Advanced Data Structures

Lecture 08: Temporal Data Structures

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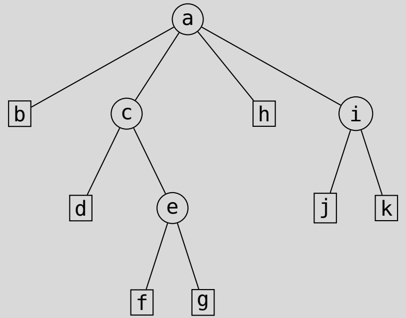
The Project

- example inputs are online
- bv: size of initial bit vector
- tree: full tree with depth *d* and *c* children
- outputs next week(?)
- representation of nodes

```
ab cd ef g   h ij k
(()(()(()()))()(()()))
```



$insertchild(T, v, i, k)$

- insert new *i*-th child of node *v* such that
- the new node becomes parent of
- the previously *i*-th to $(i + k - 1)$-th child of *v*

- boost's dynamic bit_set? yes
- for competition: space in *bits* and time in *ms*

PINGO

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https://pingo.scc.kit.edu/311809

Recap: Improving Compressed Suffix Arrays

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Lemma: Decoding Time Improved CSA

An *SA* value can be decoded in $O(\log \log n)$ time using the improved CSA

Proof (Sketch)

- on each level, odd *SA* values can be decoded using the recursive *SA*
- there are at most $\log \log n$ levels
- on each level, even *SA* values can be decoded in one step, as the next *SA* value is odd

- requires rank and select data structures

	1	2	3	4	5	6	7	8	9	10	11	12	13
T	a	b	a	b	c	a	b	c	a	b	b	a	$
SA	13	12	1	9	6	3	11	2	10	7	4	8	5
Ψ	-	1	8	9	10	11	2	6	7	12	13	4	5
NEW	13	1	9	3	11	7	5	1	10	6	7	13	4
BV	1	0	1	1	0	1	1	0	0	1	0	0	1

Temporal Data Structures

- data structure that allows updates
- queries only on the newest version
- what happens to old versions

- keep old versions around
- in a “clever” way
- lecture based on: `http://courses.csail.mit.edu/6.851/spring12/lectures/L01`

Persistence

- change in the past creates new branch
- similar to version control
- everything old/new remains the same

Retroactivity

- change in the past affects future
- make change in earlier version changes all later versions

Model of Computation

Definition: Pointer Machine

- nodes containing $d = O(1)$ fields
- one root node
- operations in $O(1)$ time
  - new node
  - x = y.field
  - x.field = y
  - x =y+z
- access nodes by root.x.y. . . .

- example on the board

- add additional functionality to existing data structures
- is this a "useful" model? **PINGO**
- balanced binary search tree
- linked list
- . . .

Persistence

- keep all versions of data structure
- never forget an old version
- updates create new versions ℹ e.g., insert/delete
- all operations are relative to specific version

Definition: Partial Persistence

Only the latest version can be updated

- versions are linearly ordered
- old versions can still be queries

Definition: Full Persistence

Any version can be updated

- versions form a tree
- updates on old versions create branch

Definition: Confluent Persistence

Like full persistence, but two versions can be combined to a new version

Definition: Functional

Nodes cannot be modified, only new nodes can be created

Partial Persistence (1/3)

Lemma: Making DS Partially Persistent

Any pointer-machine data structure with $\leq p = O(1)$ pointers to any node can be made partially persistent with

- $O(1)$ amortized factor overhead and
- $O(1)$ additional space per update

Proof (Sketch: Idea)

- store original data and pointer (read only)
- store back pointers to latest version
- store $\leq 2p$ modifications to fields
  - modification $= (\textit{version}, \textit{field}, \textit{value})$
- version v: apply modification with version $\leq v$

Proof (Sketch: Functionality)

- read version v
  - look up all modifications $\leq v$
  - if old version go through old version pointer
- write version
  - if node is not full add modification
  - if node n is full
    - create new node n'
    - copy latest version to data fields
    - copy back pointers to n'
    - for every node x such that n points to x redirect its pack pointers to n'
    - for every node x pointing to n call recursive change of pointer to n'

Partial Persistence (2/3)

Proof (Sketch: Space)

- adding only constant number of back pointers
- adding only constant number of modifications
- total additional space is $O(1)$

Proof (Sketch: Time)

- read is constant time
- write requires amortized analysis

- potential function Φ
- amortizes_cost$(n) = $ cost$(n) + \Delta\Phi$

Proof (Sketch: Time cnt.)

