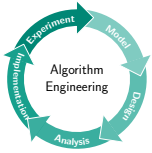
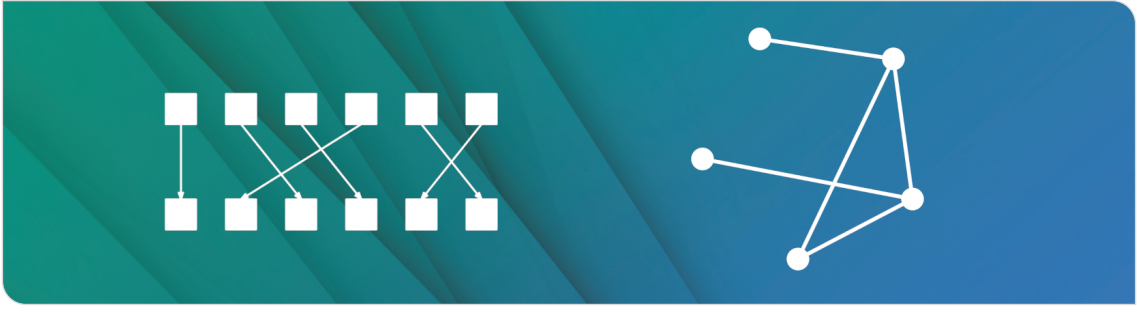
Karlsruhe Institute of Technology



ShockHash: Towards Optimal-Space Minimal Perfect Hashing Beyond Brute-Force

ALENEX 2024, Alexandria

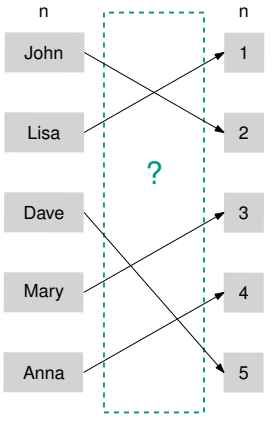
Hans-Peter Lehmann, Peter Sanders, Stefan Walzer | Jan 8, 2024



Minimal Perfect Hashing

- Static set of n keys
- Bijectively map keys to the first n integers

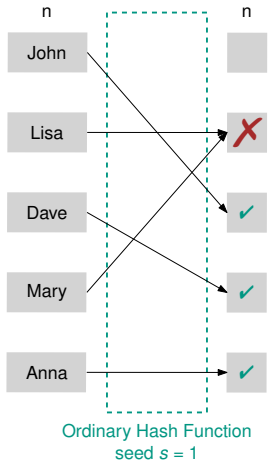
- Applications: databases, hash tables, AMQ, retrieval
- Get as close as possible to the space lower bound



Minimal Perfect Hash Function

Brute-Force Construction

- Try ordinary hash functions
- Perfect hash function data structure: store successful seed s
- Expected tries: $n^n/n! \approx e^n$
  $\Rightarrow\approx n\log_2(e)$ bits (this is optimal)



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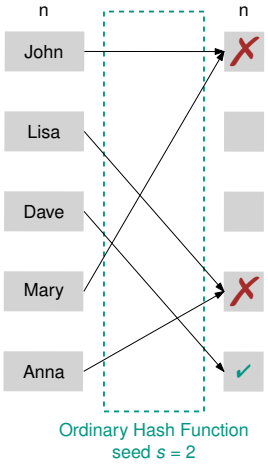

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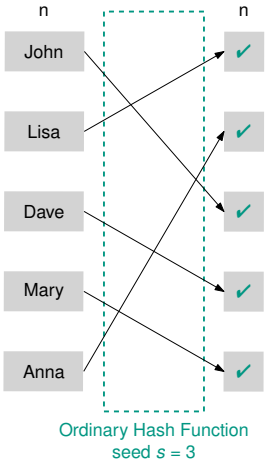
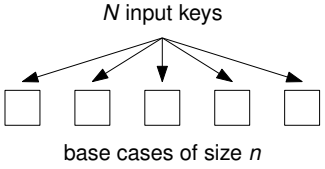


