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WeGotYouCovered:

The Winning Solver from the PACE 2019 Challenge, Vertex Cover Track

CSC'20 · February 2020

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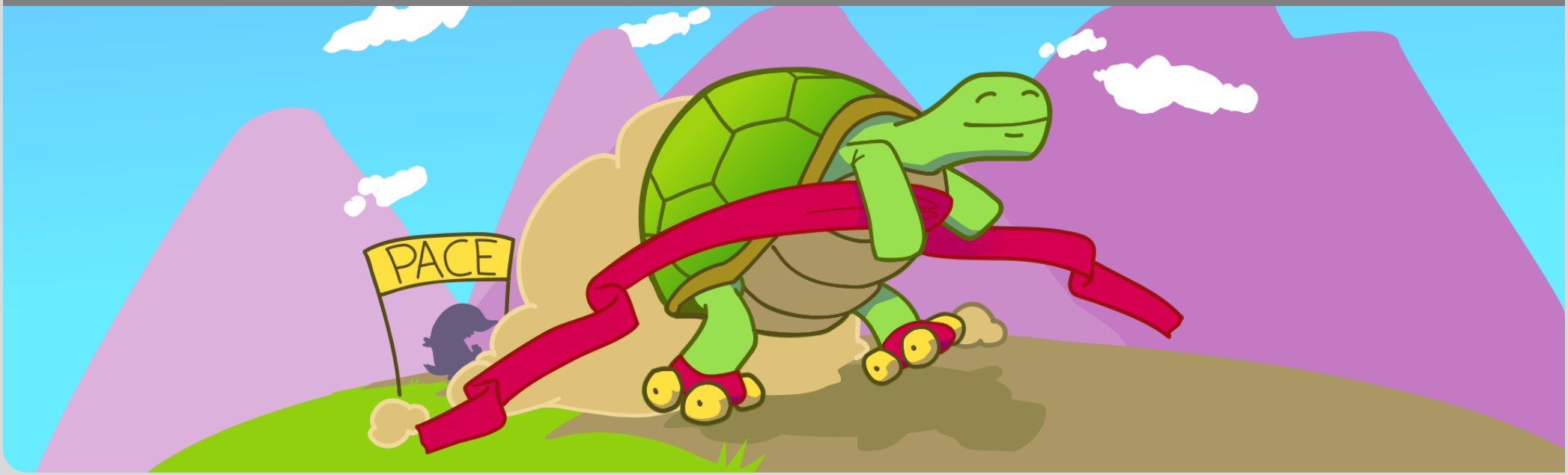
INSTITUTE OF THEORETICAL INFORMATICS · ALGORITHMICS GROUP



PACE

[Dzulfikar et al. 2019]

Parameterized **A**lgorithms and **C**omputational **E**xperiments

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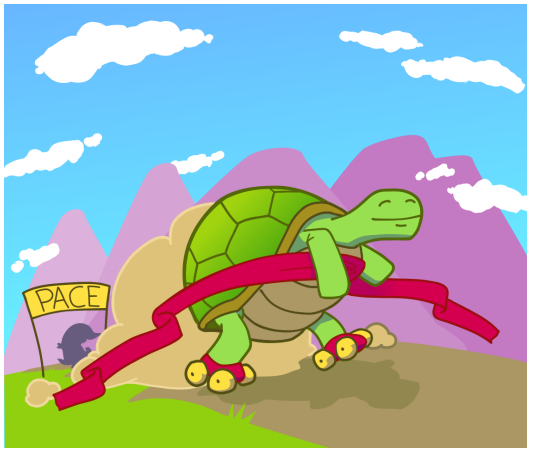
Mission

- Bridge gap between theory and practice
- Inspire new theoretical developments
- Investigate theoretical algorithms in practice
- Produce accessible implementations & benchmarks
- Encourage dissemination in scientific papers



Previous problems:

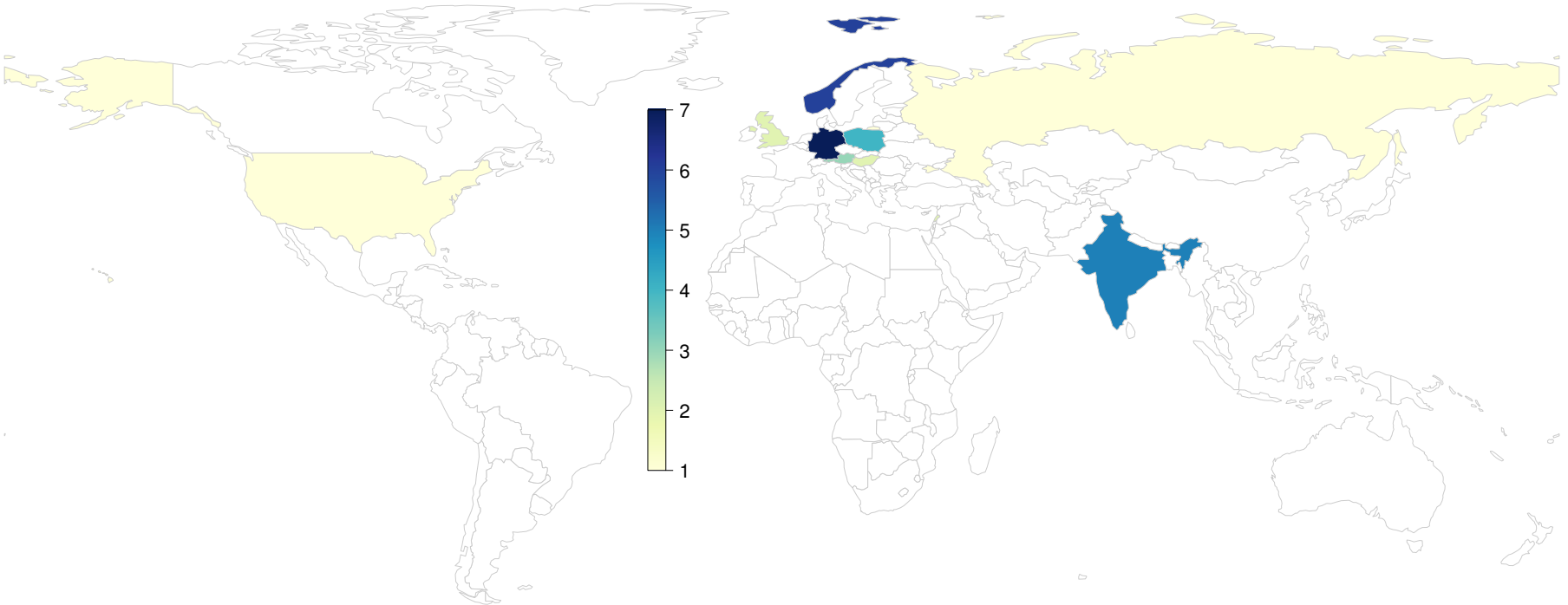
- Treewidth, Feedback Vertex Set
- Treewidth, Minimum Fill-In
- Steiner Tree (3 tracks)

PACE 2019

[Dzulfikar et al. 2019]

- Track 1: Vertex Cover
- Track 2: Hybertree Decomposition
  - Track 2a: Exact
  - Track 2b: Heuristic

