

Advanced Data Structures

Lecture 10: Orthogonal Range Searching

Florian Kurpicz

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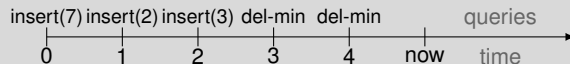
<https://pingo.scc.kit.edu/142117>

Recap: Retroactive Data Structures

Operations

- $\text{INSERT}(t, \text{operation})$: insert operation at time t
- $\text{DELETE}(t)$: delete operation at time t
- $\text{QUERY}(t, \text{query})$: ask query at time t

- for a priority queue updates are
 - insert
 - delete-min
- time is integer ⓘ for simplicity otherwise use order-maintenance data structure

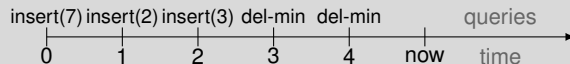


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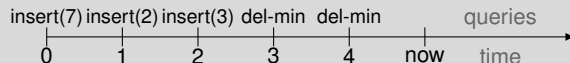
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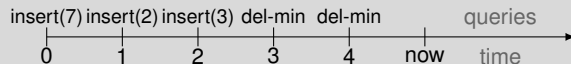
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Definition: Partial Retroactivity

QUERY is only allowed for $t = \infty$ ⓘ now

Definition: Full Retroactivity

QUERY is allowed at any time t

Definition: Nonoblivious Retroactivity

INSERT, DELETE, and QUERY at any time t but also identify changed QUERY results

Motivation: Query Set of Points

- given set of points $P = \{p_1, \dots, p_n\}$ with $p_i = (x_i, y_i)$
 - find all points in $[x, y] \times [x', y']$
 - higher dimensions are possible
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- think about database queries
 - each dimension is a property
 - searching for objects fulfilling all properties of range

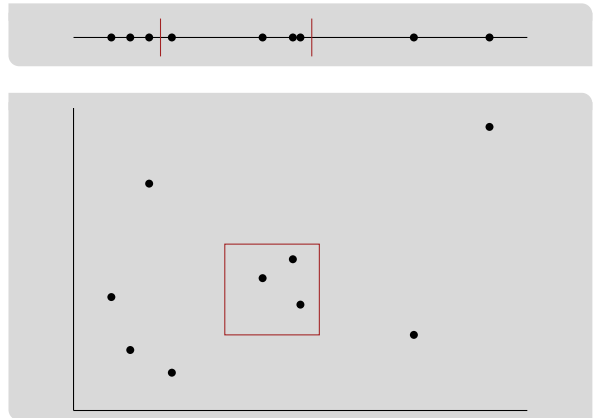
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1-Dimensional Range Searching (1/2)

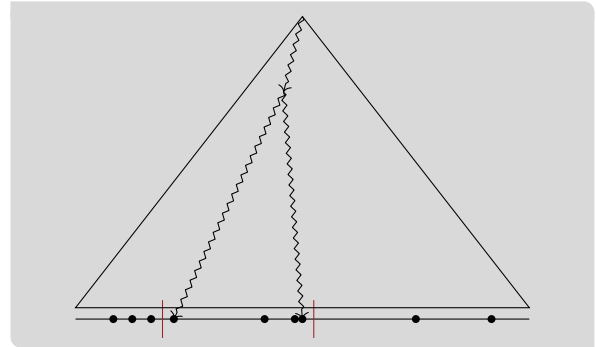
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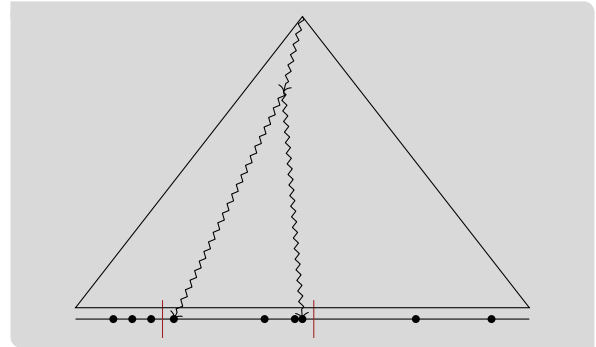
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- build BBST where each leaf contains a point
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- query for both x and x'
 - find leaves b and e for x and x'
 - let node v be node where paths to leaves split
 - report all leaves between b and e



