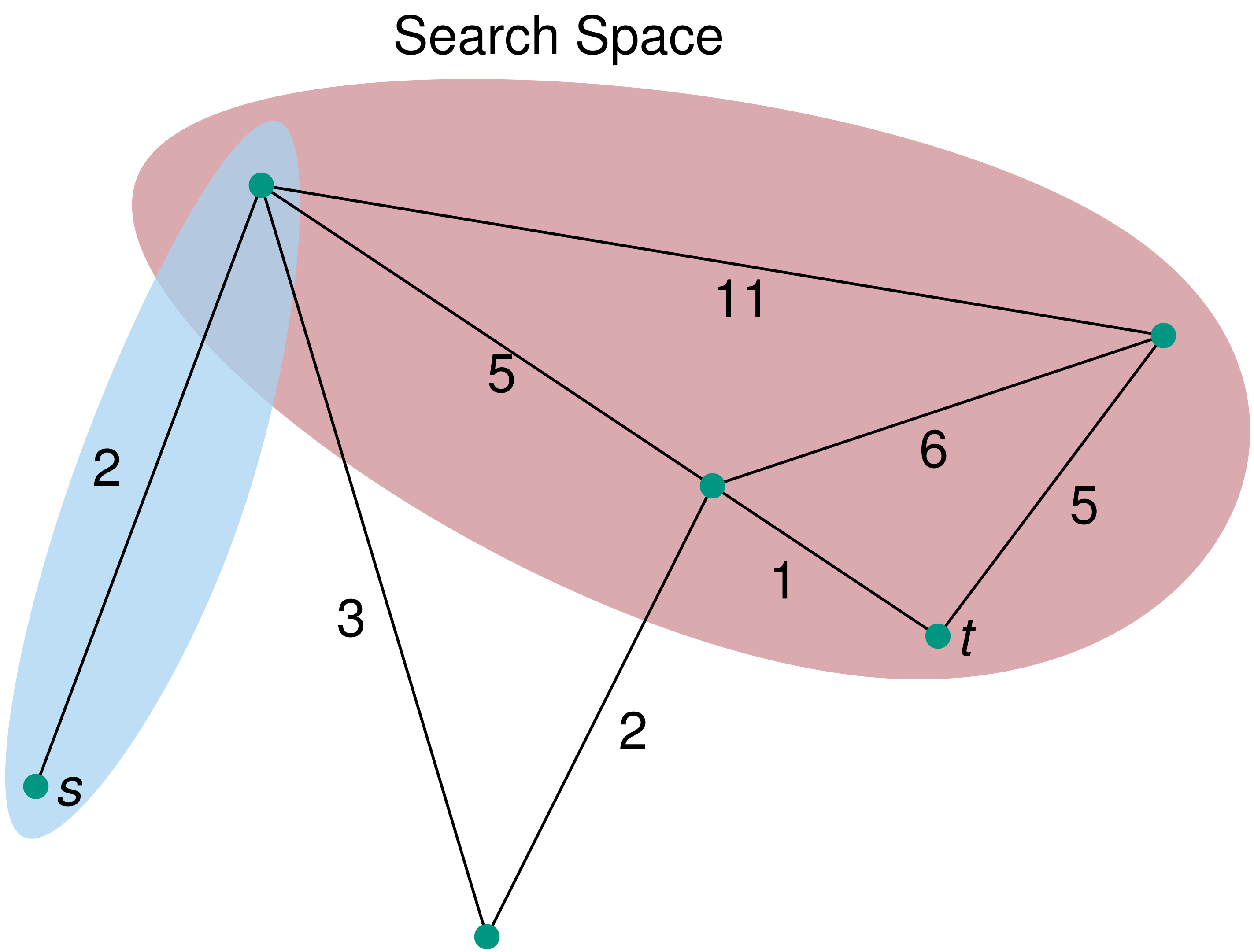

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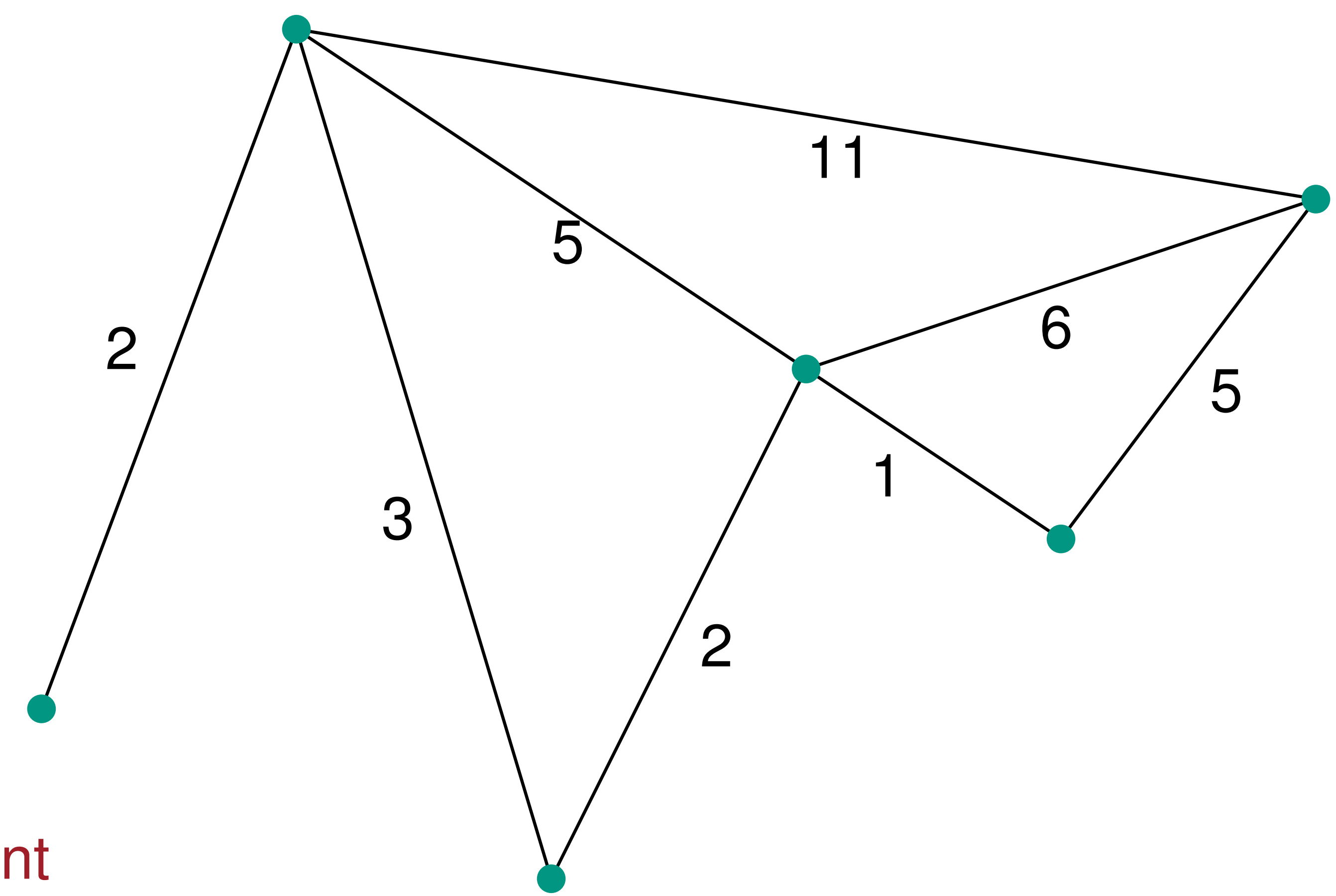

Unimportant

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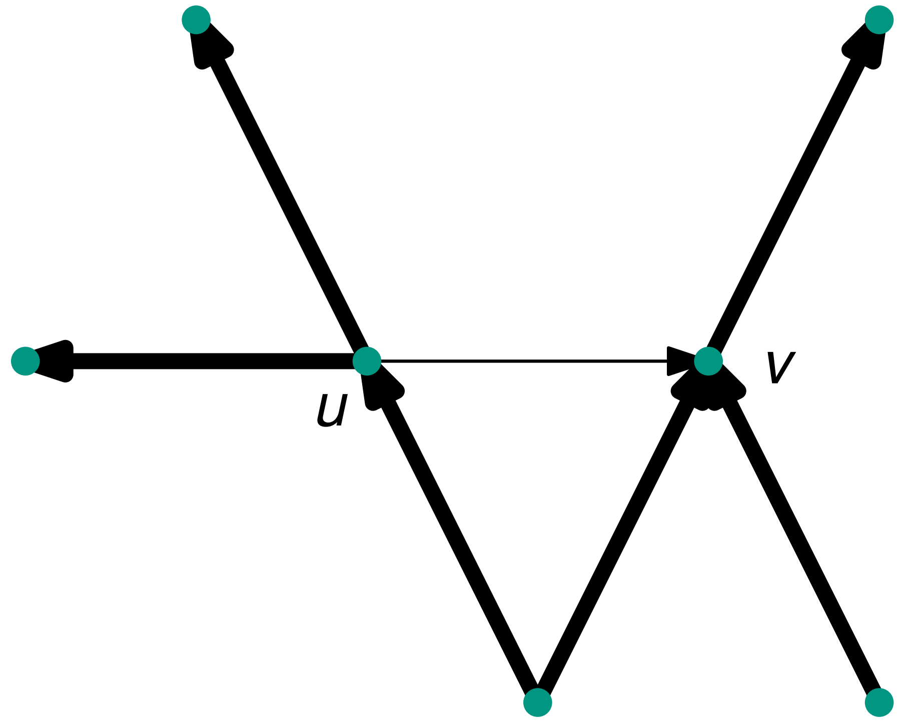