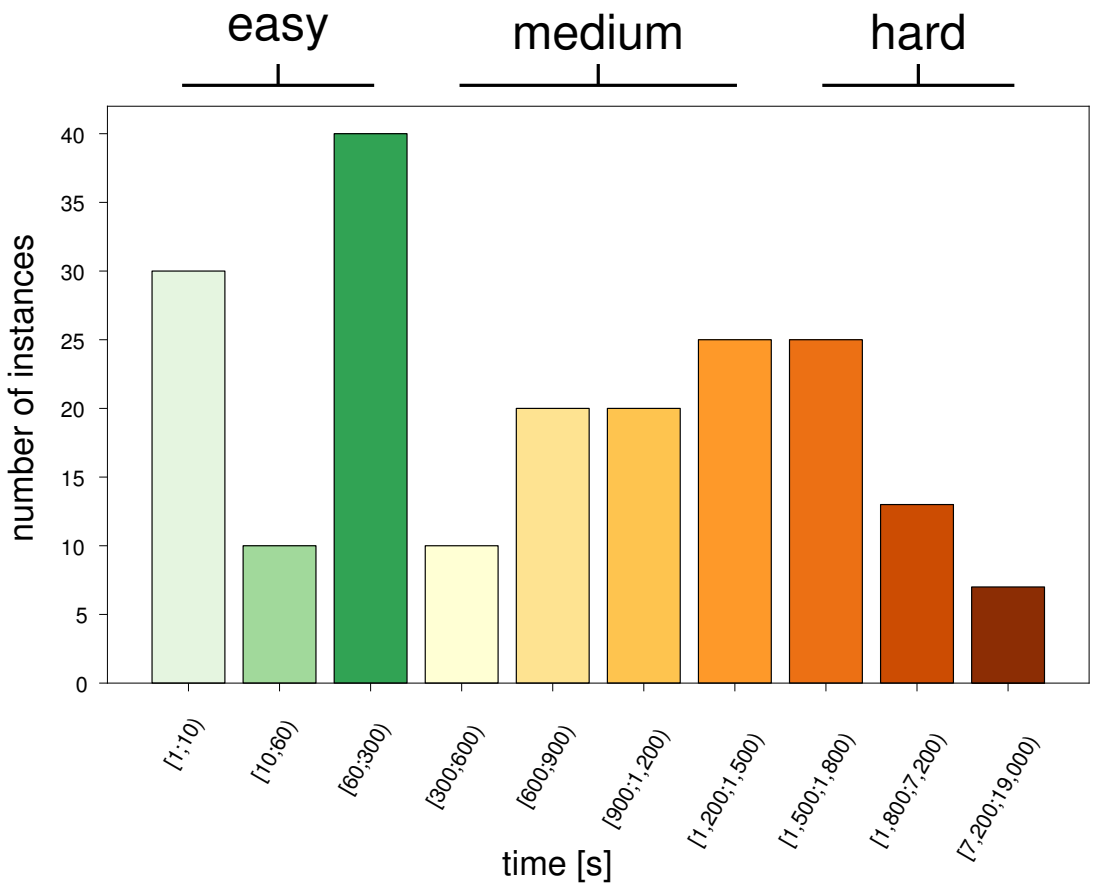
Region	Country	Teams	Participants	Tracks
Europe	Austria	3	3	1a, 2a, 2b
	Germany	4	7	1a
	Hungary	1	2	1a
	Norway	2	6	1a
	Poland	2	4	1a
	Russia	1	1	1a
	Scotland	1	2	1a, 2a, 2b
Middle East	Lebanon	1	2	1a
North America	USA	1	1	1a
South Asia	India	2	5	1a
	10	18	33	



Instances

[Dzulfikar et al. 2019]

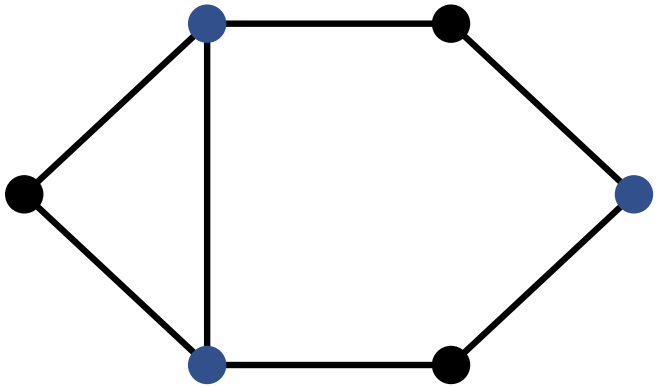
- 200 instances selected out of 9591 from various origins
- 100 public and 100 private instances
- Difficulty rated by time to solve by ILP solver



Selected Instances

- 80 easy [1s;300s)
- 80 medium [300s;1500s)
- 40 hard [1500s;19000s)

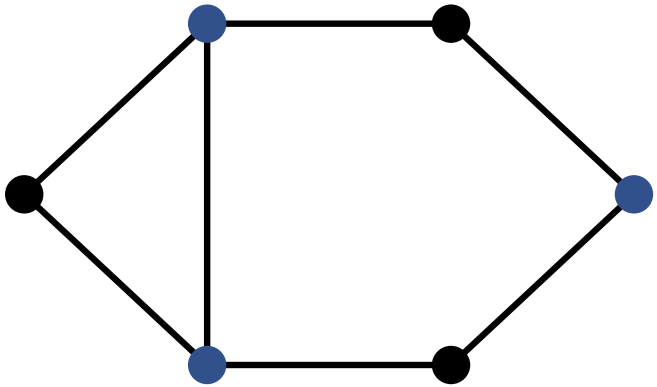
Vertex Cover



Given graph $G = (V, E)$, find $S \subseteq V$ s.t.

- Every edge $e \in E$ is connected to at least one $v \in S$
- $|S|$ is minimized

