Karlsruhe Institute of Technology

Advanced Data Structures

Lecture 11: BSP Trees and Recap

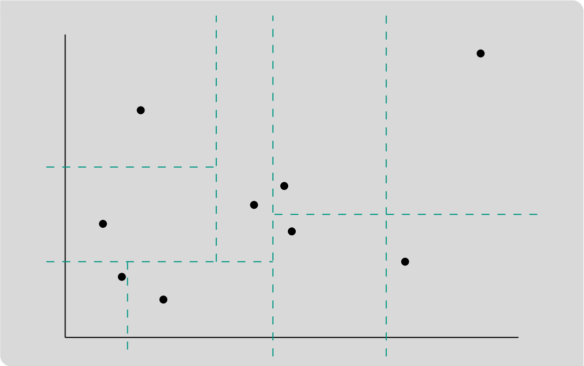
Florian Kurpicz

Recap: 2-Dimensional Rectangular Range Searching

Important

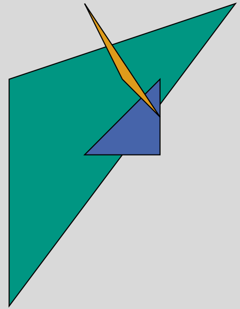
- assume now two points have the same x- or y-coordinate

- generalize 1-dimensional idea
- 1-dimensional
  - split number of points in half at each node
  - points consist of one value
- 2-dimensional
  - points consist of two values
  - split number of points in half w.r.t. one value
  - switch between values depending on depth



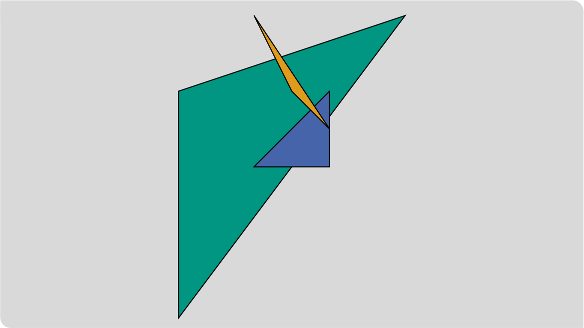
Motivation

- hidden surface removal
- which pixel is visible
- important for rendering



z-Buffer Algorithm

- transform scene such that viewing direction is positive z-direction
- consider objects in scene in arbitrary order
- maintain two buffers
  - frame buffer ℹ currently shown pixel
  - z-buffer ℹ z-coordinate of object shown
- compare z-coordinate of z-buffer and object



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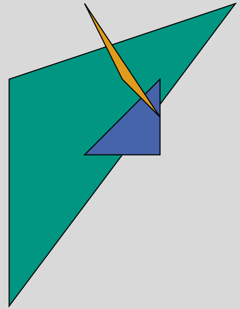
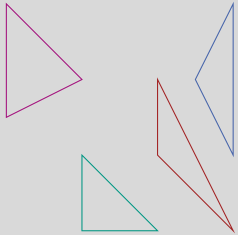
- first sort object in depth-order
- depth-order may not always exist
- how to efficiently sort objects?



BSP Trees (1/2)

- partition space using hyperplanes
- binary partition ℹ similar to kd-tree
- hyperplanes create half-spaces and cut objects into fragments



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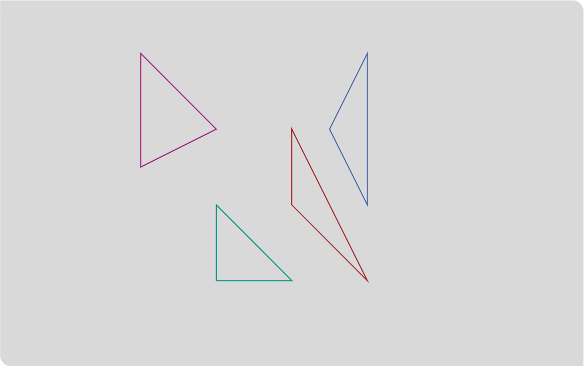

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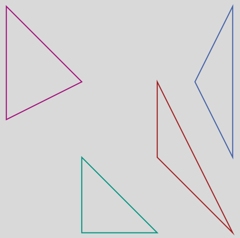

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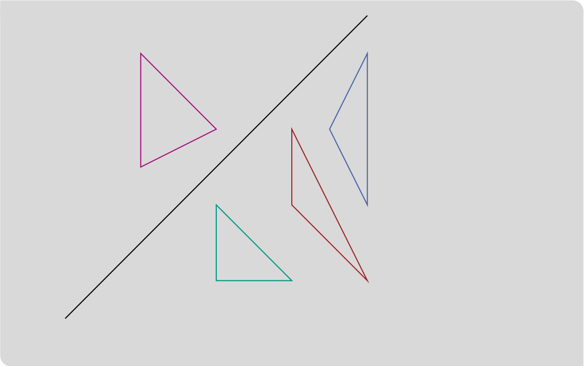