Supporting Large Input Sets

- Brute-force of e^n seeds not practical for large n
- Use as building block
- Partition input to smaller sets
- Also used in RecSplit [EGV20]



ShockHash

- Sample random graphs
- Store choice between two candidates [DHSW22a, LSW23b]

x	$h_0(x)$	$h_1(x)$
John	3	4
Lisa	2	1
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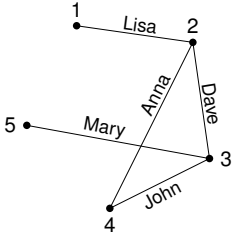

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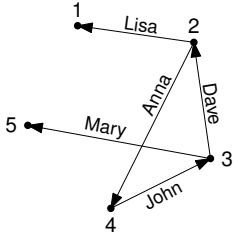

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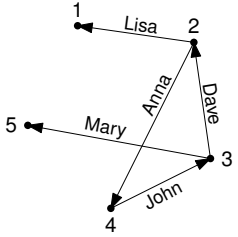

Retrieval	
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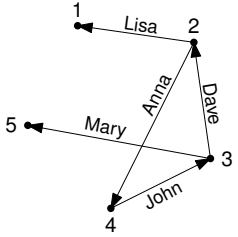

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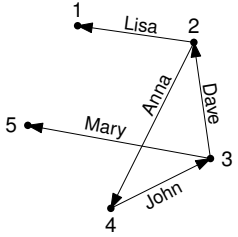

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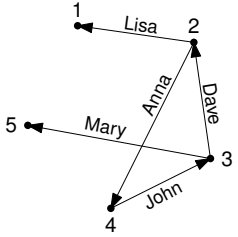

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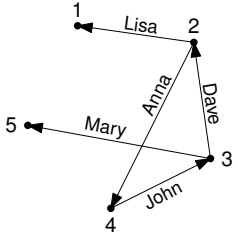

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Seed

ShockHash

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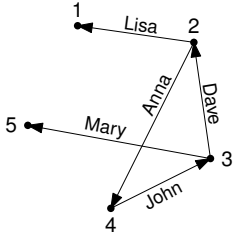

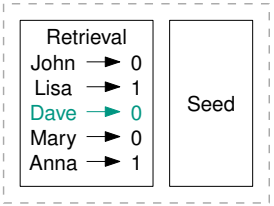

ShockHash data structure

ShockHash

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- Problem: Unlikely to work for $> n/2$ edges [PR04], here we use n
- **ShockHash**: Do it anyway, try many seeds
- Orientability check?
  Success probability?
  Space usage?

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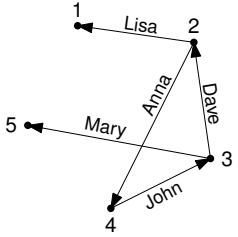
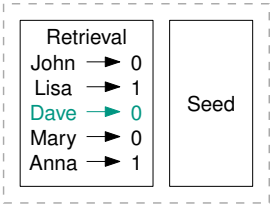
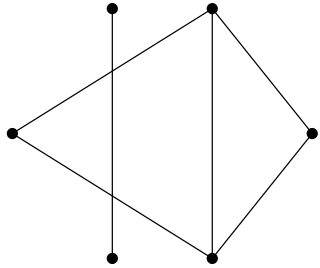




ShockHash data structure

Orientability Check

- 1-orientable if each component contains no more edges than nodes
  - Here: Tree with one additional edge
- Can be checked in linear time using connected components algorithms
- $\Rightarrow$ We check the 2^n different states of the retrieval data structure in linear time



Success Probability

$$\mathbb{P} \geq \frac{\qquad\qquad}{n^{2n}}$$

x	$h_0(x)$	$h_1(x)$
John	?	?
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Success Probability

- Labeled trees (Cayley's formula)

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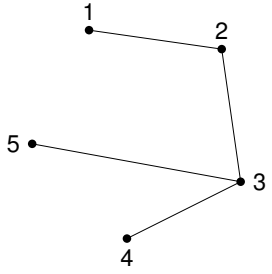