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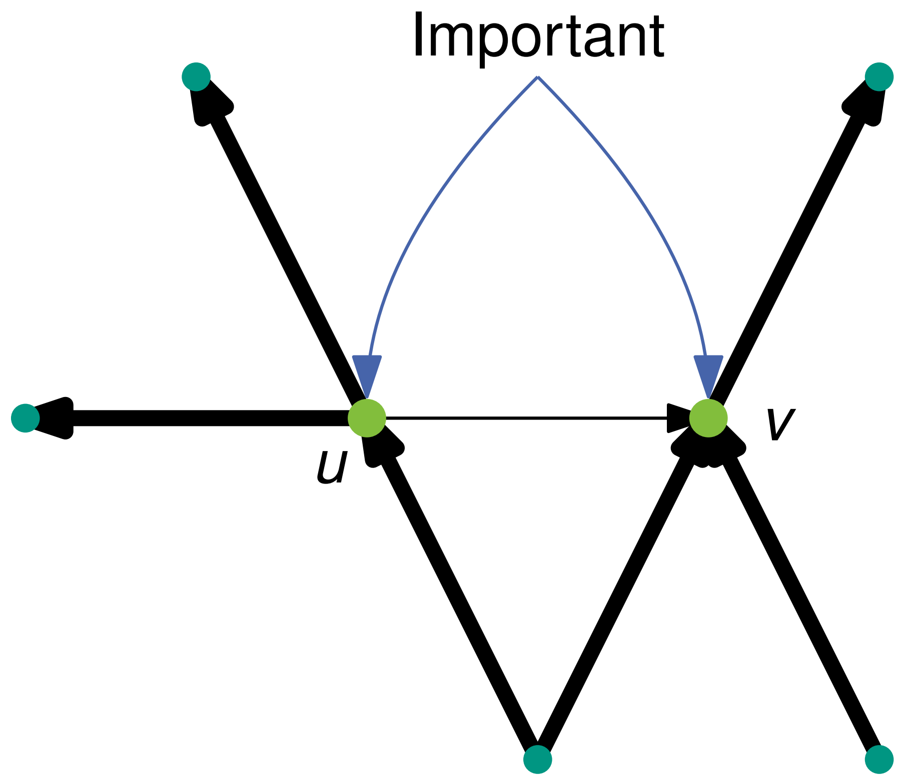

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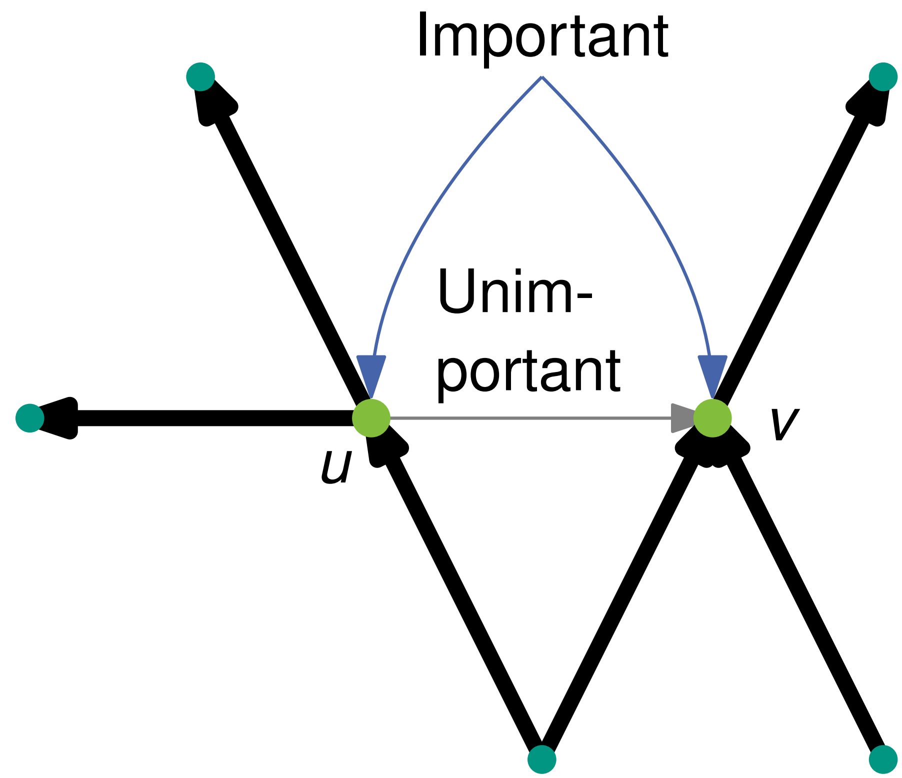

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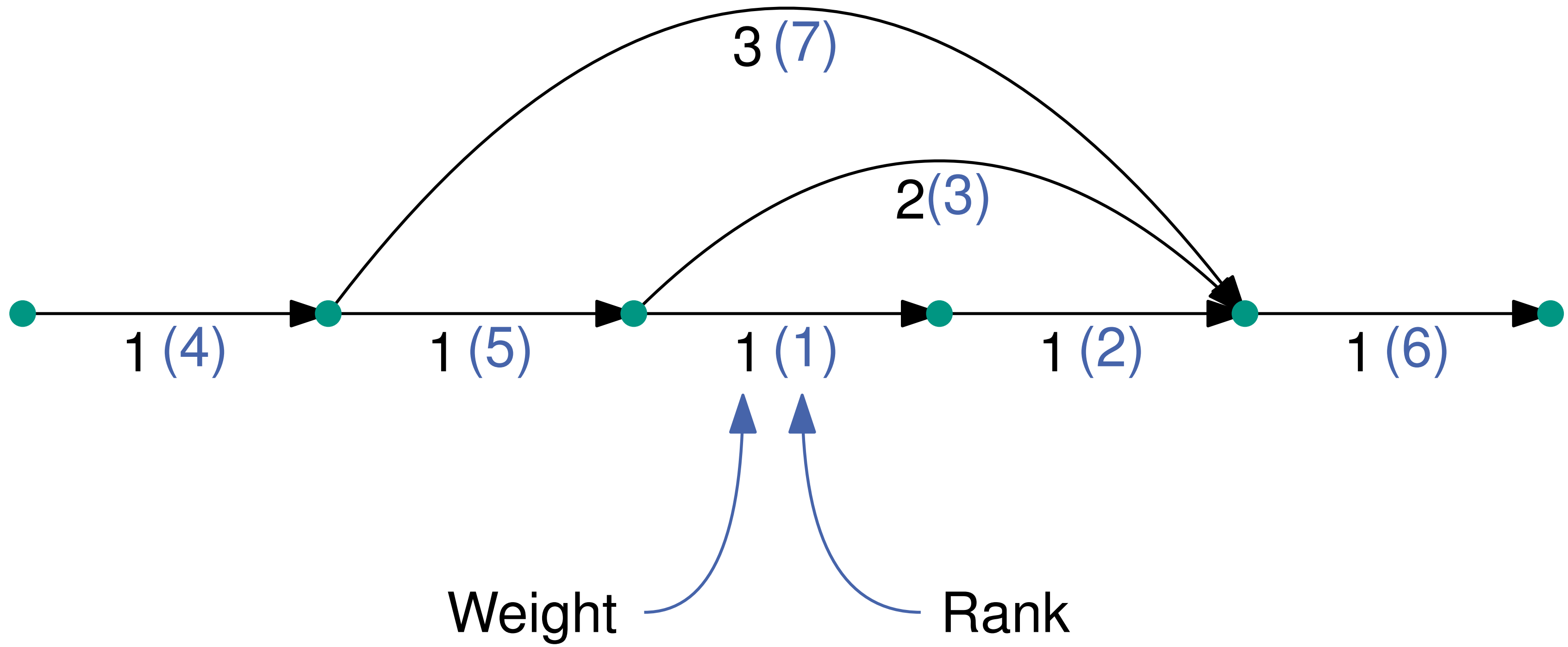

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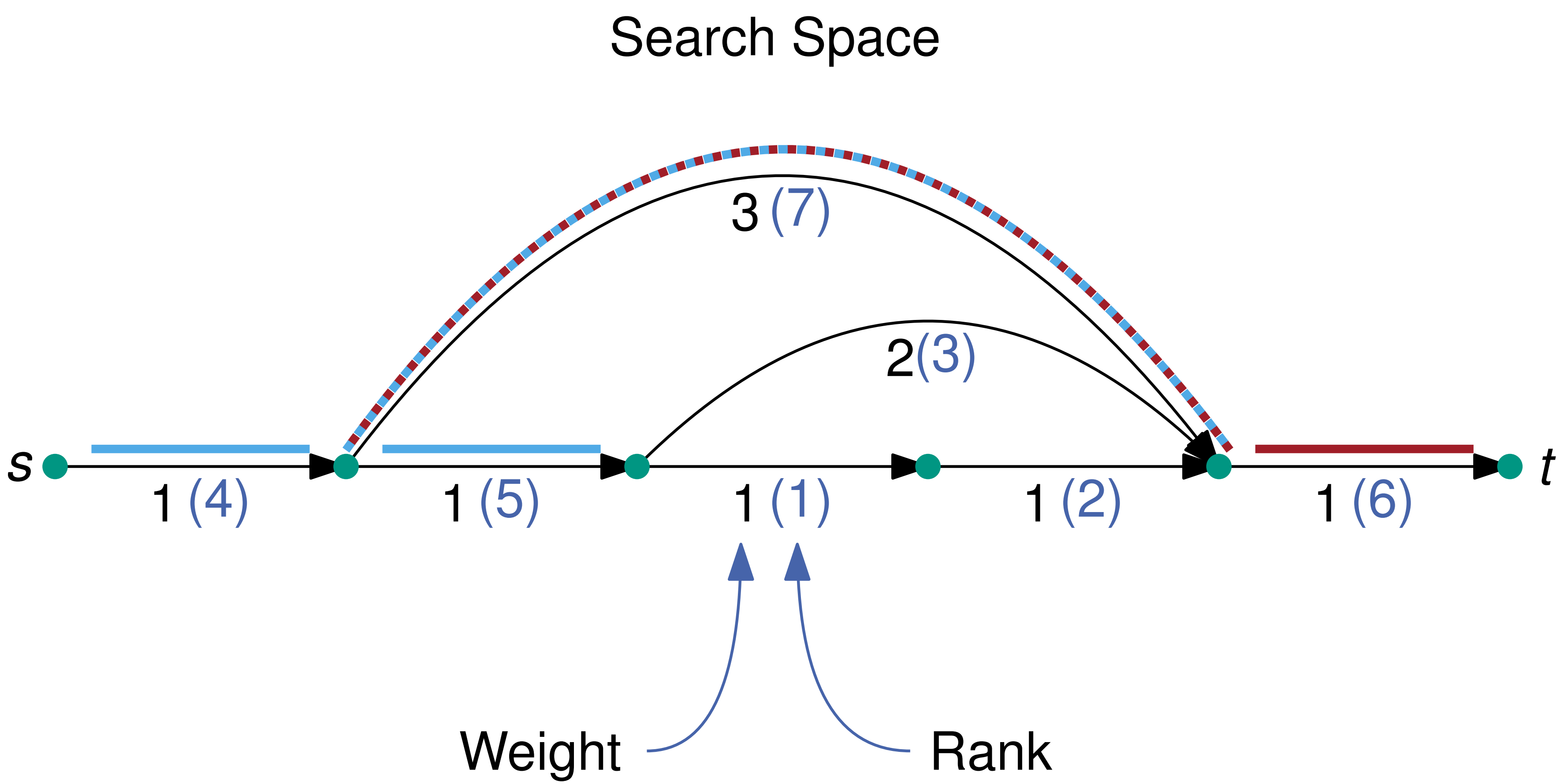