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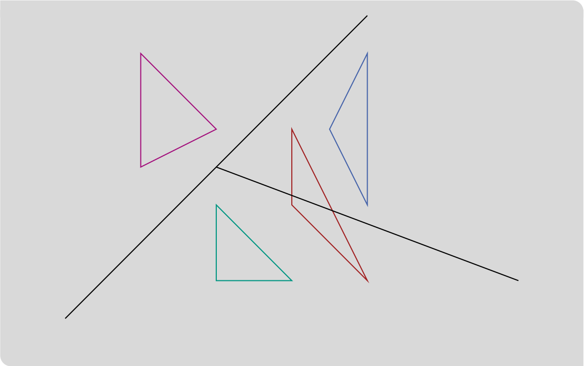

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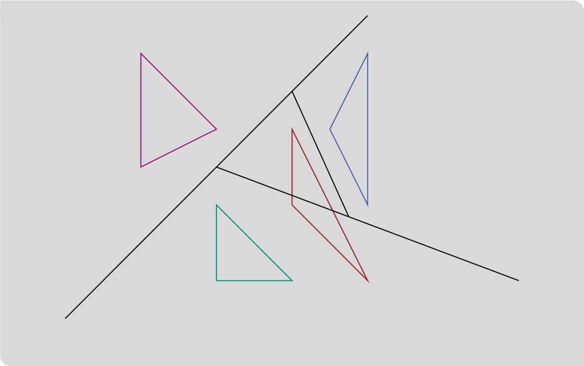

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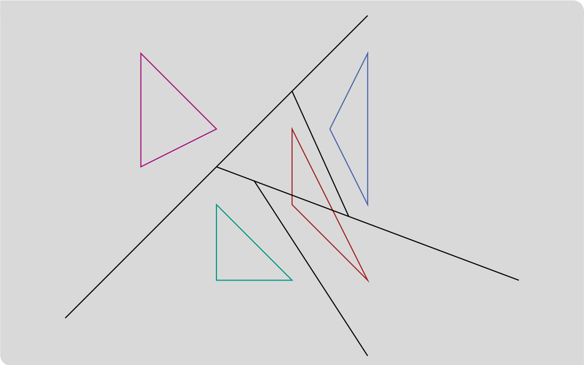

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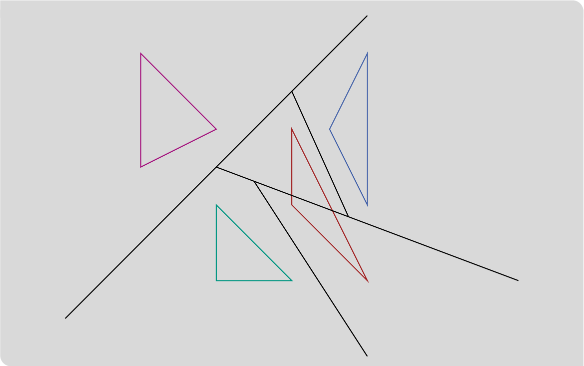
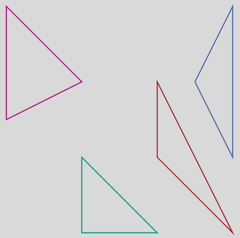
- each split creates two nodes in a tree
- if number of objects in space is one: leaf
- otherwise: inner node



BSP Trees (2/2)

- for leaf: store object/fragment
- for inner node v: store hyperplane h_v and the objects contained in h_v
- left child represents objects in upper half-space h^+
- right child represents objects in lower half-space h^-



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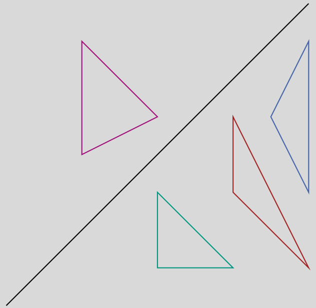