1-Dimensional Range Searching (2/2)

- how long does it take to report all children of a subtree with k leaves in a BBST?  **PINGO**


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Lemma: 1-Dimensional Range Searching

Let P be a set of n 1-dimensional points. P can be stored in a BBST that requires $O(n)$ words space, can be constructed in $O(n \log n)$ time, and can answer range searching queries in $O(\log n + occ)$ time

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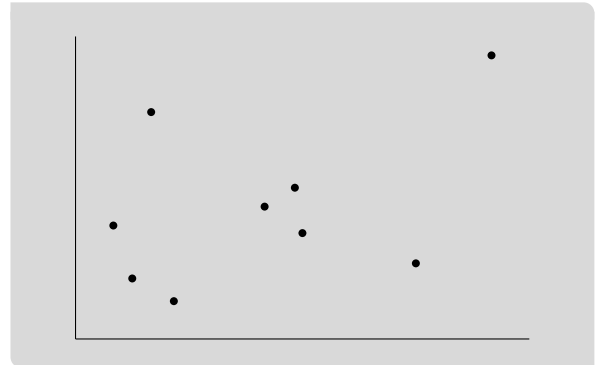
Proof (Sketch Time)

- reporting all children in a subtree requires $O(occ)$ time
- BBST has depth $O(\log n)$
- search paths starting at v have length $O(\log n)$
- report all subtrees to the right of the left path
- report all subtrees to the left of the right path

2-Dimensional Rectangular Range Searching

Important

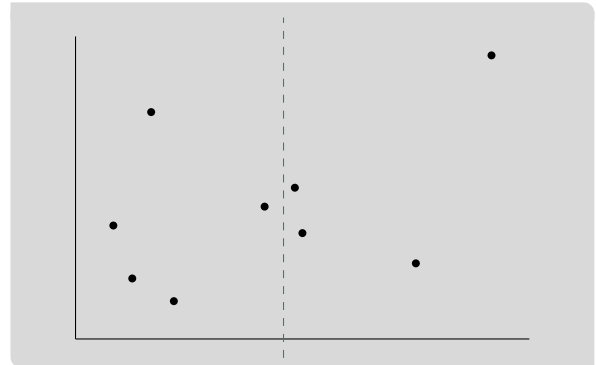
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- generalize 1-dimensional idea
- 1-dimensional
 - split number of points in half at each node
 - points consist of one value
- 2-dimensional
 - points consist of two values
 - split number of points in half w.r.t. one value
 - switch between values depending on depth