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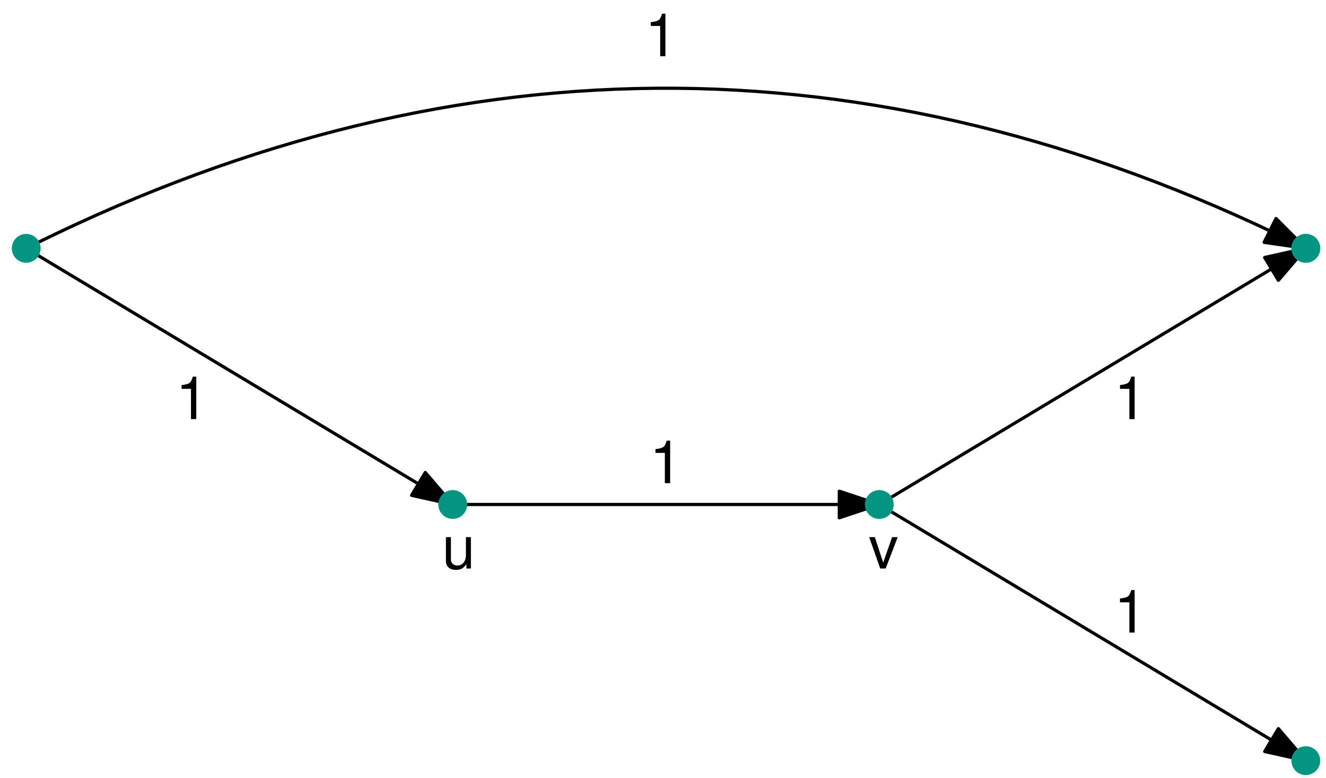

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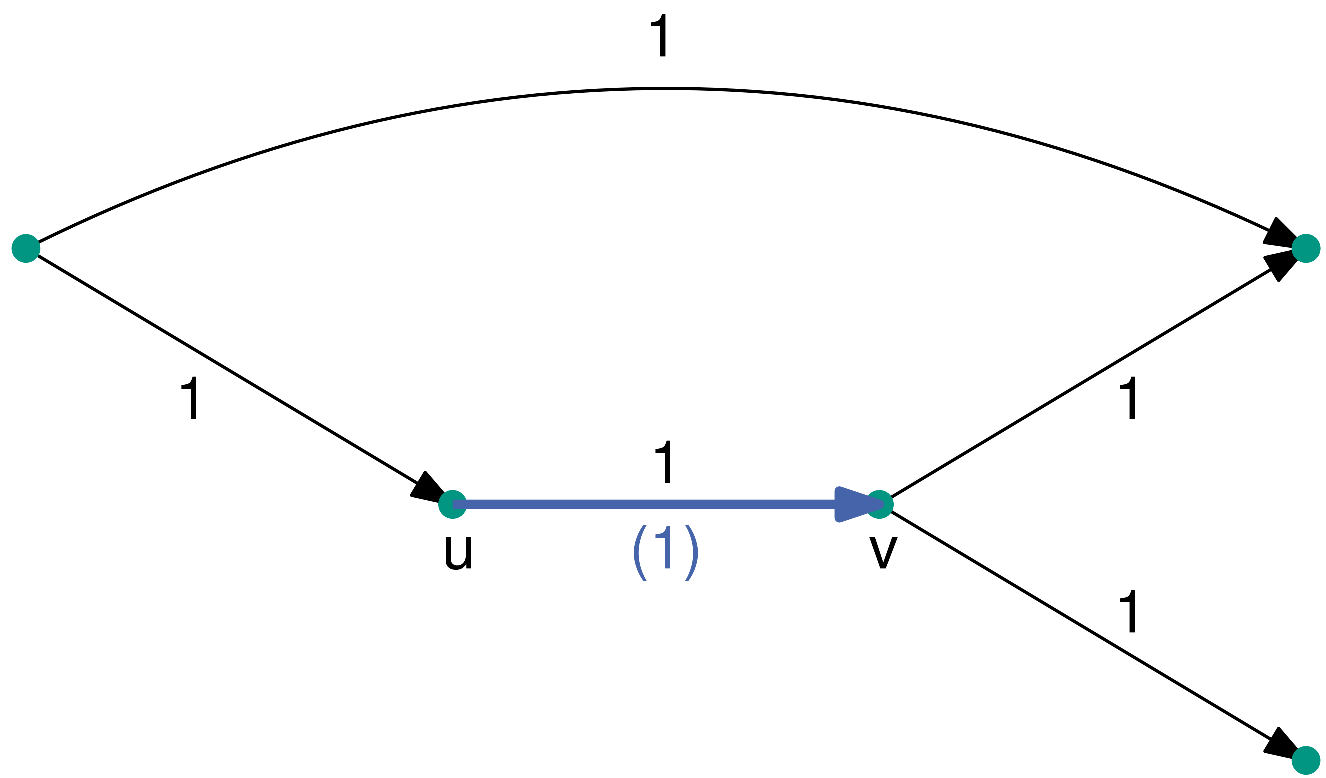

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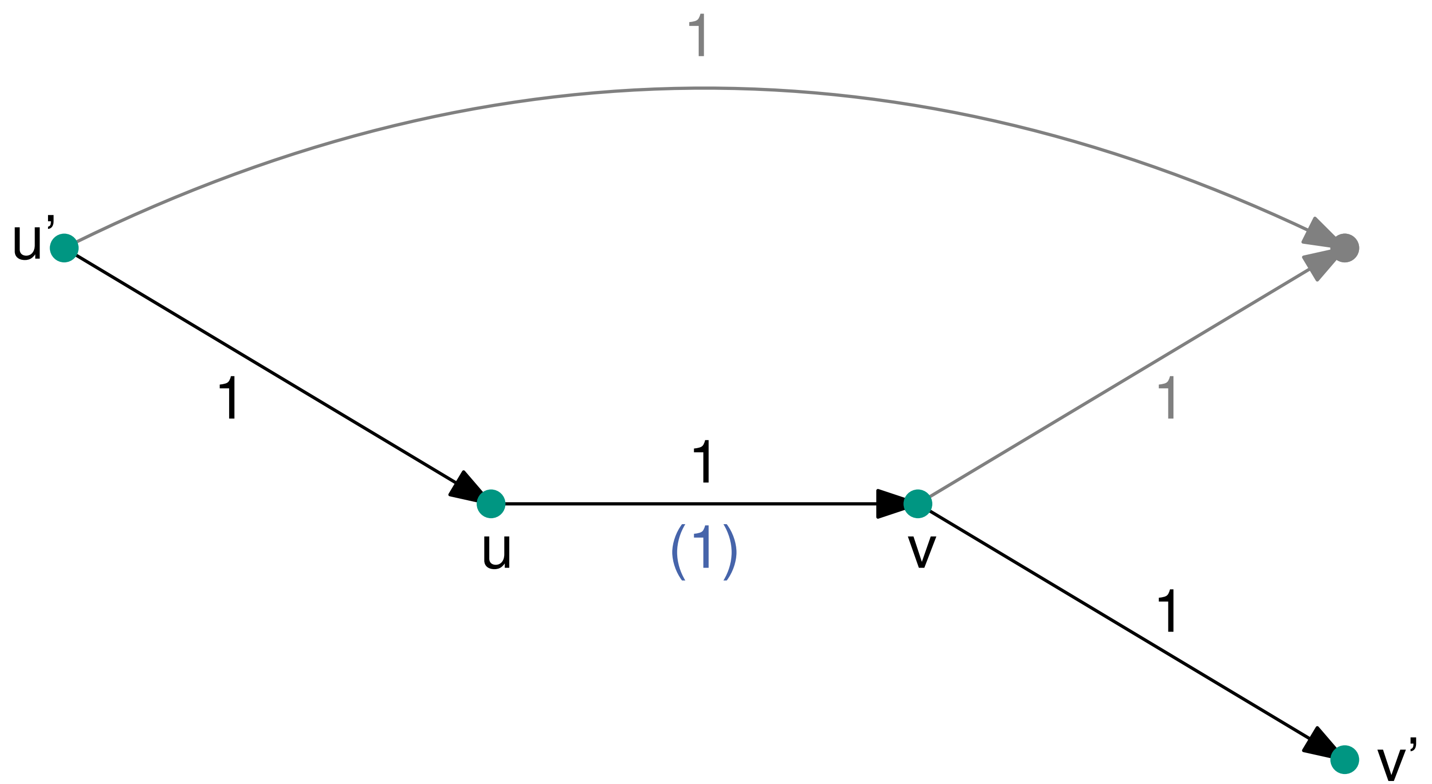

