Derandomization

Two possible views

- We already know a randomized algorithm.
  ⇝Modify it to get a deterministic algorithm
- We have a problem but no algorithm
  ⇝Probabilistics as a design and analysis tool

Two Basic Approaches

- Method of Conditional Probabilities
- Reduce the number of random bits.
  Then try all possible random choices.
  Example: Tuesday Jan. 7, 2003 14:15, Room 024.
  See also Two-Point Samplin, Motwani Raghavan Section 3.4

1 The Method of Conditional Probabilities

Example: A Set Coloring Problem

[Manber, Introduction to Algorithms, Addison Wesley 1989]

Consider a base set $S = \{x_1, \ldots, x_n n\}$,
subsets $S_1, \ldots, S_k$ of size r with $k \leq 2^{r-2}$.

Color each element of S red or blue. such that all S_i contain both a red element and a blue element.

1.1 Randomized Set Coloring

Color all elements red or blue with probability $1/2$.

$f_i = \mathbb{P}[S_i \text{ is monochromatic }] = \mathbb{P}[S_i] + \mathbb{P}[S_i] = 2^{-r+1}$

Total failure probability $\leq F(n) = \sum_{i=1}^{k} f_i = k2^{-r+1} = \frac{1}{2}$

1.2 Deterministic Set Coloring

Call S_i red iff S_i contains a red element and no blue element
Call S_i blue iff S_i contains a blue element and no red element

Define f_i:= $\begin{cases} 2^{-r} & \text{if } S_i \text{ is red or blue and has } r \text{ uncolored elements} \\ 2^{-r+1} & \text{if } S_i \text{ has } r \text{ uncolored elements} \\ 0 & \text{if } S_i \text{ has both red and blue elements} \end{cases}$

for j := 1 **to** n **do do**
  invariant $F(n) = \sum_{i=1}^{k} f_i < 1$
  δ_R := 0 −− change in F if we would color x_j red
  foreach i with $x_j \in S_i$ **do**
    if S_i is red **then** δ_R := $\delta_R + f_i$
    if S_i is blue **then** δ_R := $\delta_R - f_i$
  if $\delta_R \leq 0$ **then** color x_j red
  else color x_j blue

Why does it work?

F is an upper bound on the failure probability if we color the remaining elements randomly.

Hence, it suffices to show that the invariant is maintained

$\delta_R = -\delta_B$ Hence, F cannot increase.