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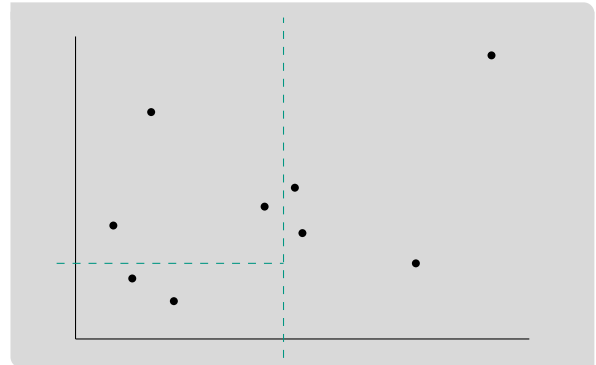
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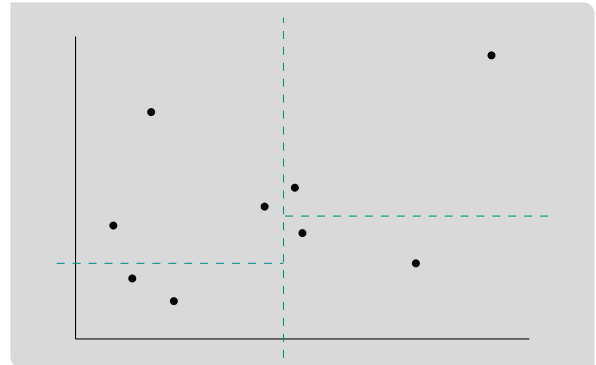
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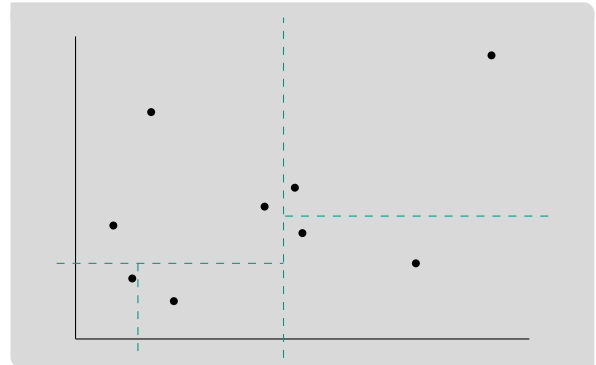
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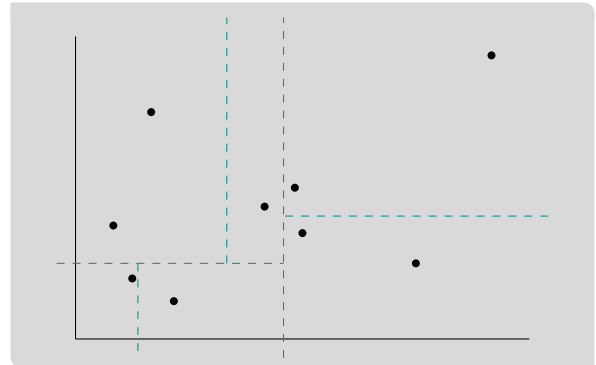
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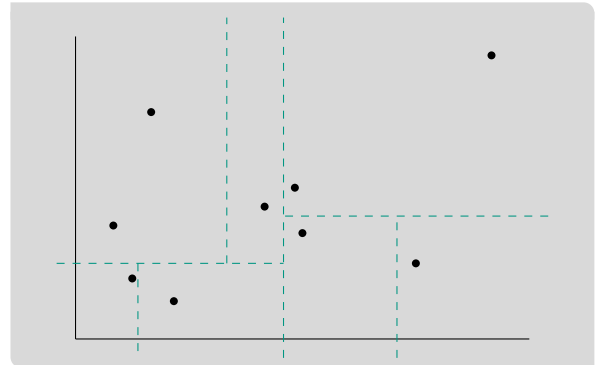


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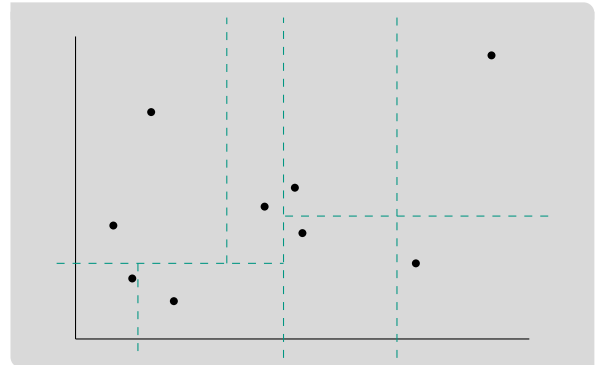
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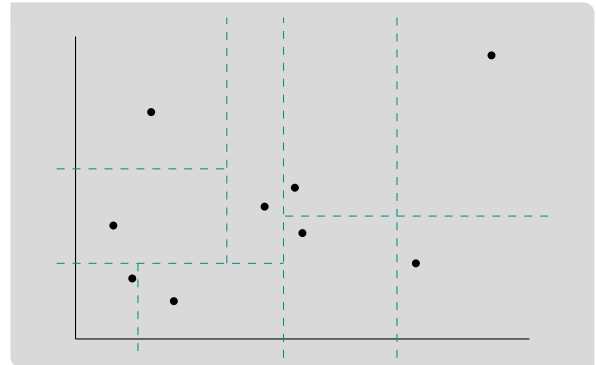
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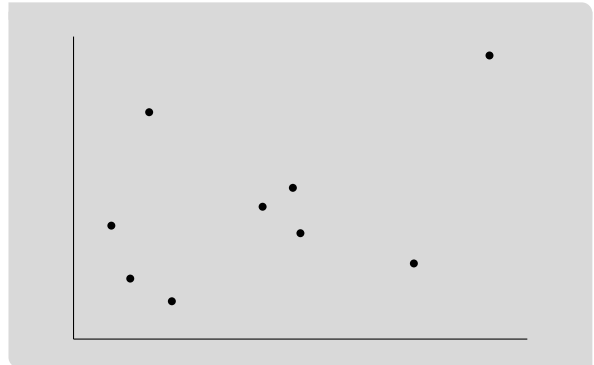
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Kd-Trees (1/4)