Vertex Cover

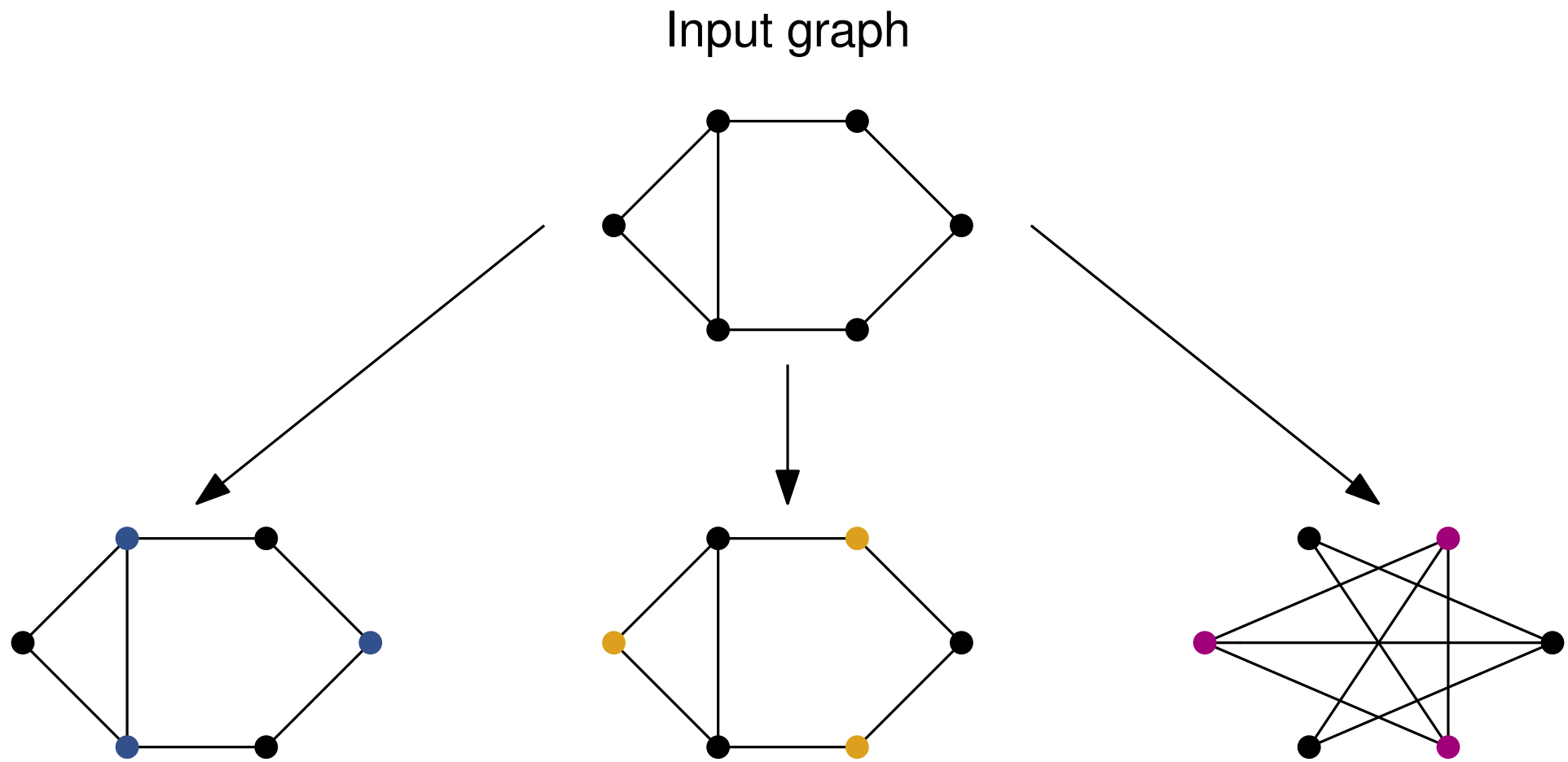


Given graph $G = (V, E)$, find $S \subseteq V$ s.t.

- Every edge $e \in E$ is connected to at least one $v \in S$
- $|S|$ is minimized

NP hard

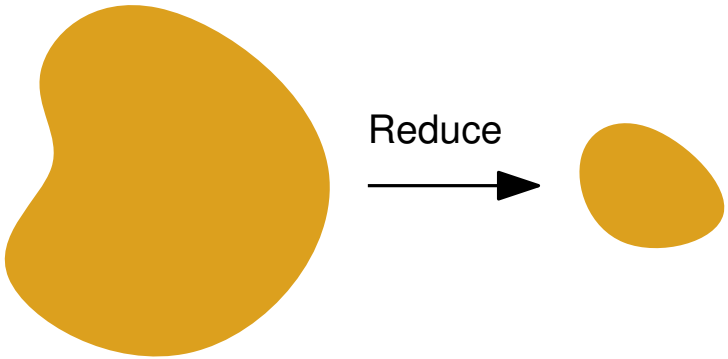
Vertex Cover and Complementary Problems



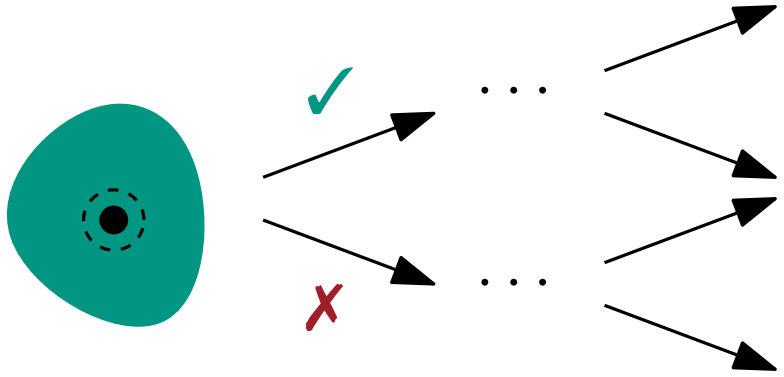
We employ algorithms from all of these problems to tackle vertex cover