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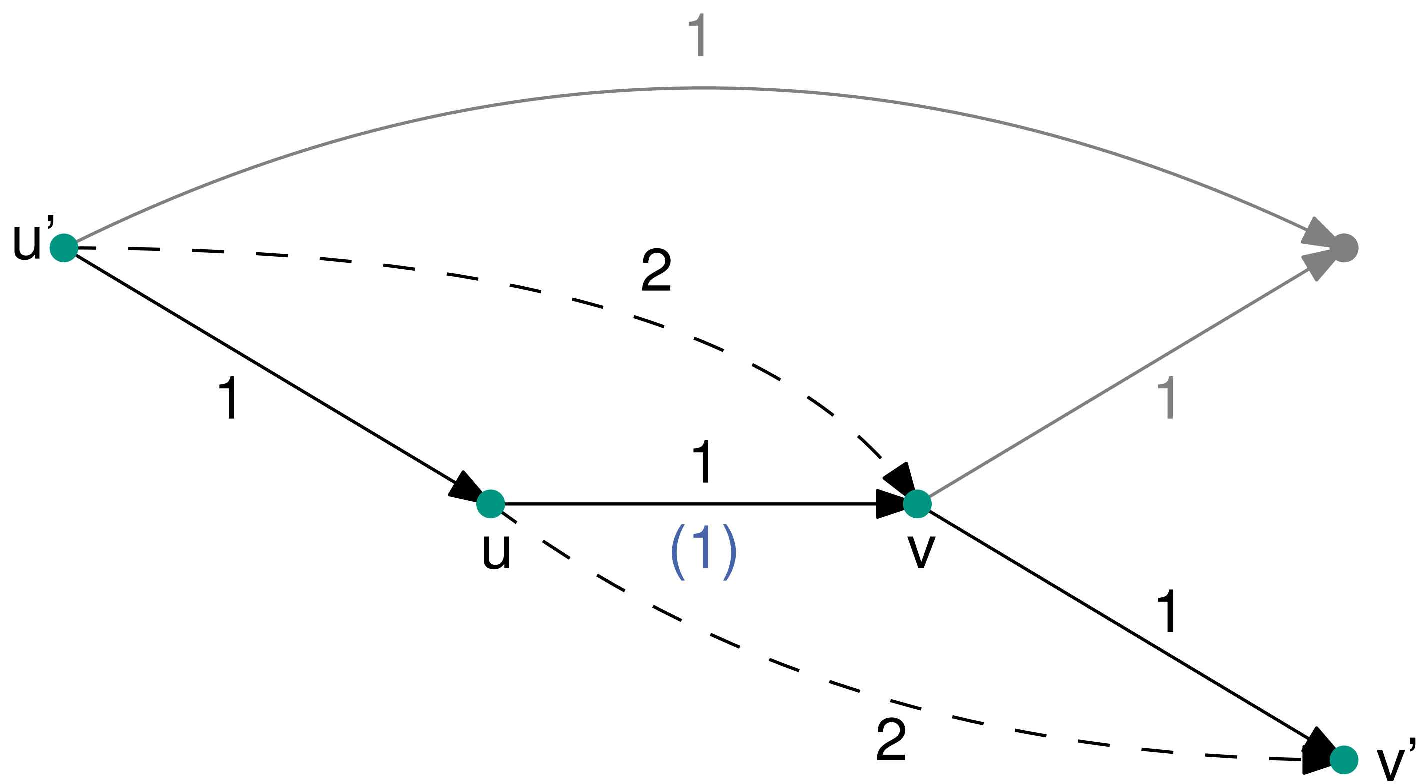

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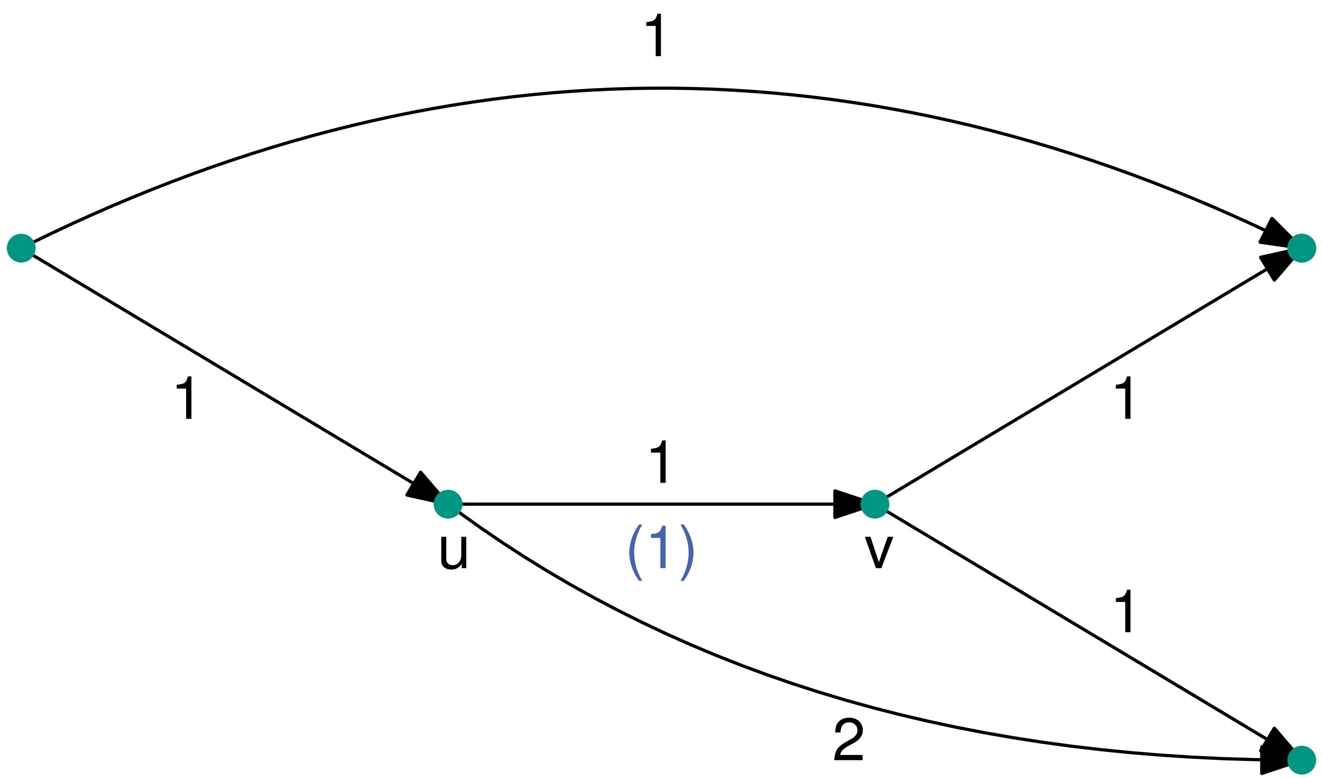

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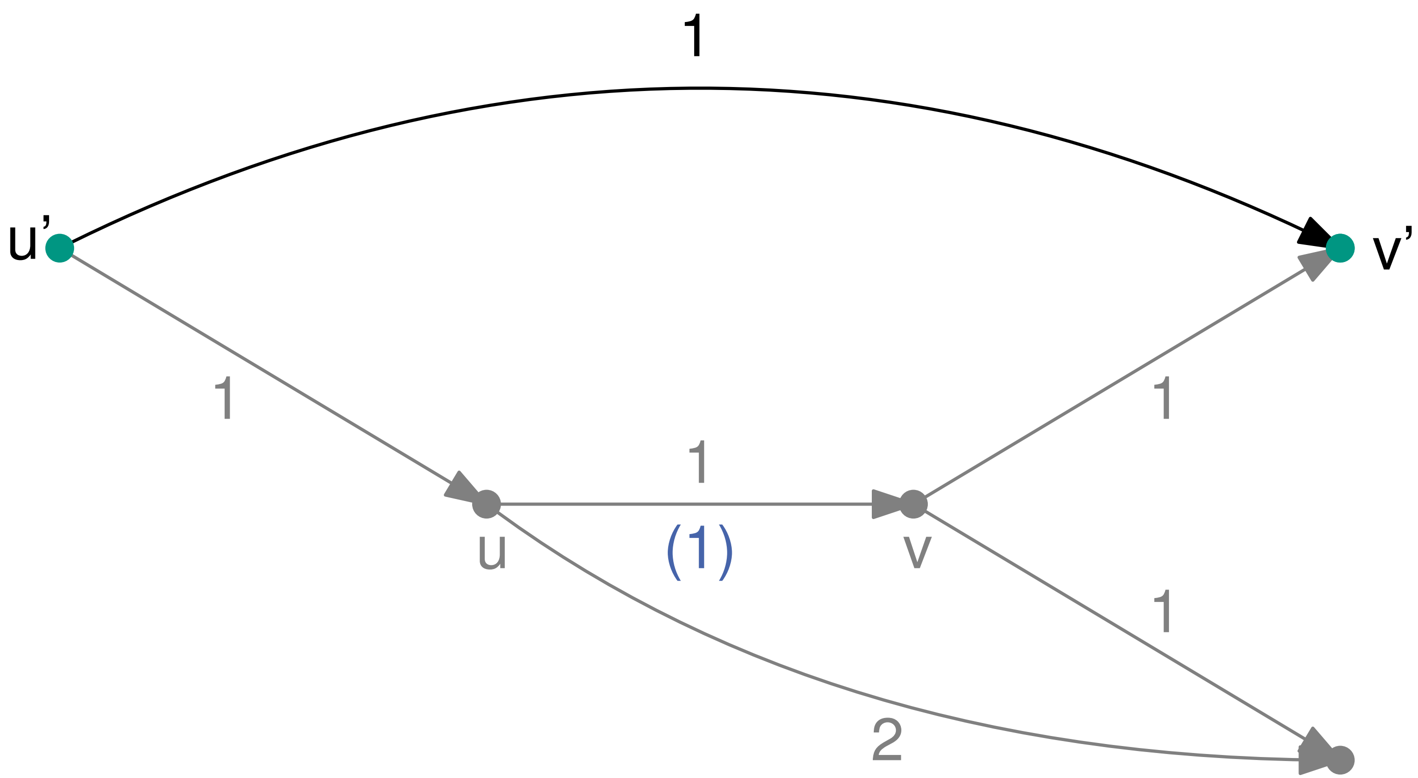