- considering the 2-dimensional case
- each inner node at an even depth
 - splits the leaves in its subtree in half
 - using the x -coordinate
- each inner node at an odd depth
 - splits the leaves in its subtree in half
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- until each region contains a single point
- each leaf represents a point

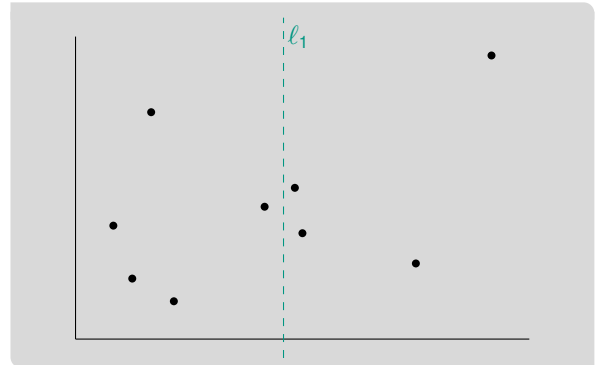
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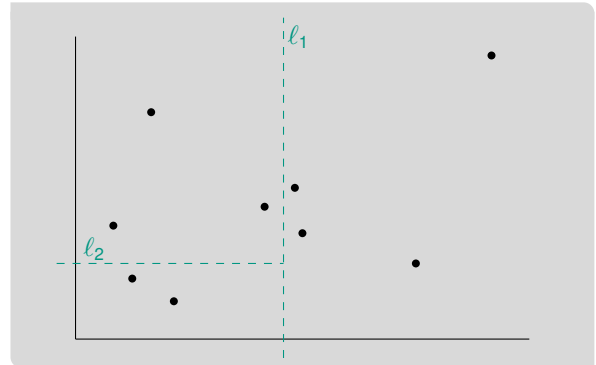
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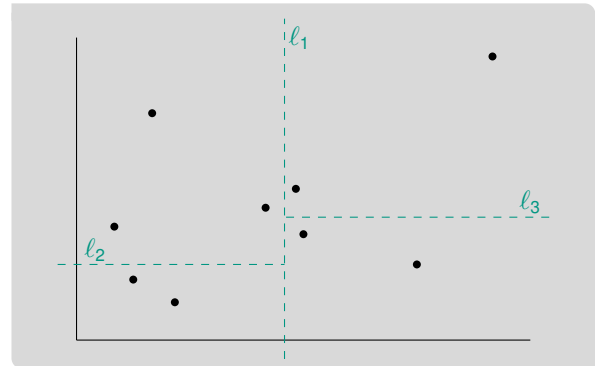
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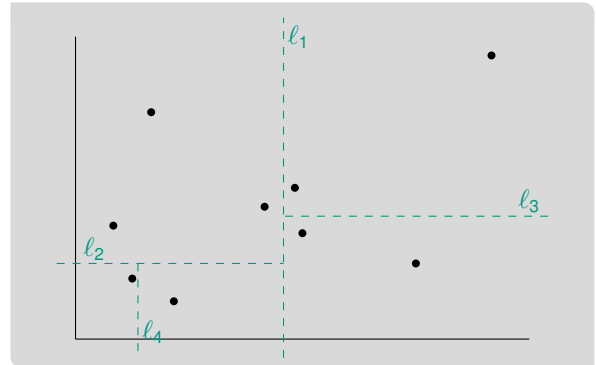
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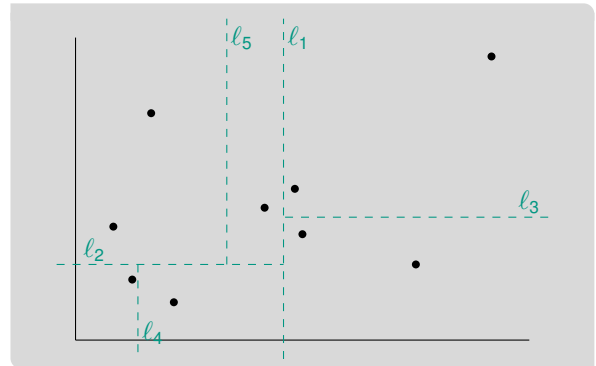
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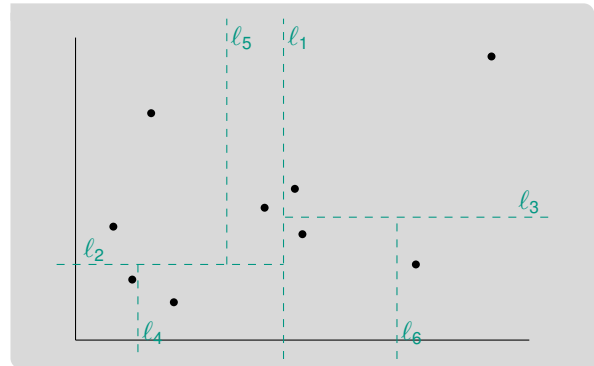
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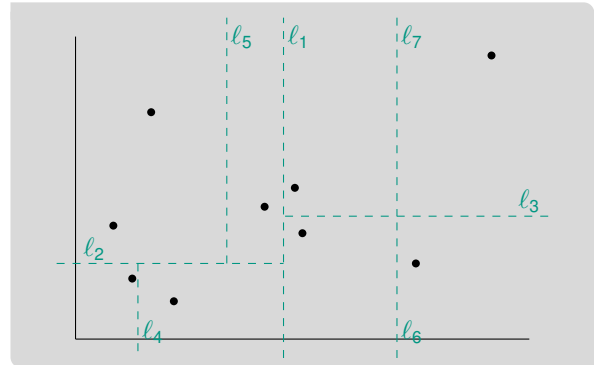
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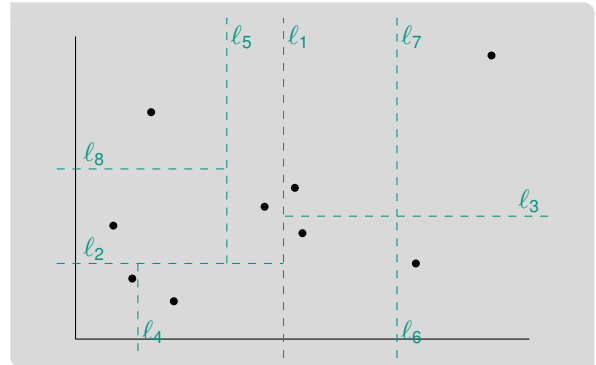
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- there are $O(n)$ leaves
- there are $O(n)$ inner nodes
- a binary tree requires $O(1)$ words per node
- $O(n)$ words total space

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Proof (Sketch: Time)

- finding the splitter is easy due to presorted points
- splitting requires $T(n)$ time with

$$T(n) = \begin{cases} O(1) & n = 1 \\ O(n) + 2T(\lceil n/2 \rceil) & n > 1 \end{cases}$$

- results in $O(n \log n)$ running time
- dominates presorting

Kd-Trees (3/4)

- use splitter depending on depth to identify paths through tree
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- example on the board 

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Proof (Sketch)

- $O(occ)$ time necessary to report points
- look at number of regions intersected by any vertical line
- upper bound for the regions intersected by query (for left and right edge of rectangle)
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Proof (Sketch, cnt.)