Success Probability

- Labeled trees (Cayley's formula)
- Table rows can be in any order

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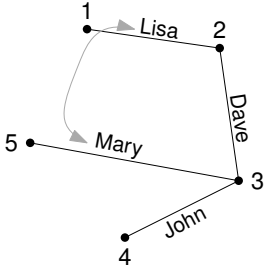

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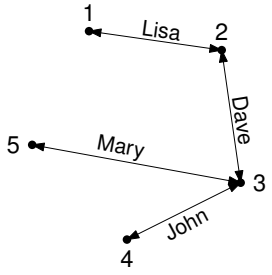

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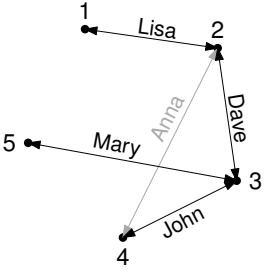

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$$= \frac{n!}{n^n} \cdot \frac{2^{n-1}}{n}$$

Brute force

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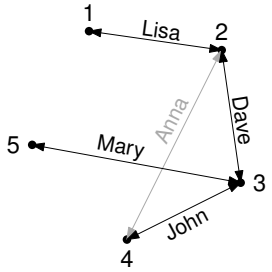

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Brute force

Almost 2^n times higher probability

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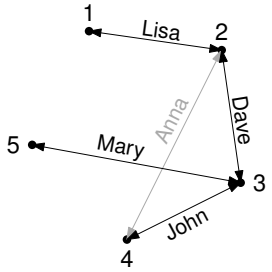
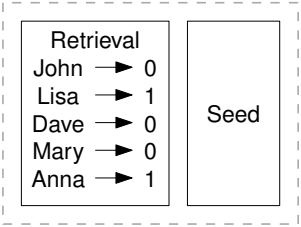


Space Usage

- Record choices in retrieval:
  BuRR [DHSW22b] solves equation system
  $f(x) = A \cdot h(x)$ to achieve $\approx n$ bits
- Expected space for seed:
  $\log_2\left(\frac{n}{2^{n-1}} \cdot \frac{n^n}{n!}\right) \approx n\log_2(e) - n$ bits
- Together: $\approx n\log_2(e)$ bits (optimal!)

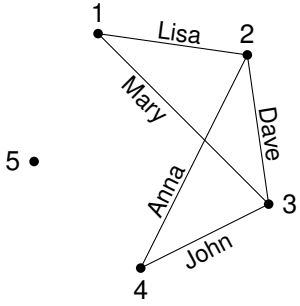


ShockHash data structure

Same space as brute-force but nearly 2^n times faster!

Filtering

- First implementation dominated by orientability check
  $\Rightarrow$Filter seeds that can't work
- Efficiently in registers
- Filter passed with probability only 0.84^n
- Main ingredient for making ShockHash feasible in practice



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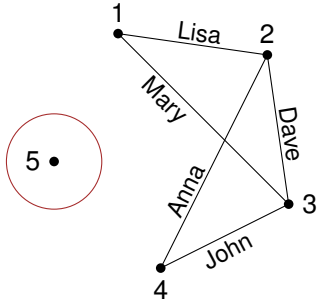

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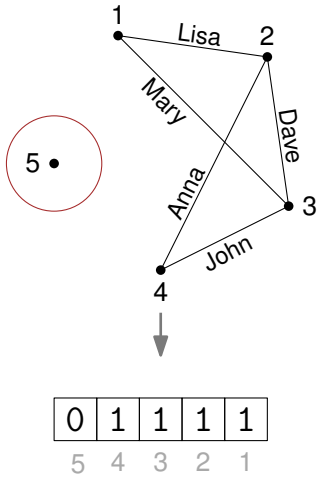
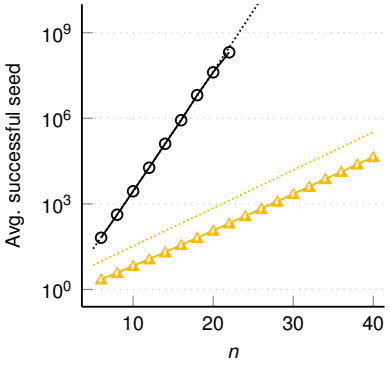
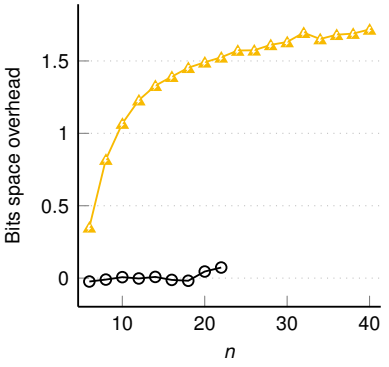
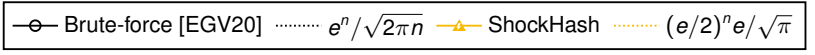