Techniques

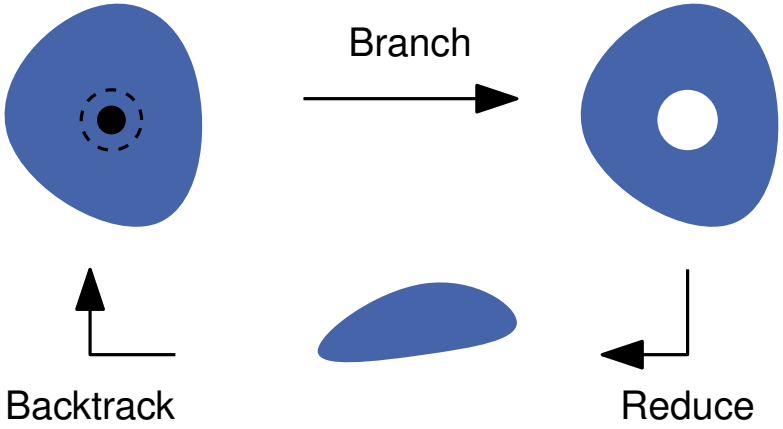
Kernelization



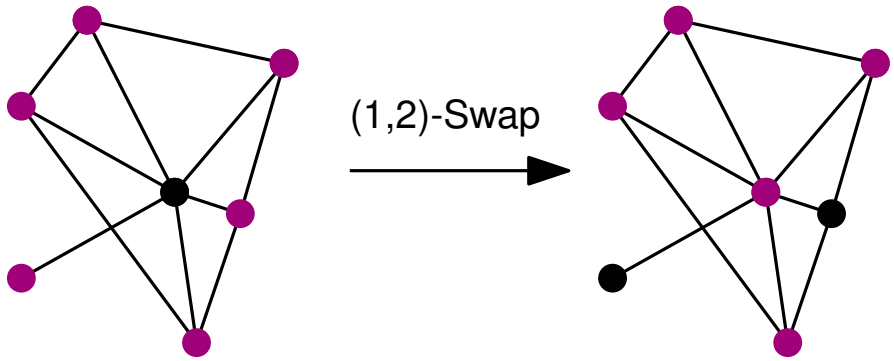
Branch and Bound



Branch and Reduce

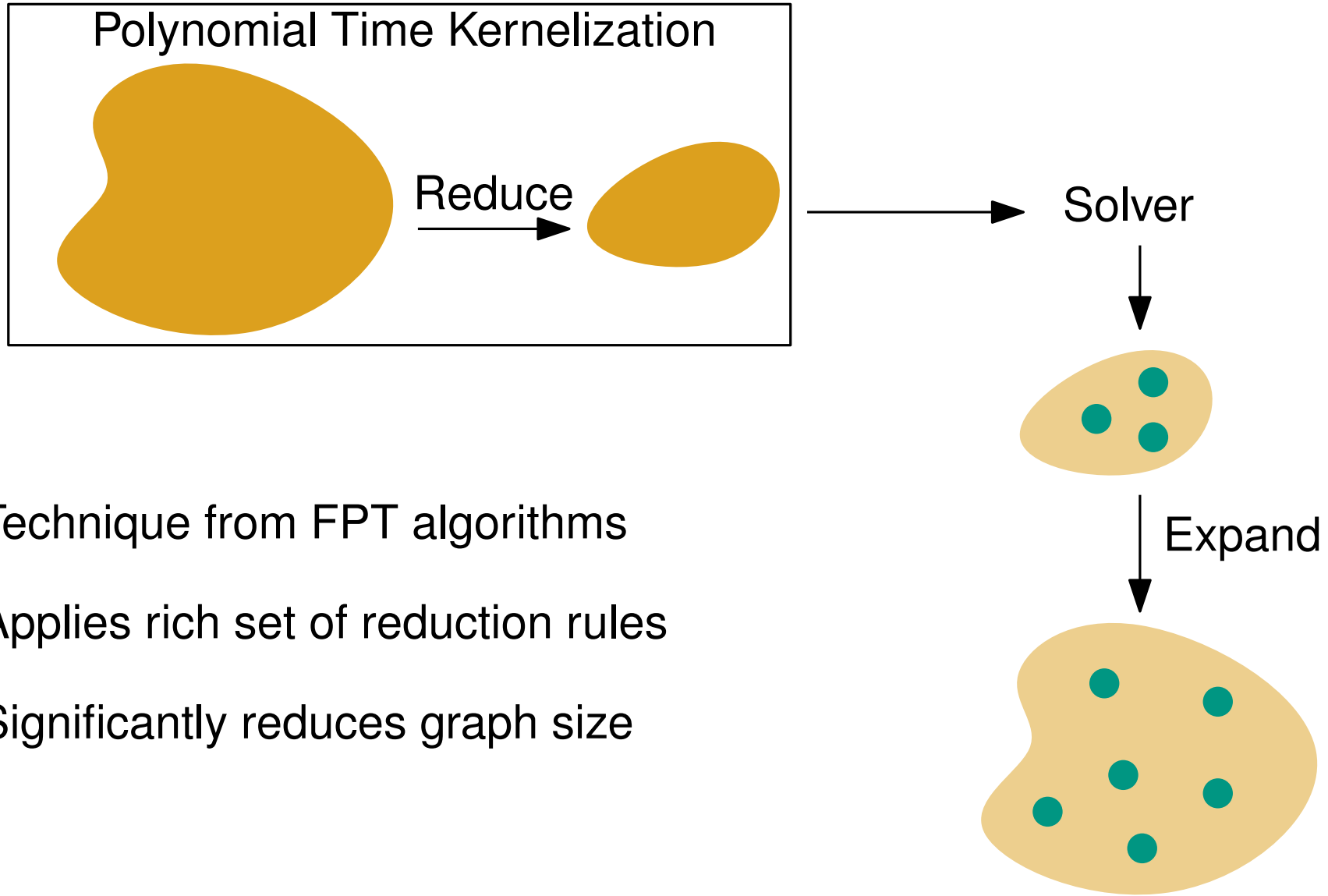


Iterated Local Search



Kernelization

[Akiba and Iwata, 2016]



- Technique from FPT algorithms
- Applies rich set of reduction rules
- Significantly reduces graph size

Reduction Rules

[Akiba and Iwata, 2016]

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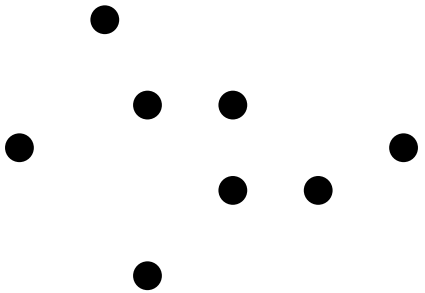
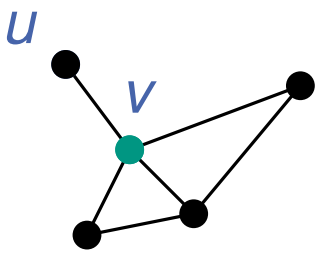
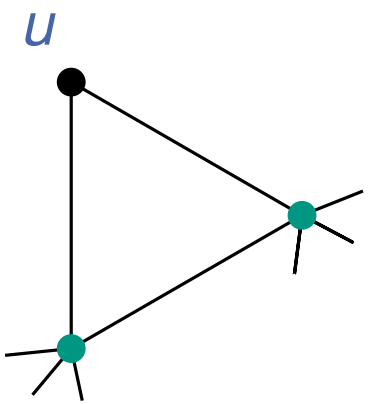
Degree 0