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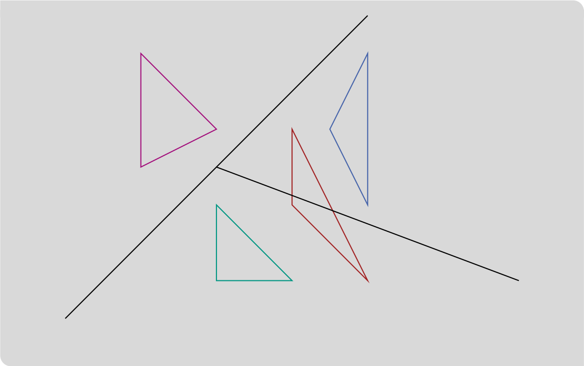

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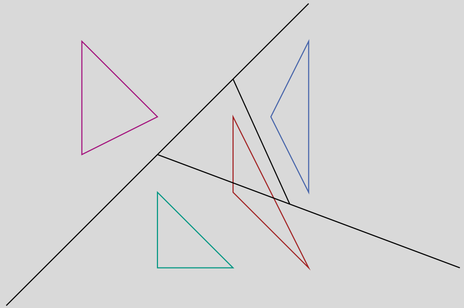

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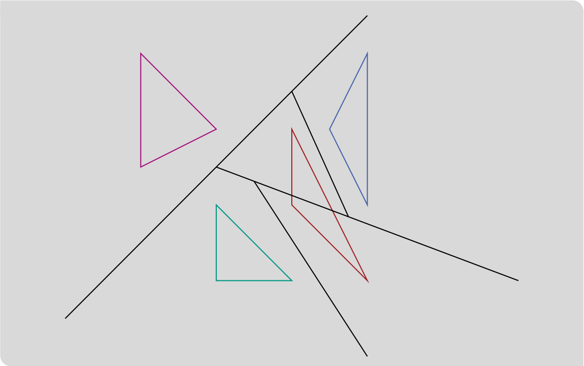

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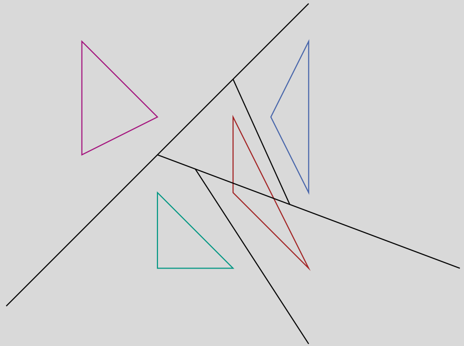
- space of BSP tree is number of objects stored at all nodes
- what about fragments?
- too many fragments can make the tree big



Auto-Partitioning

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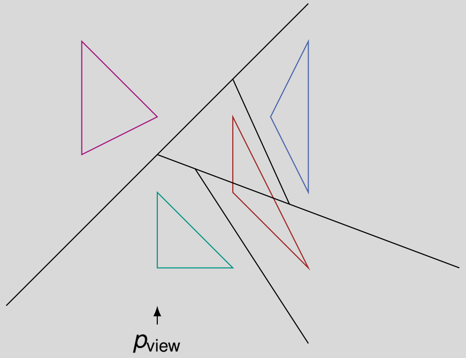
- sorting points for kd-trees worked well
- BSP-tree is used to sort objects in dept-order
- auto-partitioning uses splitters through objects
  - 2-dimensional: line through line segments
  - 3-dimensional: half-plane through polygons

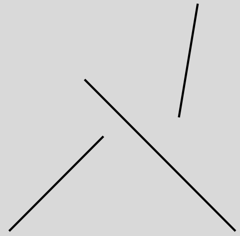
Painter's Algorithm

- consider view point p_{view}
- traverse through tree and always recurse on half-space that does not contain p_{view} first
- then scan-convert object contained in node
- then recurse on half-space that contains p_{view}



Constructing Planar BSP Trees (1/3)

- use auto-partitioning
- construction similar to construction of kd-tree
- store all necessary information
  - hyperplane
  - objects in hyperplane
- how to determine next hyperplane?
- creating fragments increases size of BSP tree



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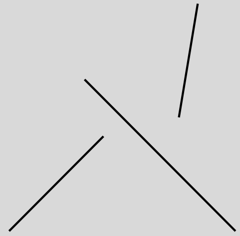
- let s be object and $\ell(s)$ line through object
- order matters



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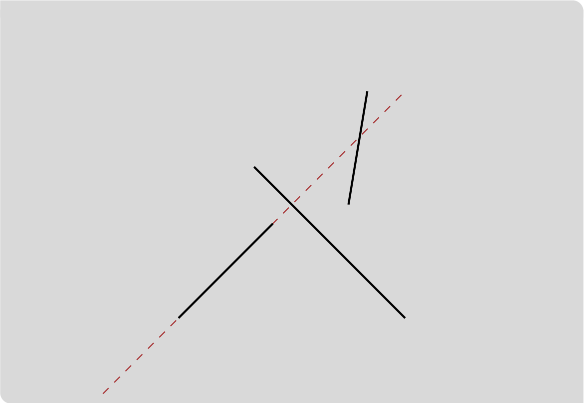