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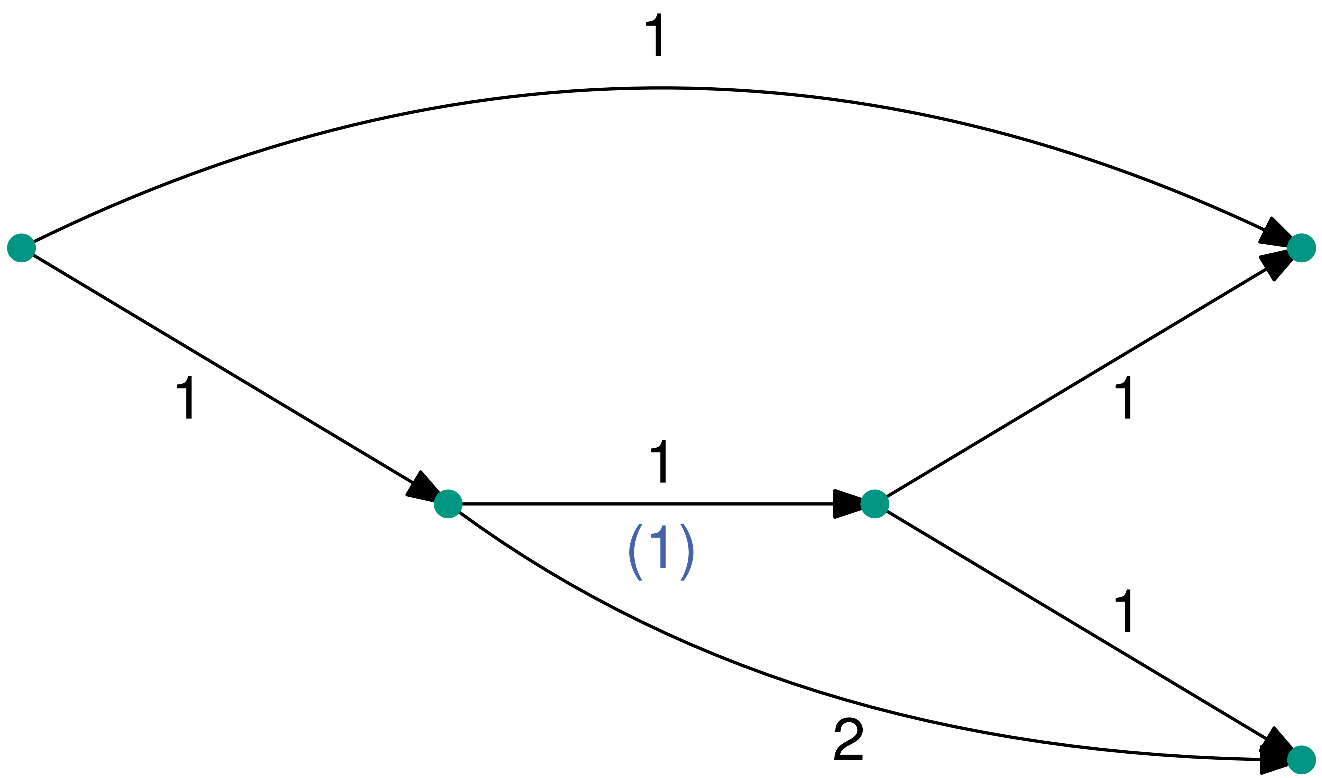

Construction Algorithm

Karlsruhe Institute of Technology

Algorithm 1: BuildEdgeHierarchy

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currentRank ← 0;
while Unranked edges remain do
    Pick unranked edge (u, v);
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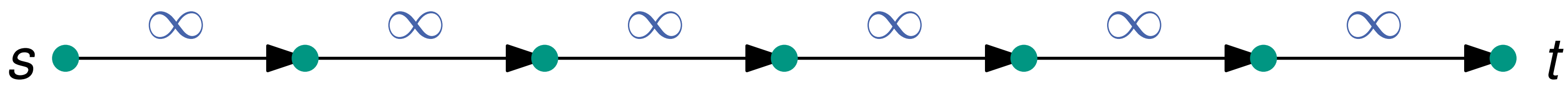
Proof Sketch



- Consider shortest s–t path and edge e being ranked
  - If e is in the middle: shortcut inserted
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    $\Rightarrow$ Edges in the middle get higher ranks

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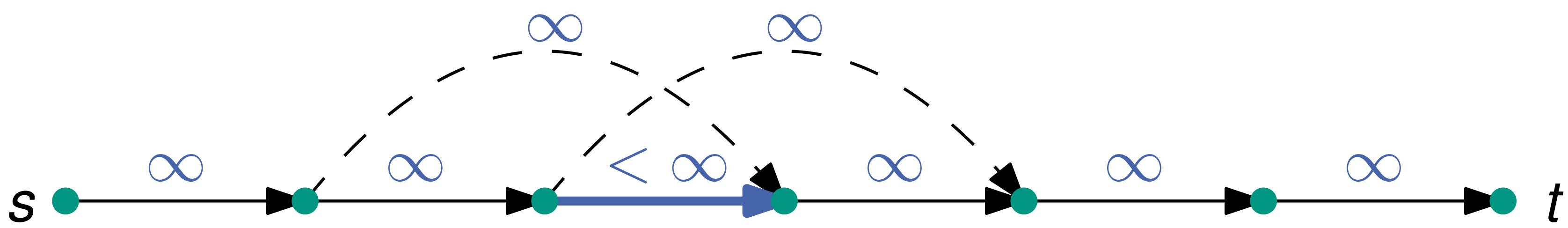

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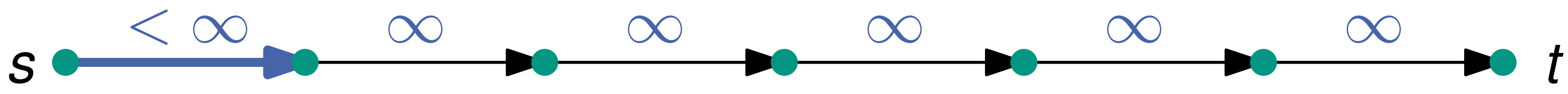

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Check whole graph

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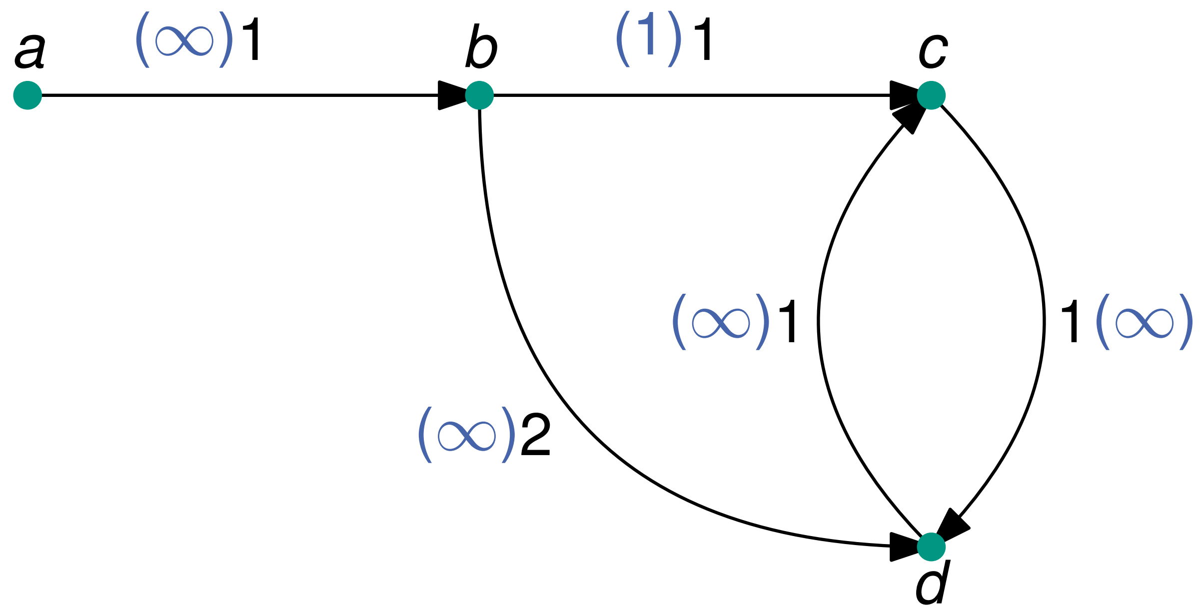

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  - Can use EH query on partially constructed EH

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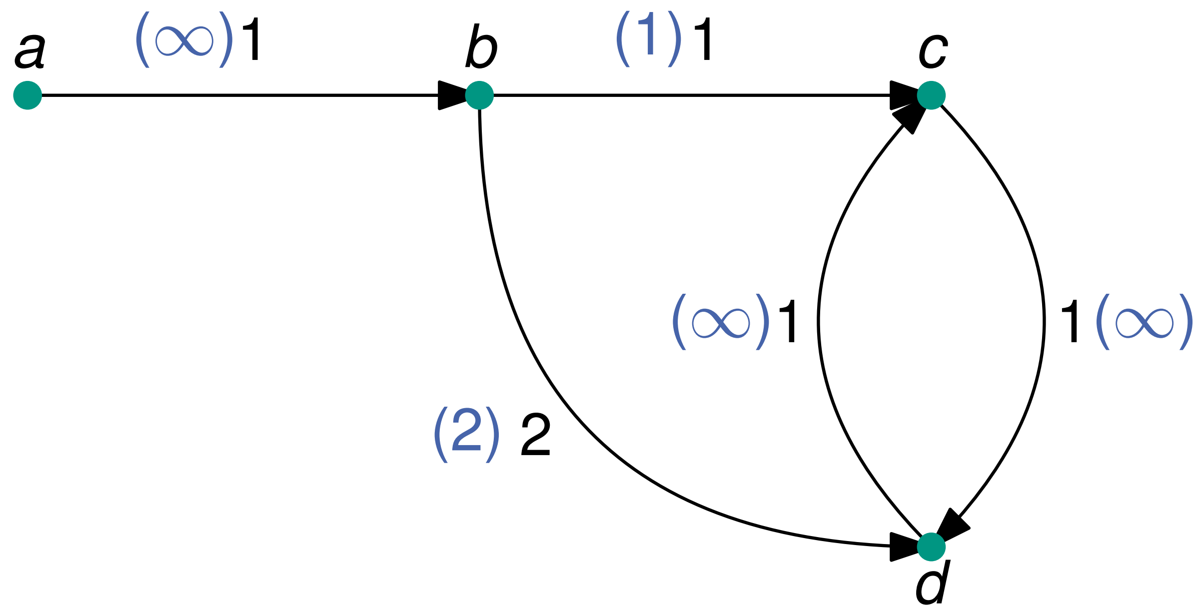