- for vertical lines consider every inner node at odd depth
- starting at root's children
- can intersect two regions
- number of nodes is $\lceil n/4 \rceil$  halved at each level
- number of intersected regions is $Q(n)$ with

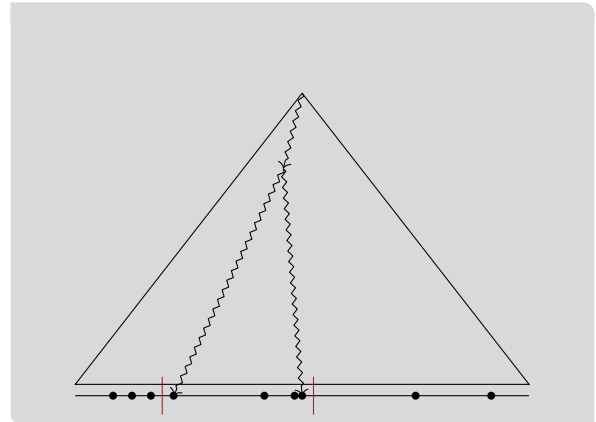
$$Q(n) = \begin{cases} O(1) & n = 1 \\ 2 + 2Q(\lceil n/4 \rceil) & n > 1 \end{cases}$$

- results in $Q(n) = O(\sqrt{n})$
- $O(\sqrt{n} + k)$ total running time

Range Trees (1/4)

- one BBST build on the x -coordinates
 - same as for 1-dimensional queries
- each inner node is associated with a set of points
- build a BBST for the y -coordinates of associated points for each inner node
 - store points in leaves not just y -coordinates
 - this BBST is used for reporting

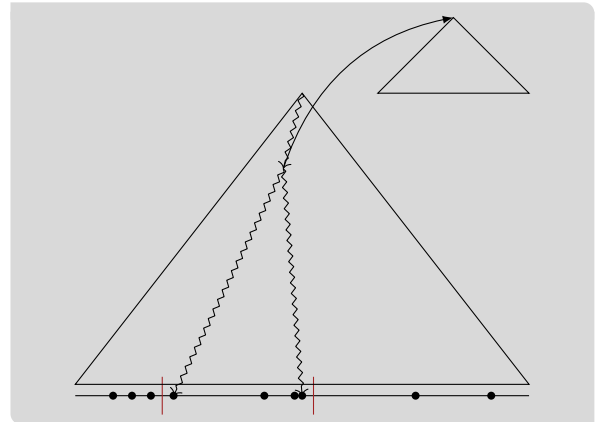
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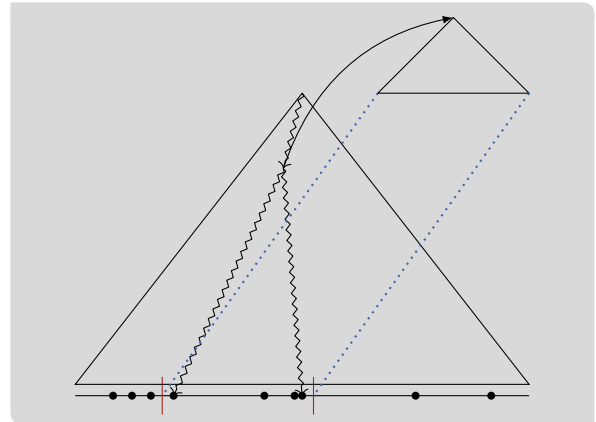
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Proof (Sketch, cnt.)

- all associated BBSTs on each depth contain every point exactly once
- total size of all BBSTs on each depth is $O(n)$
- total space $O(n \log n)$ words

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- all points are represented on **each** depth **exactly** once

Proof (Sketch, cnt.)

- all associated BBSTs on each depth contain every point exactly once
 - total size of all BBSTs on each depth is $O(n)$
 - total space $O(n \log n)$ words
- how much faster is the range tree?