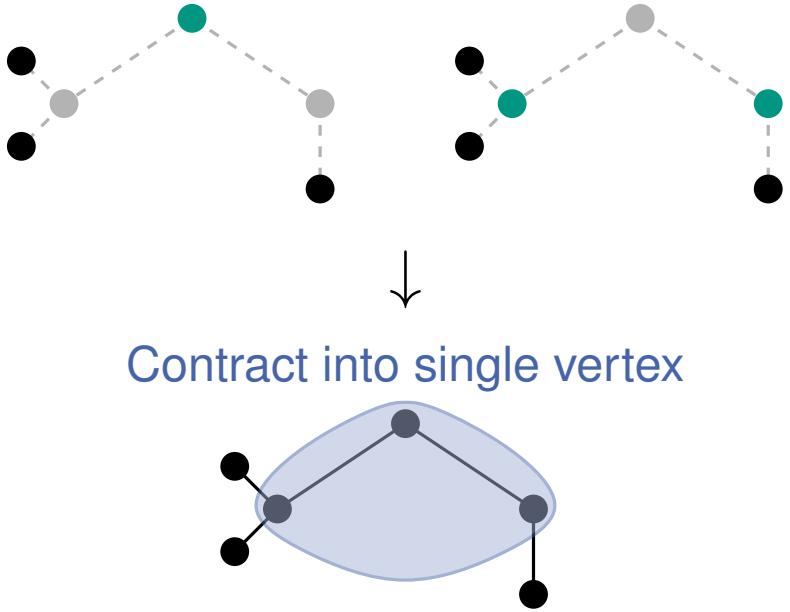
Degree 1



Degree 2



vertex folding

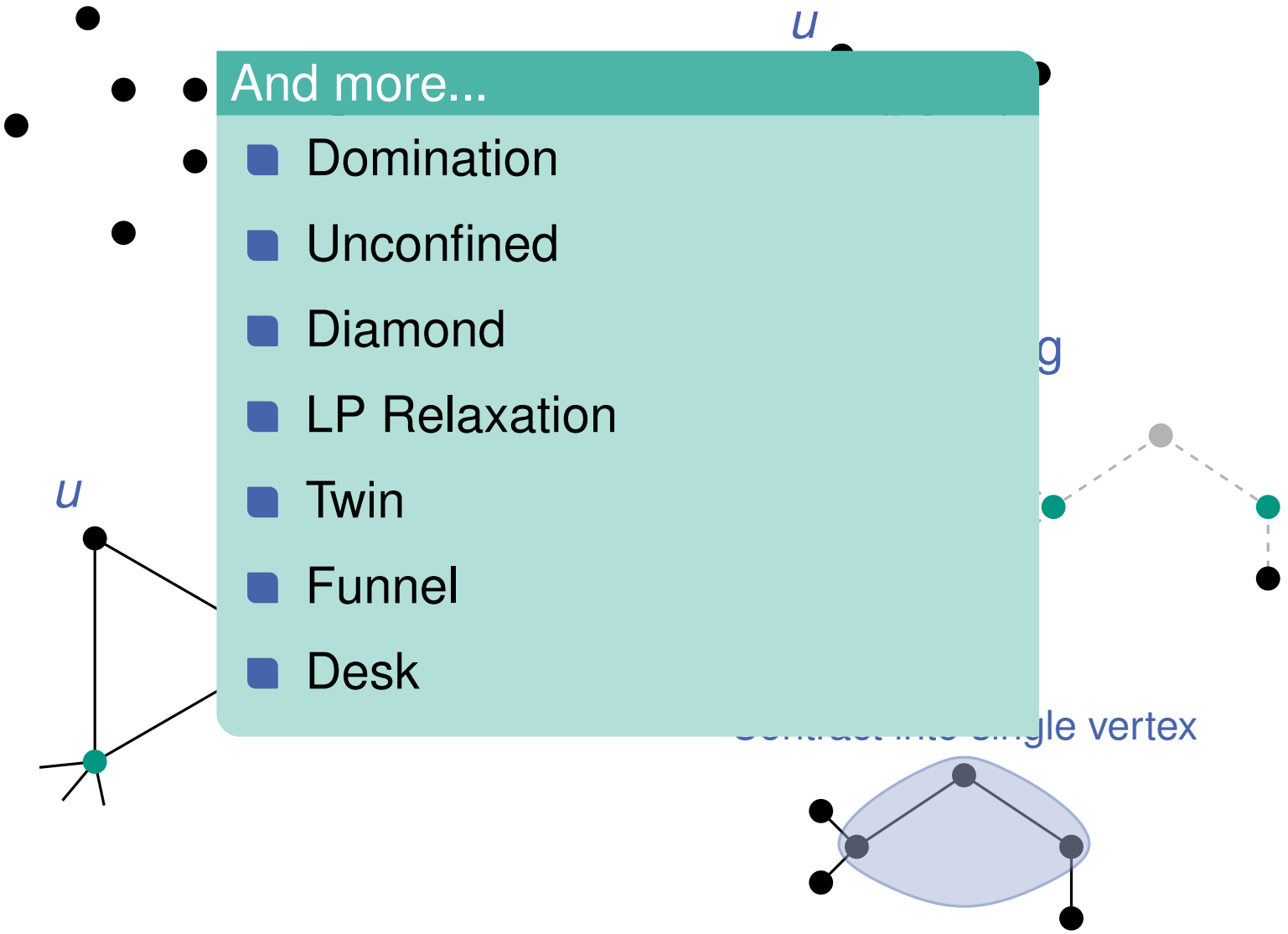


Reduction Rules

[Akiba and Iwata, 2016]

Degree 0

Degree 1

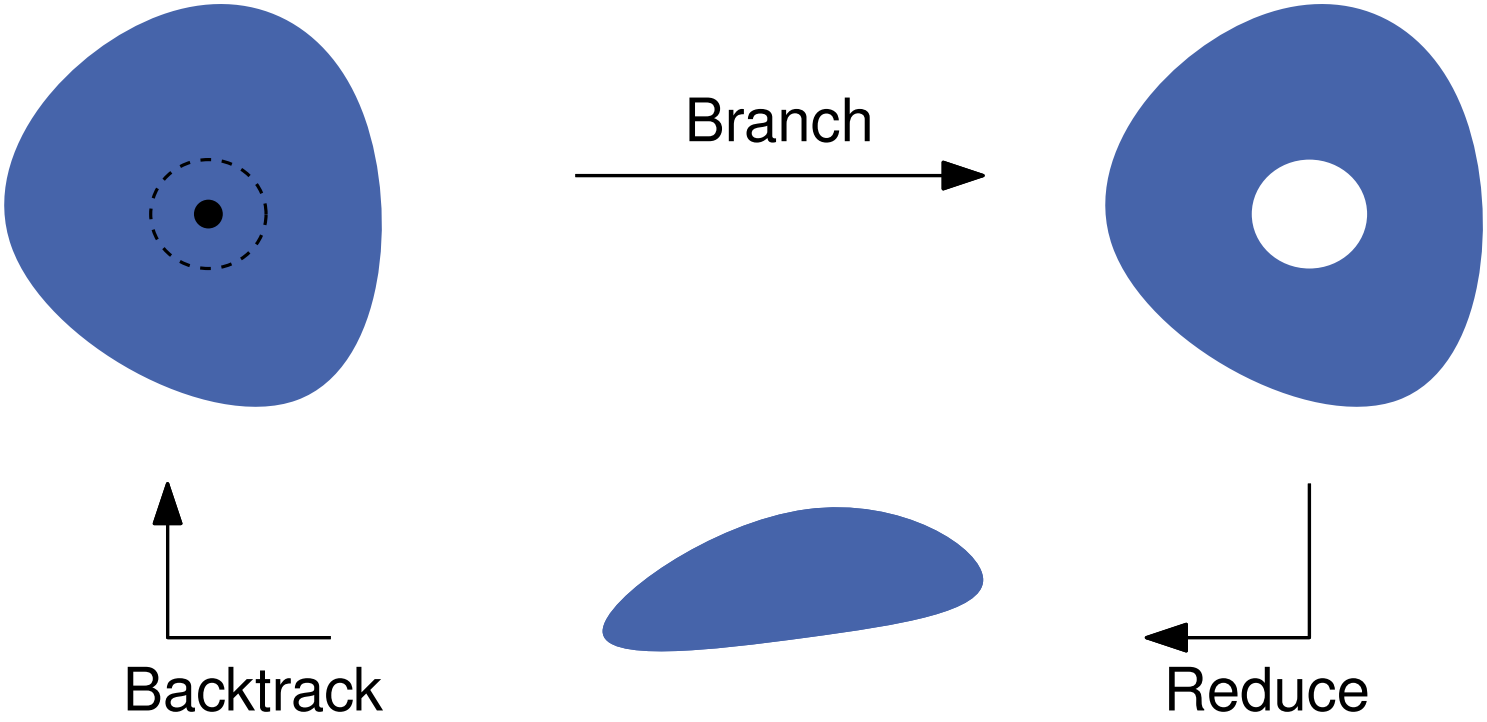


And more...