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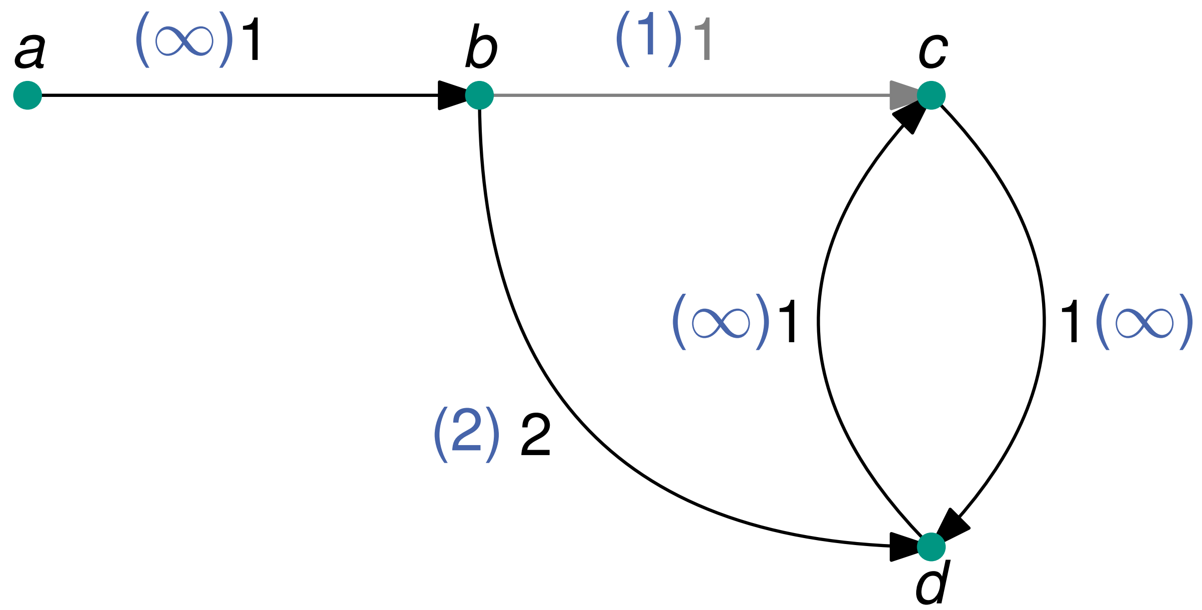

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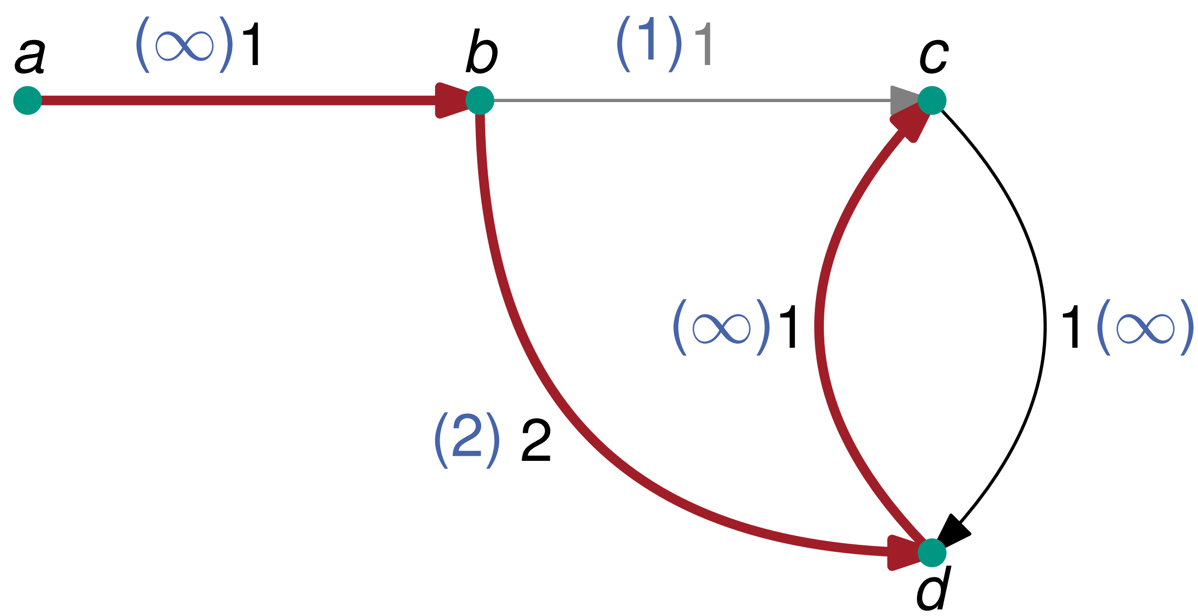

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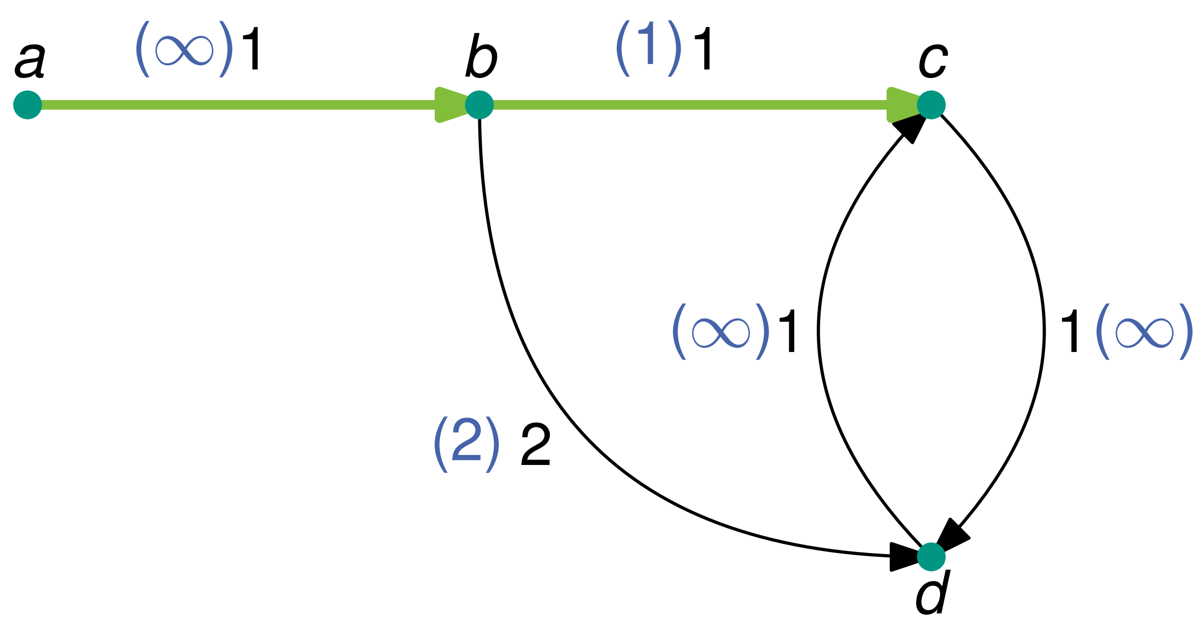

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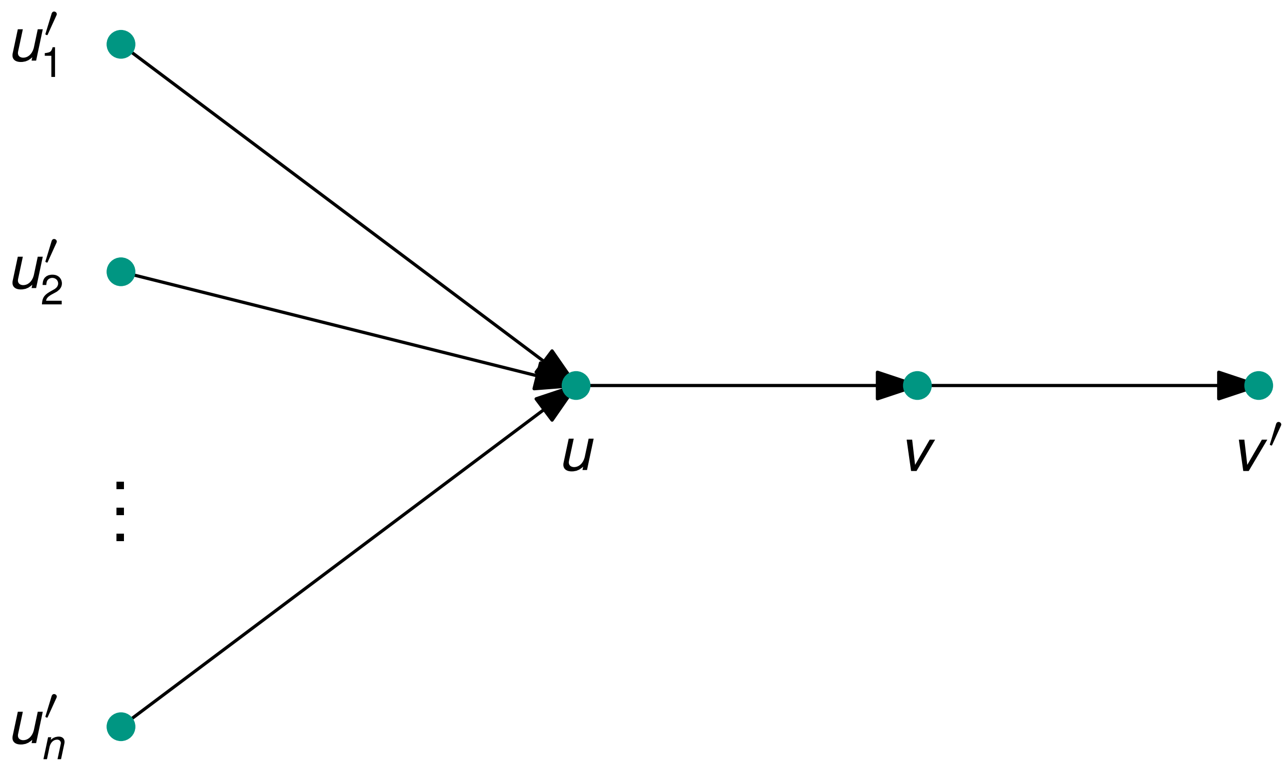