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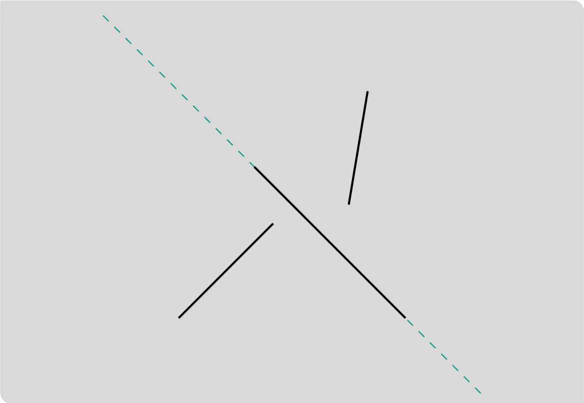

Constructing Planar BSP Trees (2/3)

Lemma: Number Line Fragments

The expected number of fragments generated when iterating through the line segments using a random permutation is $O(n \log n)$

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Proof (Sketch)

- distance of lines $dist_{s_i}(s_j) =$
$$\begin{cases} \text{\# segments inters. } \ell(s_i) \\ \text{ between } s_i \text{ and } s_j & \ell(s_i) \text{ inters. } s_j \\ \infty & \text{otherwise} \end{cases}$$
- example on the board

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Proof (Sketch, cnt.)

- let $dist_{s_i}(s_j) = k$ and $s_{j_1}, \ldots, s_{j_k}$ be segments between s_i and s_j
- what is the probability that $\ell(s_i)$ cuts s_j?
- this happens if no s_{j_x} is processed before s_i
- since order is random

$$\mathbb{P}[\ell(s_i) \text{ cuts } s_j] \leq \frac{1}{dist_{s_i}(s_j) + 2}$$

Constructing Planar BSP Trees (3/3)

Proof (Sketch, cnt.)

- expected number of cuts

$$\mathbb{E}[\text{\# cuts generated by } s_i] \leq \sum_{j \neq i} \frac{1}{dist_{s_i}(s_j) + 2} \leq 2 \sum_{k=0}^{n-2} \frac{1}{k+2} \leq 2 \ln n$$

- all lines generate at most $2n \ln n$ fragments

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Proof (Sketch)

- computing permutation in linear time
- construction is linear in number of fragments to be considered
- number of fragments in subtree is bounded by n
- number of recursions is $n \log n$

Conclusion and Outlook

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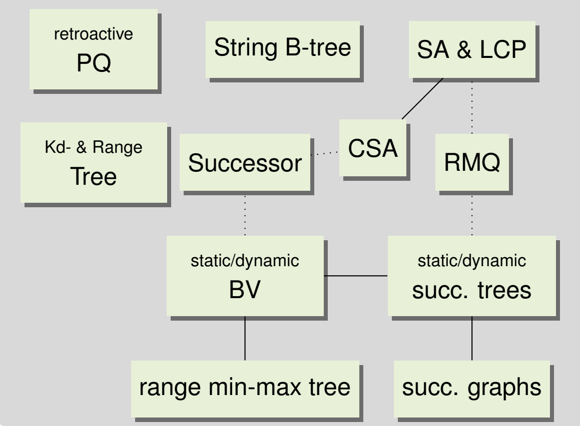
This Lecture

- BSP trees

Advanced Data Structures



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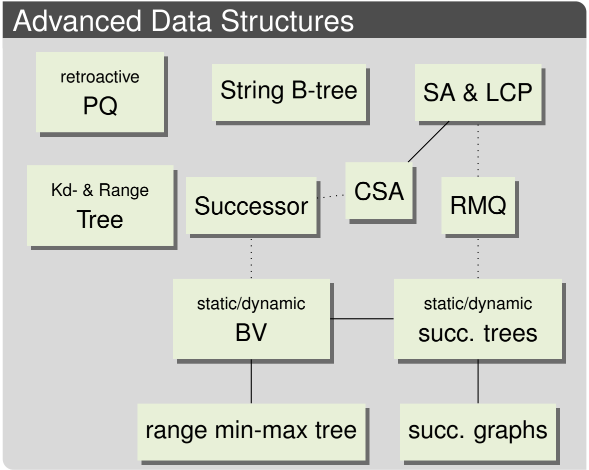
This Lecture

- BSP trees

Next Lecture

- your presentations



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