- Domination
- Unconfined
- Diamond
- LP Relaxation
- Twin
- Funnel
- Desk

Branch and Reduce

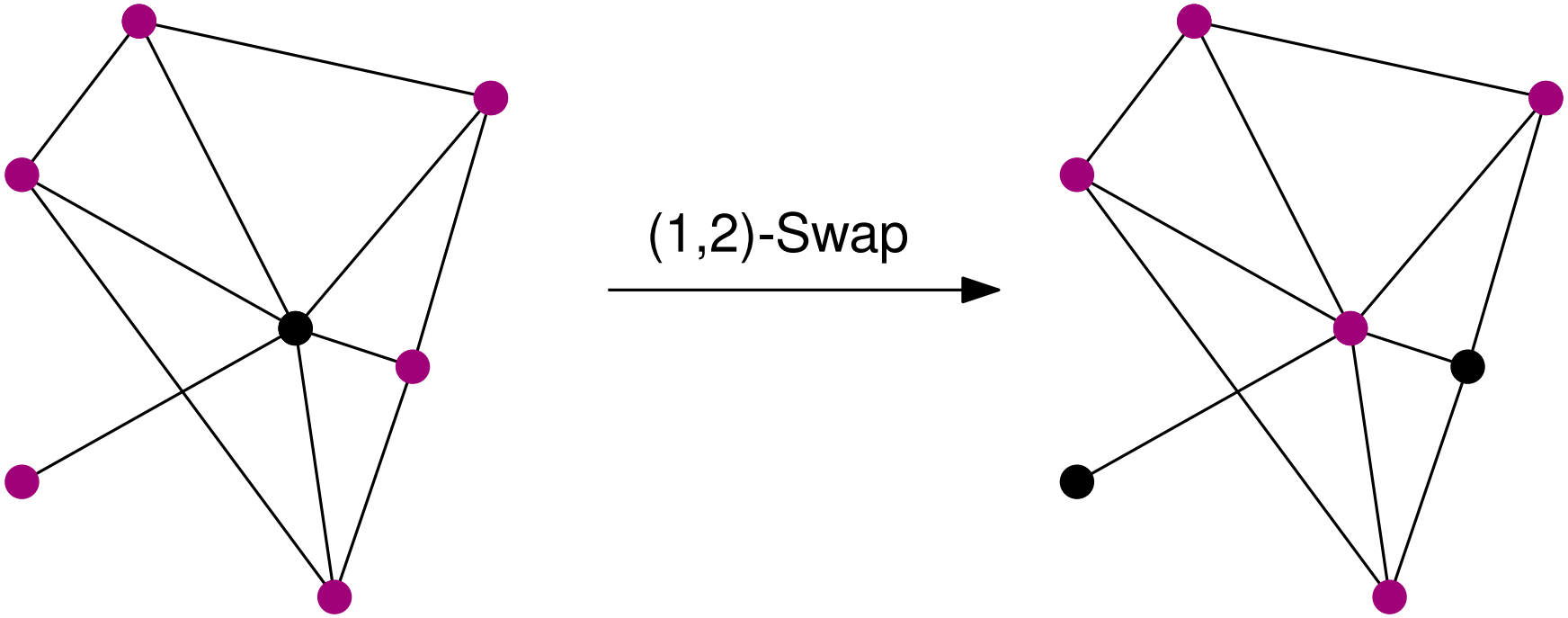
[Akiba and Iwata, 2016]



- Reduce graph after each branch
- Additional branching rules to reduce graph size
- Prune search based on lower bounds

Iterated Local Search

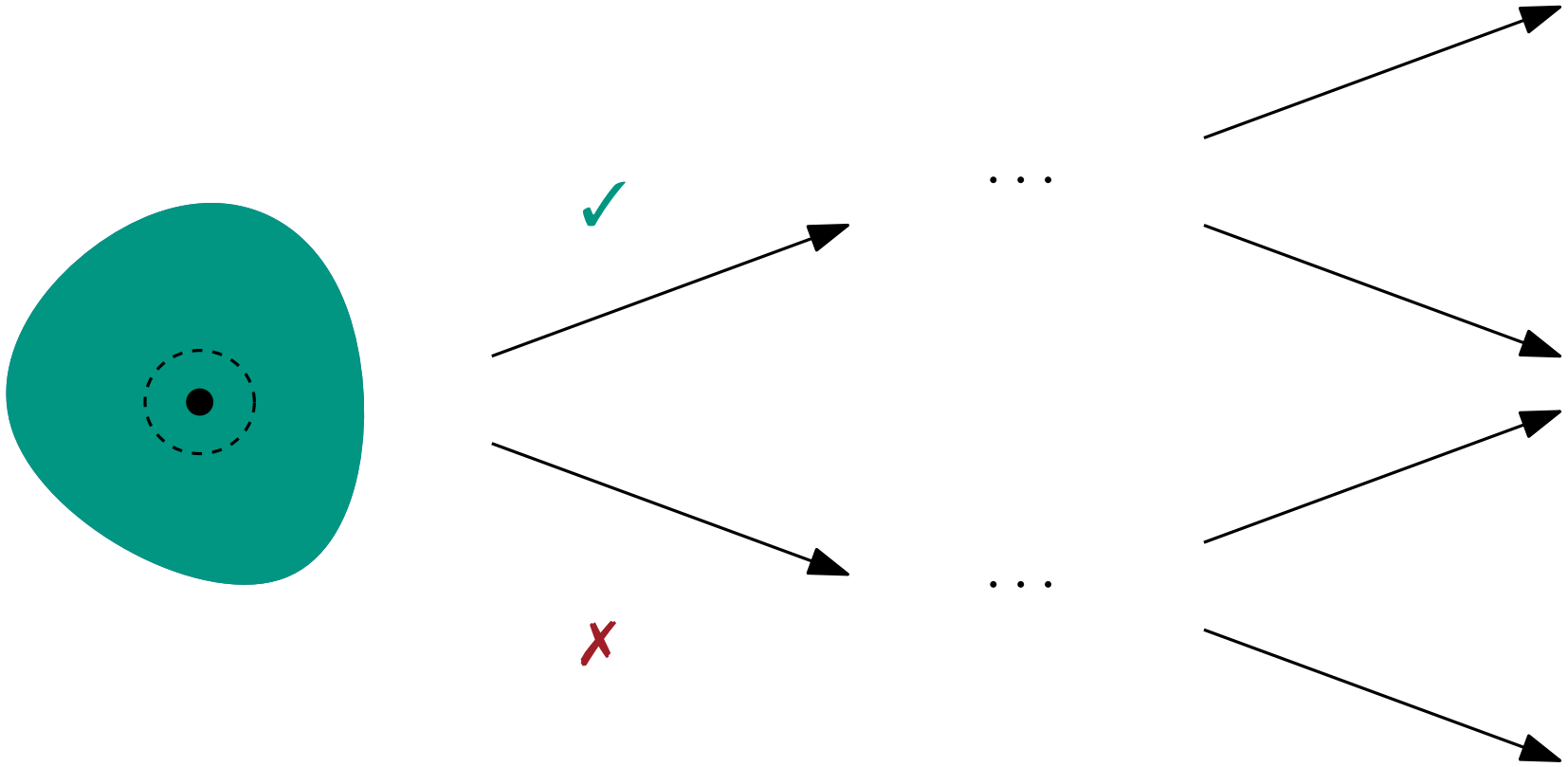
[Andrade et al. 2012]



- Originally developed for independent sets
- Perturbation and tabu lists to escape local optima
- Can often find (near-)optimal solutions