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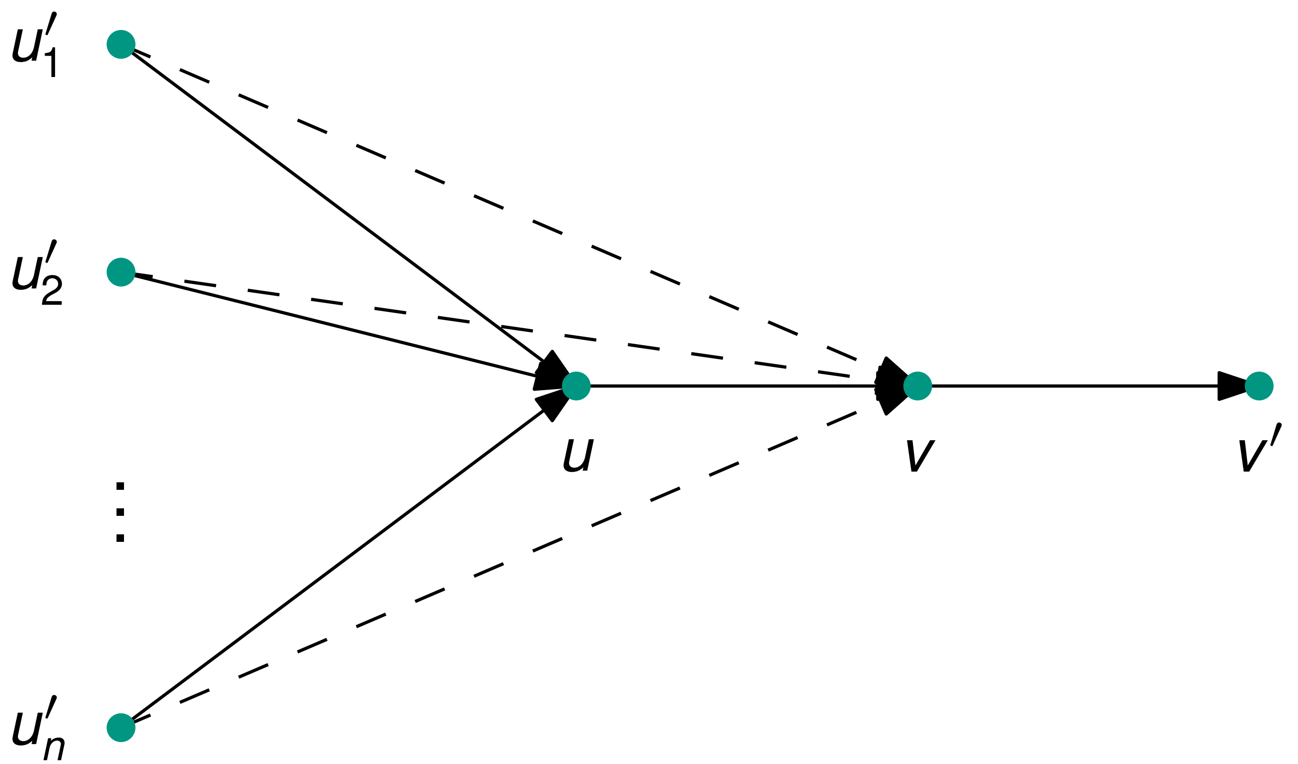

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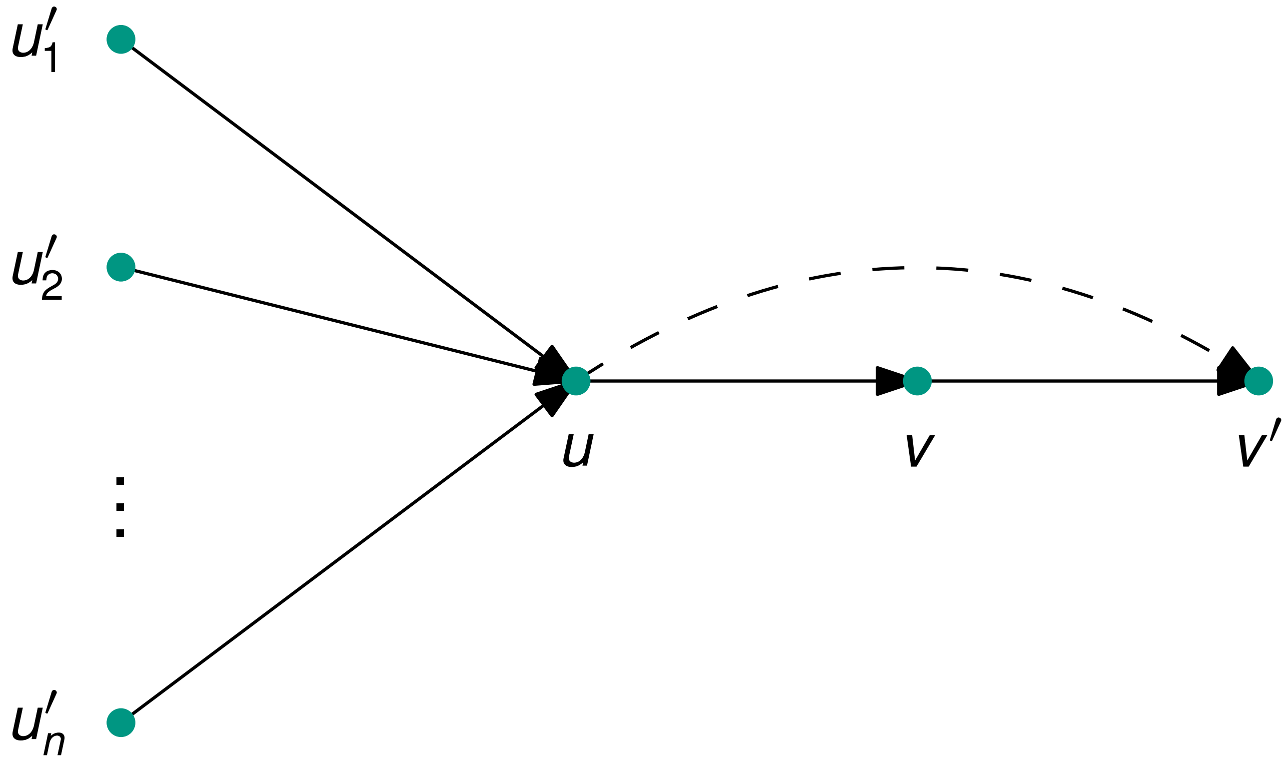

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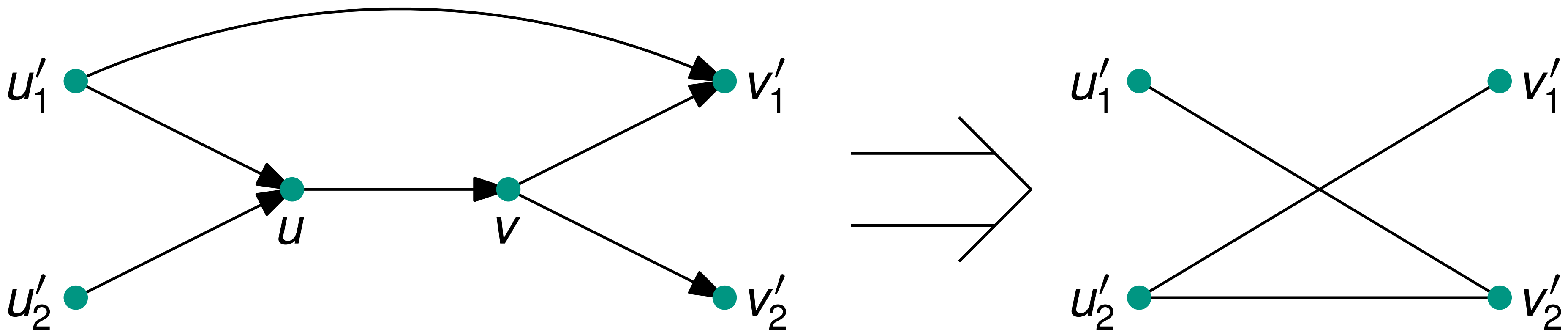

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- Add shortcuts to vertices in MVC
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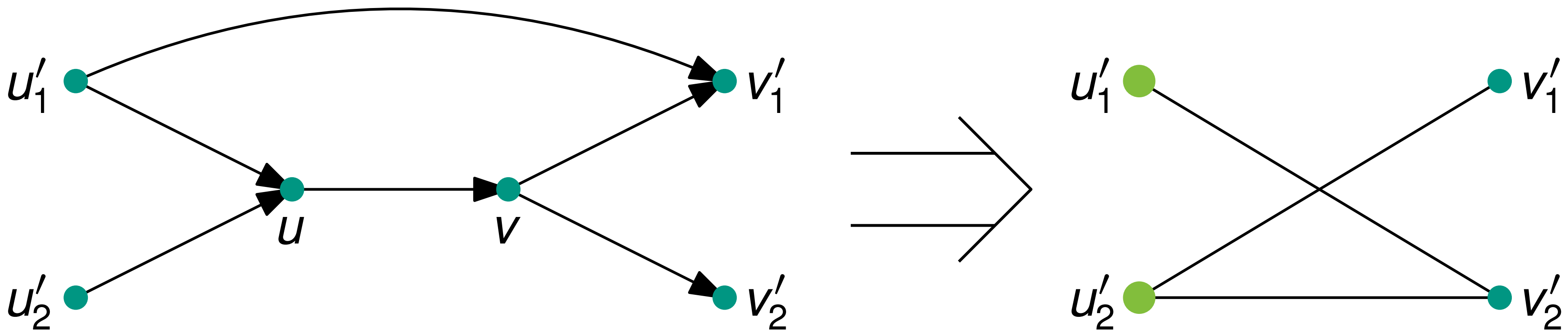