Range Trees (3/4)

- 2-dimensional rectangular range search reduced to two 1-dimensional range searches
- look in BBST for x -coordinates \textcircled{i} same as 1-dimensional case
- instead of reporting subtrees to the right/left of paths search associated BBSTs
- report results in leaves of associated BBSTs

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Proof (Sketch)

- each search in an associated BBST t requires $O(\log n + occ_t)$ time
- $O(\log n)$ associated BSSTs T are searched \textcircled{i} as seen in 1-dimensional case
- total query time $\sum_{t \in T} O(\log n + occ_t)$
- $\sum_{t \in T} O(occ_t) = O(occ)$
- $\sum_{t \in T} O(\log n) = O(\log^2 n)$
- total time: $O(\log^2 n + occ)$

Range Trees (4/4)

- range trees can be generalized to higher dimensions
- for each dimension add an additional associated BBST
- reporting in final BBST
- d -dimensional queries are d 1-dimensional queries

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- recursive query time $Q_d(n)$ with $Q_2(n) = O(\log^2 n)$
- $Q_d(n) = O(\log n) + O(\log n) \cdot Q_{d-1}(n)$
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Proof (Sketch Construction Space)

- recursive space $S_d(n)$ with $S_2(n) = O(n \log n)$ words
- $T_d(n) = O(n \log n) + O(\log n) \cdot T_{d-1}(n)$
- solves to $S_d(n) = O(n \log^{d-1} n)$


Fractional Cascading (1/2)

- sorted sets S_1, \dots, S_m
- $|S_1| = n$ and $S_{i+1} \subseteq S_i$
- report elements in range $[x..x']$ in S_1, \dots, S_m

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- in addition to S_i store pointers to S_{i+1}
- for each element in S_i store pointer to successor in S_{i+1}
- possible because $S_{i+1} \subseteq S_i$ 

Fractional Cascading (2/2)

Lemma: Fractional Cascading

Given sets S_1, \dots, S_m with $|S_1| = n$ and $S_{i+1} \subseteq S_i$, find a range in all S_i 's using fractional cascading requires $O(m + \log n + occ)$ time

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Proof (Sketch)

- binary search on S_1 requires $O(\log n)$ time
- following pointer to S_2 requires $O(1)$ time
- scanning S_2 requires $O(occ)$ time
- following pointer to S_3 requires $O(1)$ time
- repeat m times
- total: $O(m + \log n + occ)$ time

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- how to apply to range trees?
- instead of associated BBSTs store leaf data in arrays for all nodes but root
- each node has associated data
- store **two** successor pointers to the associated data in the left and right child
- two pointers to cover all possible paths
- this is a **layered range tree**

Query Layered Range Trees

- search in BBST for x -coordinates remains the same
- to search y -coordinates first search associated BBST of root
- same as initial binary search for fractional cascading
- continue to follow pointers in associated data and scan to report queries

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Proof (Sketch)

- the initial search requires $O(\log n)$ time
- the search in the associated BBST of the root requires $O(\log n)$ time
- remaining searches in associated data a requires $O(1 + occ_a)$ time
- each point is reported once
- total time: $O(\log n + occ)$

General Sets of Points

- all solutions requires unique x and y -coordinates
- big limitation for applications
- remember database motivation

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 - compare points using
 $(x|k) < (x'|k') \iff x < x' \text{ or } (x = x' \text{ and } k < k')$

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- range queries $[x..x'] \times [y..y']$ become

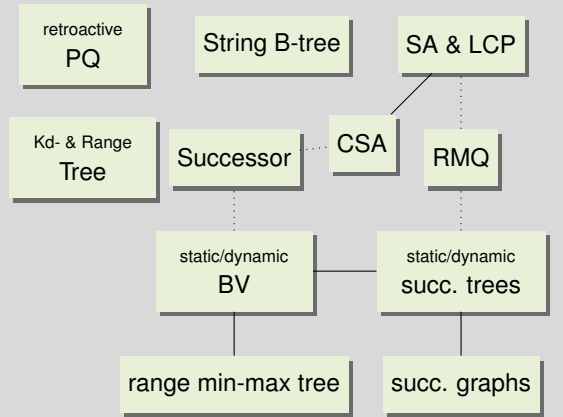
$$[(x|-\infty)..(x'|\infty)] \times (y|-\infty)..[(y'|\infty)]$$

Conclusion and Outlook

This Lecture

- orthogonal range searching

Advanced Data Structures



Conclusion and Outlook

This Lecture

- orthogonal range searching

Next Lecture

- geometric data structures
- Q&A
- results of evaluation

Advanced Data Structures

