Randomized Algorithms, WS 2002/03
Problem Set 3
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Lecturers:
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This homework is due on Friday, Dec. 13.

Problem 3-1 (Multiple Choice Allocation)
$n = km$ balls are to be allocated to m bins for some integer k. For each ball we have the choice between *two* bins choosen independently and uniformly at random. We assume that k is a constant, i.e., independent of m and we analyze the situation as $m \longrightarrow \infty$. Show that with probability $1 - o(1)$ it is *not* possible to allocate the balls to the bins such that the maximum occupancy is k. Hint: Think about what happens if there is a bin that cannot receive any balls.

Problem 3-2 (Buffered writing)
We want to design a simplified version of the writing component of an emulation of the multihead model on the parallel disk model. Suppose we have D physical disks. In one output step we can physically *output* one data block to each of the physical disks. In each round of emulation, we want to logically *write* $3D$ logical blocks. A block that is logically written must be stored in internal memory until it is physically *output* to two target disks. Assume that all written blocks have independent and uniformly distributed random target disks.

Explain how to implement writing such that T rounds of the emulation algorithm are executed in an expected number of at most $9T$ output steps. You are allowed to use an internal buffer capable of storing $O(D \log D)$ blocks. What is the constant factor in the $O(\cdot)$? Hint: group logarithmically many rounds in a similar way as the parallel priority queue insertion algorithm.