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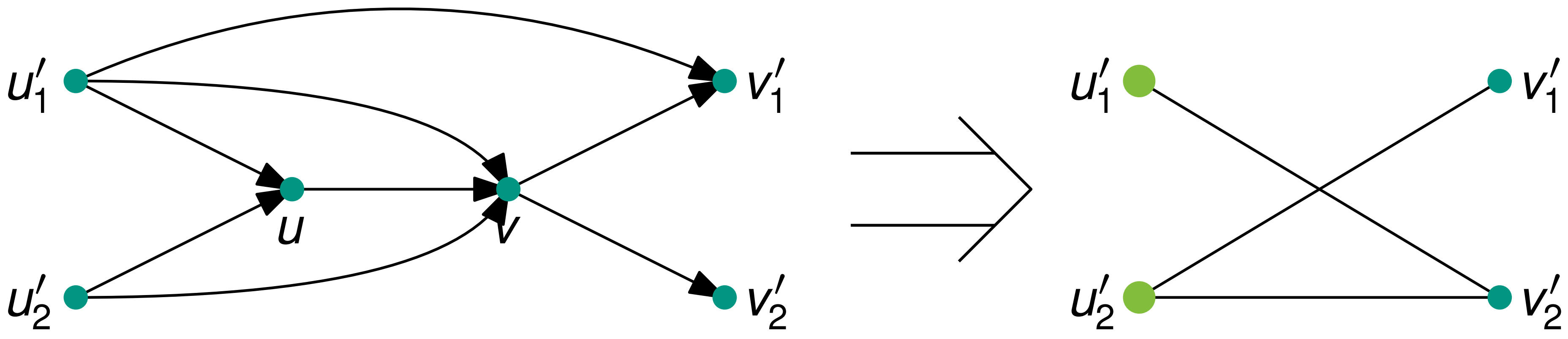

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Preprocessing

Travel Time

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Orig.	USA	7145	674	104.5	104.0
Orig.	EUROPE	3171	453	70.3	70.3
Turns	USA	45904	15462	270.3	404.3
Turns	EUROPE	17822	4743	194.0	249.1

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* EH Preprocessing uses CH queries

Intel Xeon E5-4640, 2.4 GHz

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Preprocessing

Travel Time

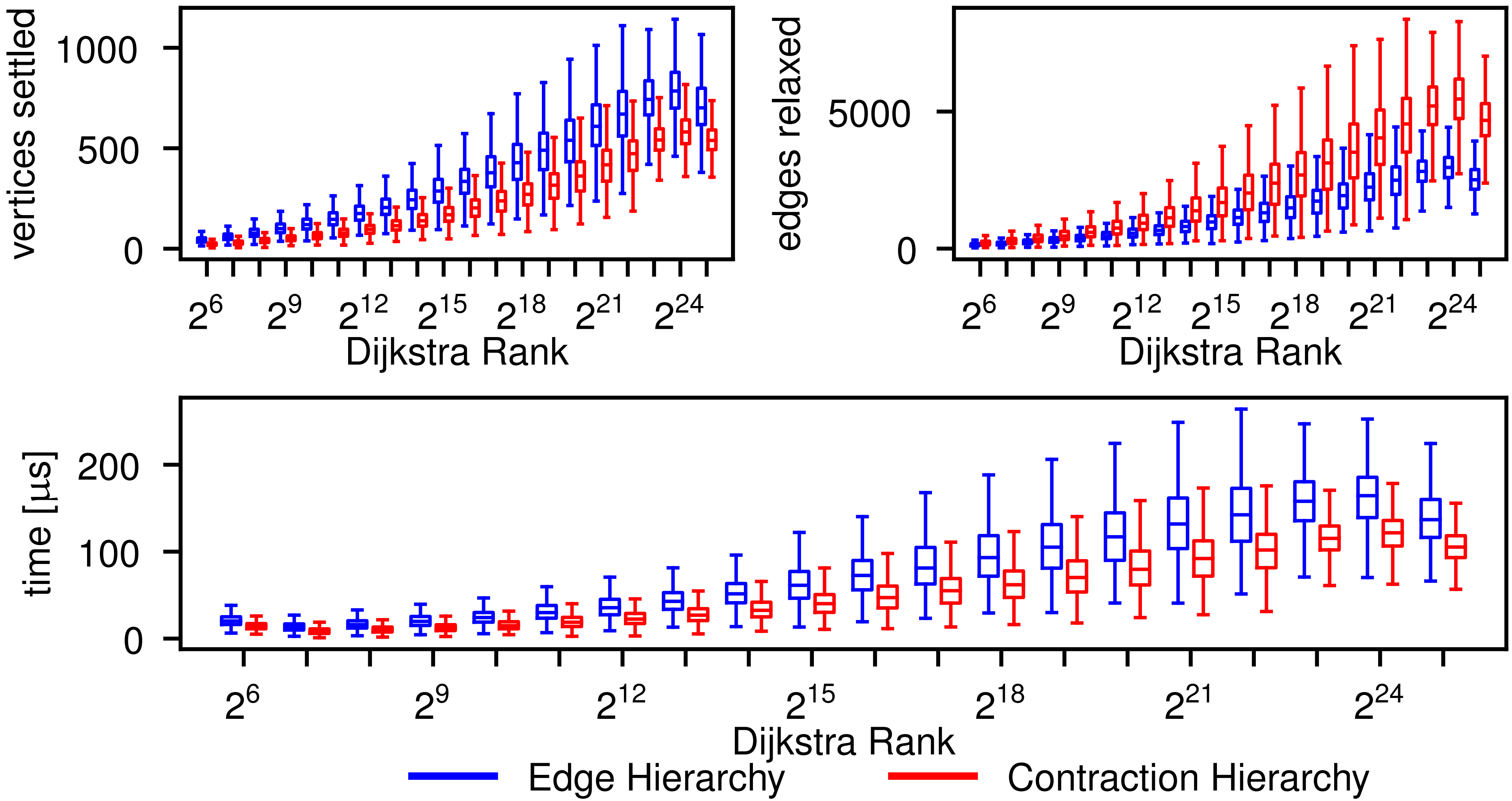
	Graph	Prepr. [s] EH	Prepr. [s] CH	$\lvert E \rvert$ [M] EH	$\lvert E \rvert$ [M] CH
Orig.	USA	7145	674	104.5	104.0
Orig.	EUROPE	3171	453	70.3	70.3
Turns	USA	45904	15462	270.3	404.3
Turns	EUROPE	17822	4743	194.0	249.1

Distance

	Graph	Prepr. [s] EH	Prepr. [s] CH	$\lvert E \rvert$ [M] EH	$\lvert E \rvert$ [M] CH
Orig.	USA	21041	1537	110.8	110.7
Orig.	EUROPE	14487	2152	79.6	79.6
Turns	USA	58025	30869	251.1	398.3
Turns	EUROPE	24757	13266	172.3	267.2

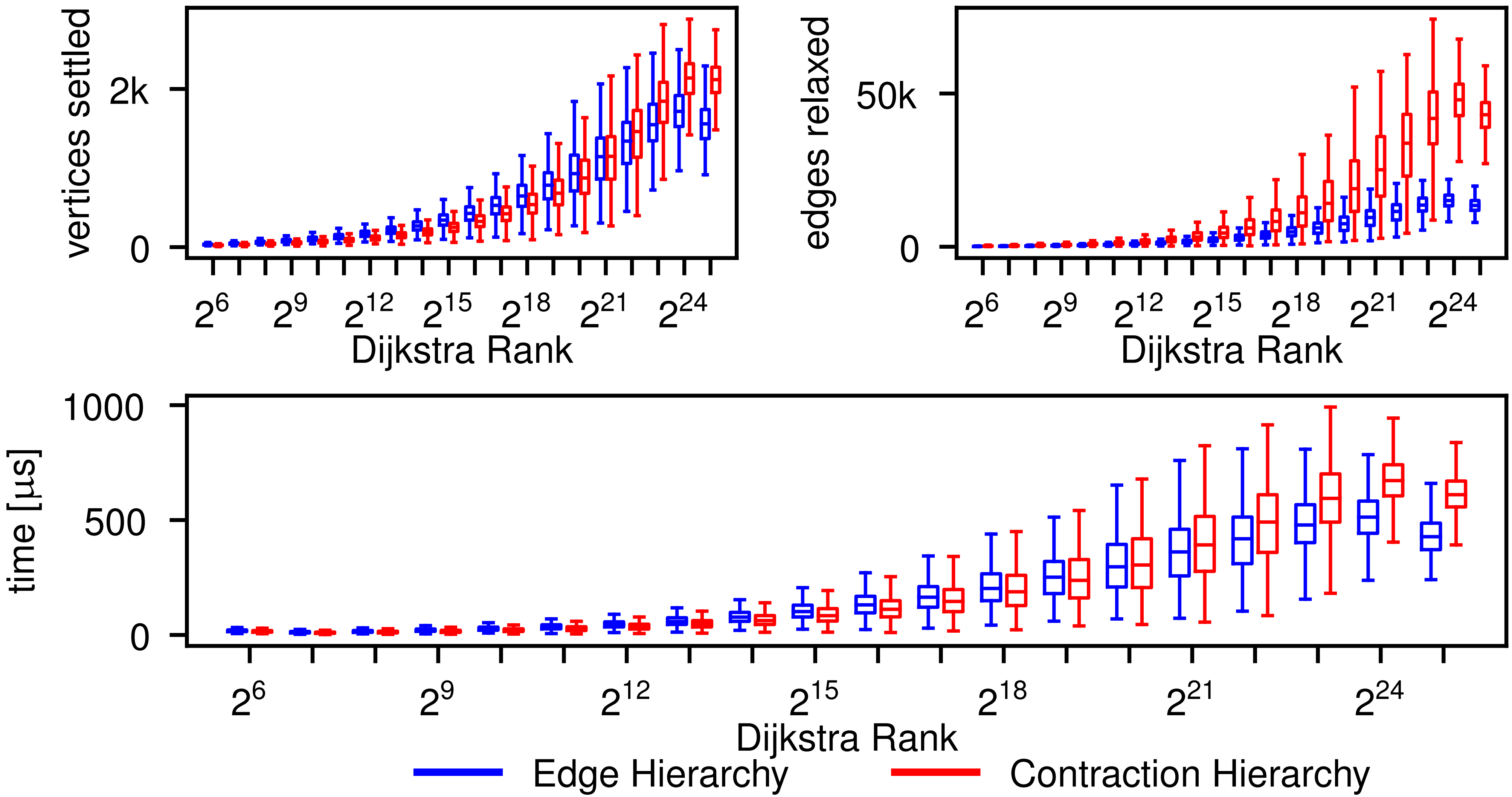
* EH Preprocessing uses CH queries
Intel Xeon E5-4640, 2.4 GHz

Queries (Travel Time + Turns)



Instance: Western Europe by PTV

Queries (Distance + Turns)



Instance: Western Europe by PTV

The End

- **In the Paper**
  - Edge Selection
  - Partial Stall on Demand
- **Future Work**
  - Cache efficient distance labels
  - Applications with more expensive edge relaxations
- **Questions?**

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References

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