- potential
  $\Phi = c \cdot \sum$ #modifications in latest version
- change of potential by adding new modification
- change of potential by creating new node
- combined:

$$\text{amortized\_cost} \leq c + c - 2cp + p \cdot \text{recursion}$$

- first c: constant time checking
- second c: adding new modification
- remaining part if new node is created
- total amortized time: $O(1)$

Partial Persistence (3/3)

Lemma: Making DS Partially Persistent

Any pointer-machine data structure with $\leq p = O(1)$ pointers to any node can be made partially persistent with

- $O(1)$ amortized factor overhead and
- $O(1)$ additional space per update

- possible in $O(1)$ worst case time [Bro96]

- also possible for full persistence? **PINGO**

Full Persistence (1/4)

Differences

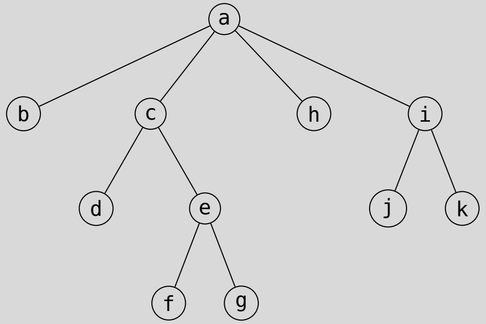
- versions are no longer numbers
- versions are nodes in a tree

- can we represent versions in a linear fashion?

PINGO

```
ab cd ef g   h ij k
(()(()(()()))()(()()))
```

$b_a b_b e_b b_c b_d e_d \ldots$



- versions change
- update in constant time?

Order-Maintenance Data Structure

Linked List

- insert before or after element in $O(1)$ time
- check if u is predecessor of v in n time

Balanced Search Tree

- insert before or after element in $O(\log n)$ time
- check if u is predecessor of v in $O(\log n)$ time

Order-Maintenance DS [DS87]

- insert before or after element in $O(1)$ time
- check if u is predecessor of v in $O(1)$ time
- how is 

- linearized version tree in order-maintenance DS
- insert in $O(1)$ time
  - new version v of u
  - after b_u
  - before e_u
- check order of versions in $O(1)$ time
- maintain and check linearized version tree in $O(1)$ time
- important for applying modifications to fields

Full Persistence (2/4)

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Lemma: Making DS Fully Persistent

Any pointer-machine data structure with $\leq p = O(1)$ pointers to any node can be made fully persistent with

- $O(1)$ amortized factor overhead and
- $O(1)$ additional space per update

Proof (Sketch: Idea)

- store original data and pointer (read only)
- store back pointers to all versions
- store $\leq 2(d + p + 1)$ modifications to fields
  - modification $= (\mathit{version}, \mathit{field}, \mathit{value})$
- version v: look at ancestors of v

Proof (Sketch: Functionality)

- read version v
  - look up all modifications $\leq v$
  - if old version go through old version pointer
- write version
  - if node is not full add modification
  - the same if node is full? **PINGO**
  - if node n is full
    - split node into two
    - each new node contains half of modifications
    - modifications are tree
    - partition tree
    - apply all modifications to “subtree”
    - recursively update pointers

Full Persistence (3/4)

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Proof (Sketch: Space)

- if no split no additional memory
- if split $O(1)$ memory

Proof (Sketch: Time)

- applying versions in $O(1)$ time
- there are $\leq 2(d+p)+1$ recursive pointer updates
- potential

$$\Phi = -c \cdot \sum \#\text{empty modification slots}$$

Proof (Sketch: Time cnt.)

- if node is split $\Delta\Phi = -c \cdot 2(d+p+1)$
- if node is not split $\Delta\Phi = c$
- combined:

$$\begin{aligned}\text{amortized\_cost} = c &+ c \\ &- 2c(d+p+1) \\ &+ (2(d+p)+1) \cdot \text{recursions}\end{aligned}$$

- if node is split constants cancel each other out

Full Persistence (4/4)

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Lemma: Making DS Fully Persistent

Any pointer-machine data structure with $\leq p = O(1)$ pointers to any node can be made fully persistent with

- $O(1)$ amortized factor overhead and
- $O(1)$ additional space per update

- versions are represented by tree
- tree has pointers to order-maintenance DS
- order-maintenance DS has pointers to tree

- de-amortization is open problem

Confluent Persistence

- hard because concatenation
- linked list concatenate with itself
- after u version length 2^u

- more information:
  https://ocw.mit.edu/courses/6-851-advanced-data-structures-spring-2012/pages/calendar-and-notes/

Conclusion and Outlook

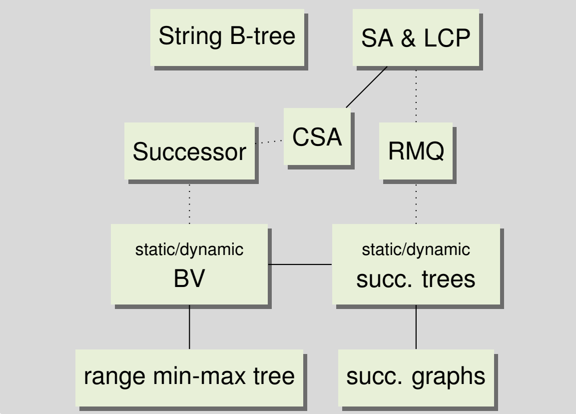
This Lecture

- partial and full persistent data structures

Next Lecture

- retroactive data structures

Advanced Data Structures



Bibliography I

[Bro96] Gerth Stølting Brodal. "Partially Persistent Data Structures of Bounded Degree with Constant Update Time". In: *Nord. J. Comput.* 3.3 (1996), pages 238–255.

[DS87] Paul F. Dietz and Daniel Dominic Sleator. "Two Algorithms for Maintaining Order in a List". In: *STOC*. ACM, 1987, pages 365–372. DOI: 10.1145/28395.28434.