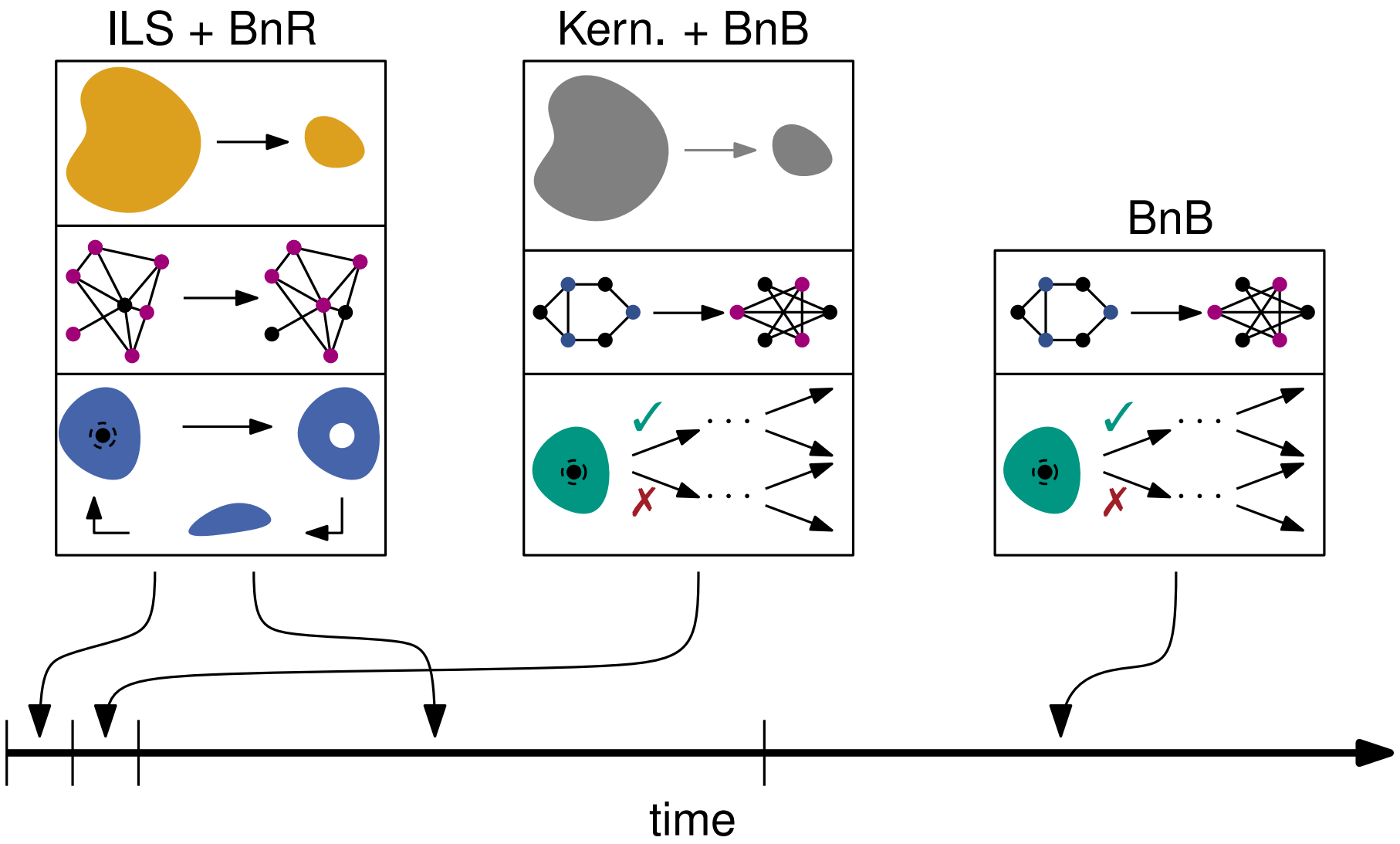
Branch and Bound

[Li et al. 2017]

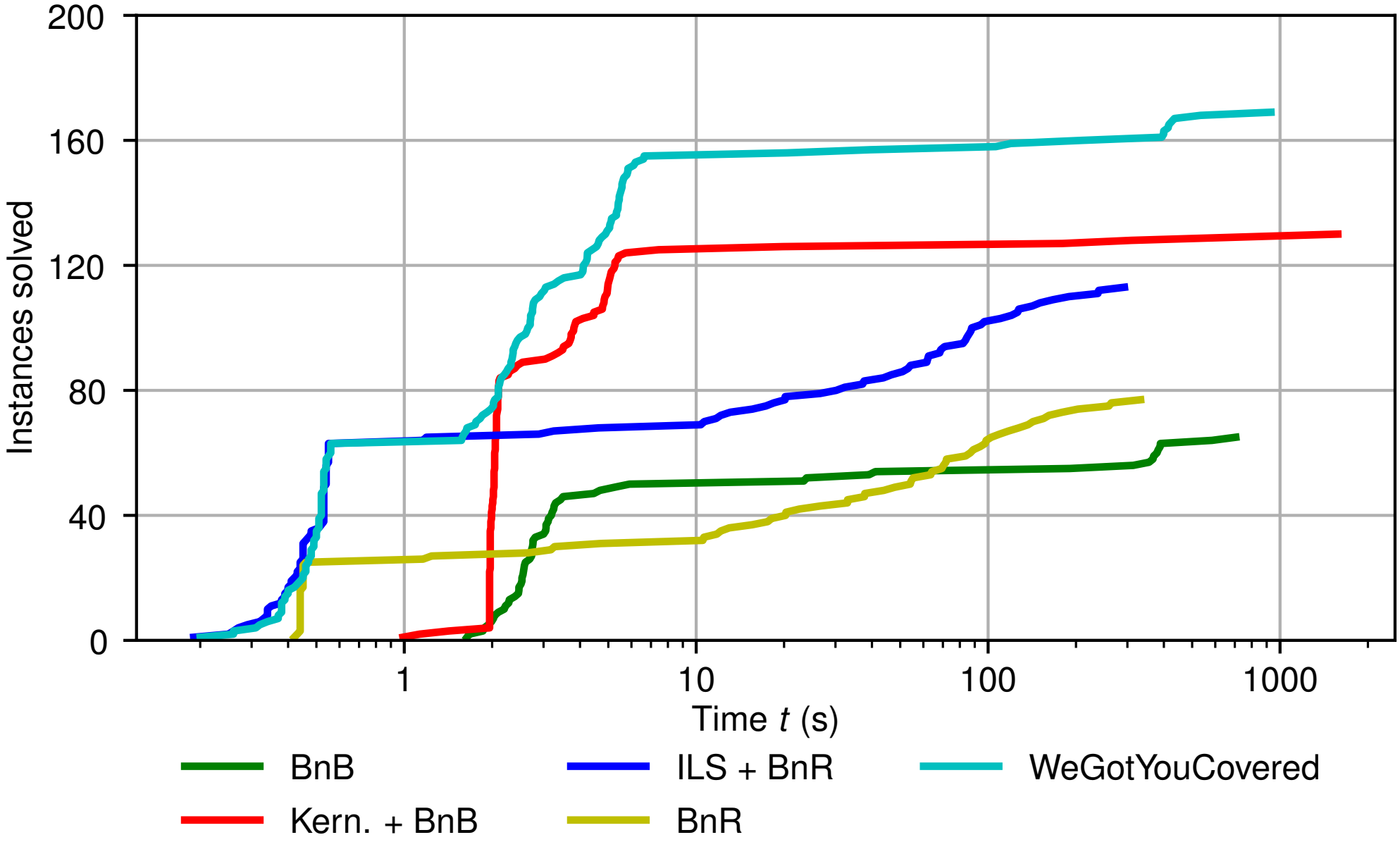


- Originally developed for maximum cliques
- Incremental MaxSAT reasoning to prune search
- Combination of static and dynamic vertex ordering

