Randomized Algorithms, WS 2002/03
Problem Set 2
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Lecturers:
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This homework is due on Friday, Nov. 29.

Problem 2-1 (Game Theory)
Determine the value V of the following game and give optimal mixed strategies for the two players.

$$\begin{pmatrix} 5 & 6 \\ 7 & 4 \end{pmatrix}$$

Problem 2-2 (Lower Bounding)
Use Yao's Minimax Principle to prove a lower bound on the expected running time of any Las Vegas algorithm for sorting n numbers.
Hint: Any deterministic comparison-based sorting algorithm corresponds to a decision tree in which each leaf corresponds to a different input order. Argue about the number of leaves of this tree and the average height of a leave.

Problem 2-3 (Number of Empty Bins)
m balls are randomly thrown into n bins. Let the random variable Y denote the number of empty bins. Show that $\mathbf{E}[Y] \leq ne^{-\frac{m}{n}}$

Problem 2-4 (Maximum Occupancy)
m balls are randomly thrown into n bins. Let the random variable $L_{\max}$ denote the maximum occupancy of any bin. Show that $\mathbf{E}[L_{\max}] = \frac{m}{n} + O(\sqrt{\frac{m}{n}\log n})$ if $m = \Omega(n \log n)$.