Theory and Practice

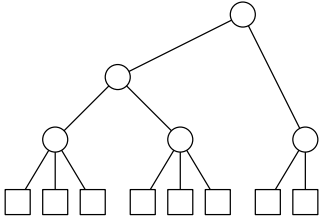




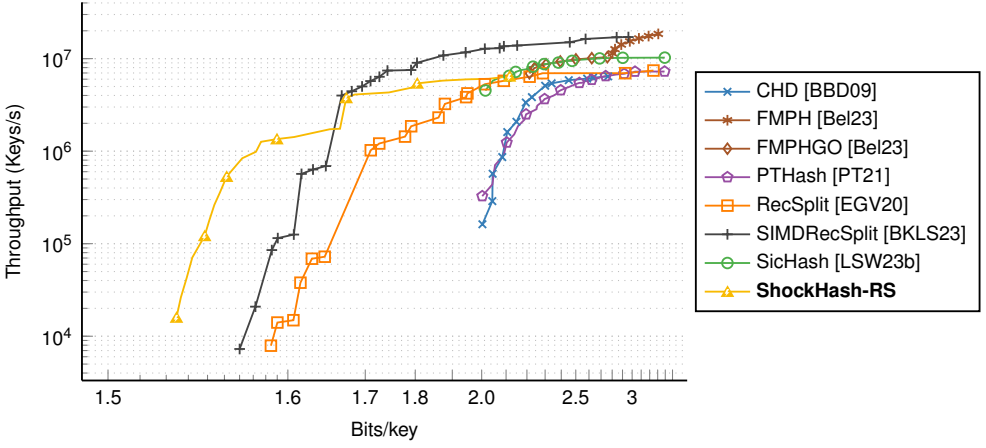


ShockHash-RS

- ShockHash as building block
- RecSplit [EGV20] for partitioning
  $\Rightarrow$ ShockHash-RS
- Practical building block size:
  16 (Brute-force) $\Rightarrow$ 50 (ShockHash)



Construction Throughput



Performance Overview

Method	Space	Constr.	Query
PTHash, c=7.0, α=0.99, C-C	3.524	198 ns	20 ns
SIMDRecSplit, n=14, b=2000	1.585	11 749 ns	108 ns
SIMDRecSplit, n=16, b=2000	1.560	137 902 ns	100 ns
ShockHash-RS, n=30, b=2000	1.583	787 ns	114 ns
ShockHash-RS, n=39, b=2000	1.556	1 805 ns	118 ns
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Lower bound: 1.443 bits/key

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ShockHash-RS, n=58, b=2000	1.523	111 593 ns	115 ns

Lower bound: 1.443 bits/key

Performance Overview

Method	Space	Constr.	Query
PTHash, c=7.0, α=0.99, C-C	3.524	198 ns	20 ns
SIMDRecSplit, n=14, b=2000	1.585	11 749 ns	108 ns
SIMDRecSplit, n=16, b=2000	1.560	137 902 ns	100 ns
ShockHash-RS, n=30, b=2000	1.583	787 ns	114 ns
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Conclusion

- ShockHash is a building block for perfect hash functions
- Samples random graphs
- Optimal space consumption but 2^n construction speedup compared to brute-force
- Up to 76 times faster in practice
- /ByteHamster/ShockHash

- Future work
  - Improve query performance
  - Sample bipartite graphs [LSW23a]

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