Algorithm Overview

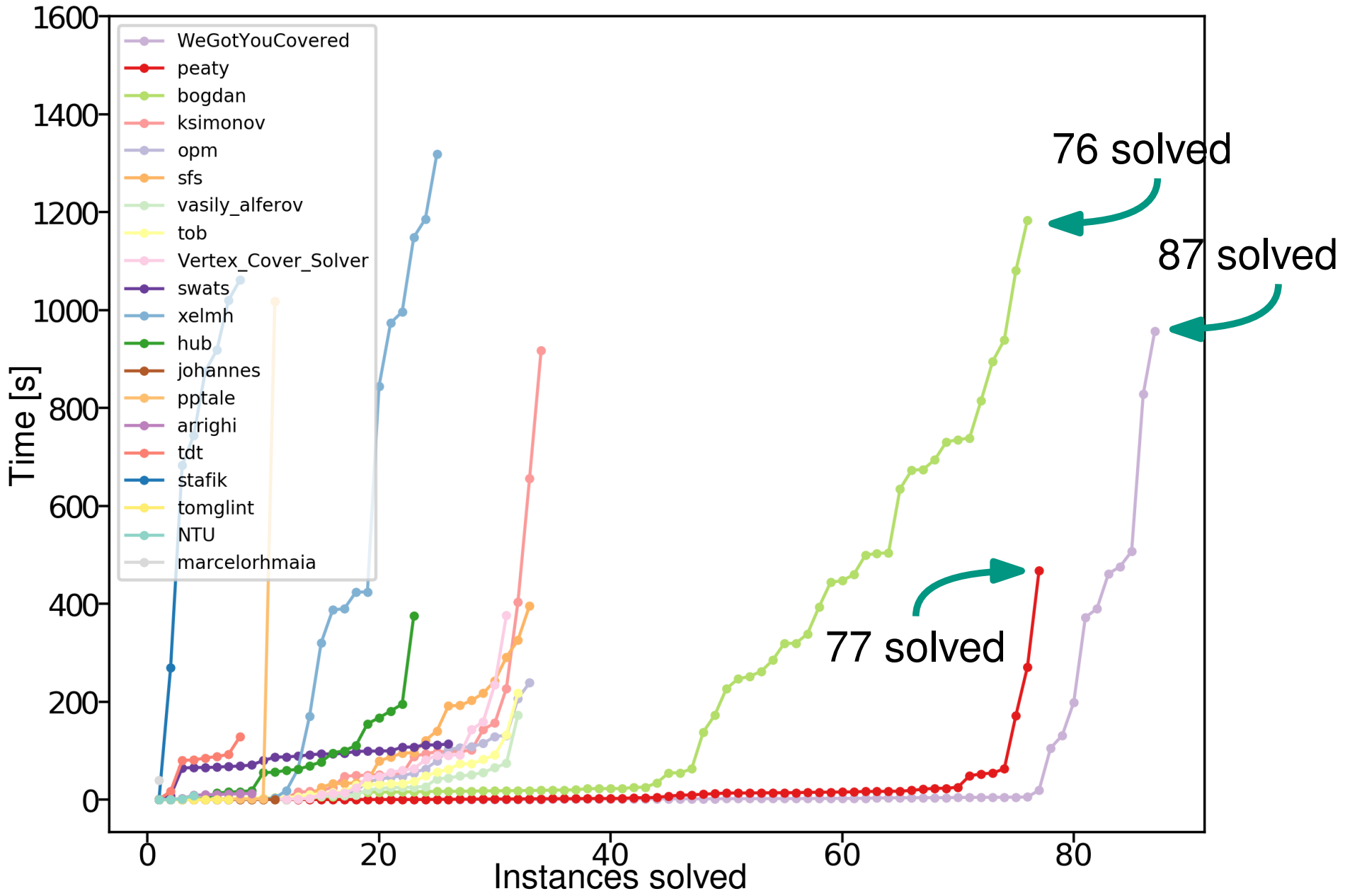


Instances Solved Over Time



PACE Results

[Dzulfikar et al. 2019]



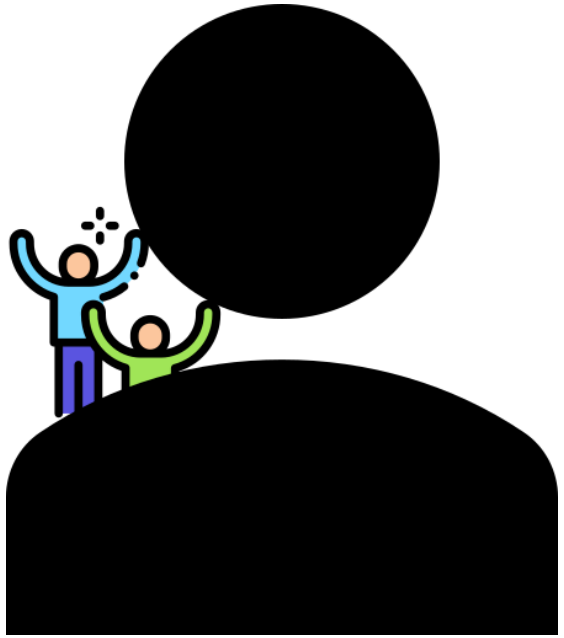
Conclusion

Lessons learned / future work

- Heuristics can help branch and bound algorithms
- Algorithm selection is hard!
  - What makes an instance hard for one algorithm but easy for another?
  - When does kernelization “fail”?

Acknowledgements

- Takuya Akiba and Yoichi Iwata
- Diogo V. Andrade, Mauricio G. C. Resende and Renato F. Werneck
- Chu-Min Li, Hua Jiang and Felip Manyà
- Johannes K. Fichte and Markus Hecher



Icons made by Those Icons and Freepik from www.flaticon.com

Code: github.com/sebalamm/pace-2019 (algo2.iti.kit.edu/kamis)

References

- Akiba, Takuya, and Yoichi Iwata. “Branch-and-reduce exponential/FPT algorithms in practice: A case study of vertex cover.” Theoretical Computer Science 609 (2016): 211-225.
- Andrade, Diogo V., Mauricio G. C. Resende, and Renato F. Werneck. “Fast local search for the maximum independent set problem.” Journal of Heuristics 18.4 (2012): 525-547.
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- Li, Chu-Min, Hua Jiang, and Felip Manyà. “On minimization of the number of branches in branch-and-bound algorithms for the maximum clique problem.” Computers & Operations Research 84 (2017): 1-15.

More Reduction Rules

[Akiba and Iwata, 2016]

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Reductions

- LP-relaxation

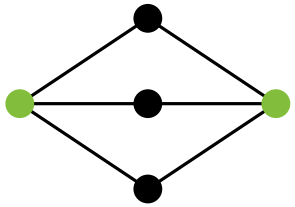
  $\rightarrow$ Minimize $\sum x_v$ where $x_u + x_v \geq 1$.
  If $x_v = 1$, then **in some MVC**.

- Unconfined

  $\rightarrow$ **Some MVC** exists containing "unconfined" vertices

- Twin

  $\rightarrow$ Generalization of vertex folding



- Diamond, alternative, . . .