Randomized Algorithms, WS 2002/03
Problem Set 5
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Lecturers:
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This homework is due on Friday, Jan. 24.

Problem 5-1 Show that random walks on nonbipartite connected undirected graphs define an aperiodic Markov chain.

Problem 5-2 Specify a family of *directed* graphs with n nodes that have a cover time that is exponential in n. Explain why the cover time is exponential.

Problem 5-3 Explain in detail how the deterministic set coloring algorithm from the lecture can be implemented to run in time $O(\sum_{i=1}^{k} |S_i|)$.

Problem 5-4 Devise a linear time Monte Carlo algorithm for testing whether two rooted trees are isomorphic. Exercise 7.2 in Motwani Raghavan gives a more detailed explanation and some hints. What remains to be done:

- explain why the polynomials defined there are identical iff the two trees are isomorphic
- invoke a theorem from the lecture that allows you to reduce testing of polynomial identities to polynomial evaluation.
- explain how the polynomials defined there can be evaluated efficiently.

Problem 5-*** (optional — if you solve this *graph isomorphism problem* you will get famous ;-) Generalize Problem 5-4 for arbitrary graphs, i.e., define a mapping from arbitrary undirected graphs to polynomials such that the resulting polynomials are identical iff the two graphs are isomorphic. It should be possible to evaluate the polynomials in time polynomial in the size of the graphs.