Karlsruhe Institute of Technology

# Advanced Data Structures

## Lecture 09: Temporal Data Structures

Florian Kurpicz

# PINGO

https://pingo.scc.kit.edu/551581

# Recap: Improving Compressed Suffix Arrays

**Lemma: Decoding Time Improved CSA**

An *SA* value can be decoded in $O(\log \log n)$ time using the improved CSA

**Proof (Sketch)**

- on each level, odd *SA* values can be decoded using the recursive *SA*
- there are at most $\log \log n$ levels
- on each level, even *SA* values can be decoded in one step, as the next *SA* value is odd

- requires rank and select data structures

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *T* | a | b | a | b | c | a | b | c | a | b | b | a | $ |
| *SA* | 13 | 12 | 1 | 9 | 6 | 3 | 11 | 2 | 10 | 7 | 4 | 8 | 5 |
| $\Psi$ | - | 1 | 8 | 9 | 10 | 11 | 2 | 6 | 7 | 12 | 13 | 4 | 5 |
| NEW | 13 | 1 | 9 | 3 | 11 | 7 | 5 | 1 | 10 | 6 | 7 | 13 | 4 |
| *BV* | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |

# Temporal Data Structures

- data structure that allows updates
- queries only on the newest version
- what happens to old versions

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- keep old versions around
- in a “clever” way
- lecture based on: `http://courses.csail.mit.edu/6.851/spring12/lectures/L01`

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## Persistence

- change in the past creates new branch
- similar to version control
- everything old/new remains the same

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## Retroactivity

- change in the past affects future
- make change in earlier version changes all later versions

# Model of Computation

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**Definition: Pointer Machine**

- nodes containing $d = O(1)$ fields
- one root node
- operations in $O(1)$ time
  - new node
  - x = y.field
  - x.field = y
  - x =y+z
- access nodes by root.x.y. . . .

- example on the board

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- is this a “useful” model? **PINGO**

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- . . .

# Persistence

- keep all versions of data structure
- never forget an old version
- updates create new versions ℹ e.g., insert/delete
- all operations are relative to specific version

**Definition: Partial Persistence**

Only the latest version can be updated

- versions are linearly ordered
- old versions can still be queries

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**Definition: Confluent Persistence**

Like full persistence, but two versions can be combined to a new version

**Definition: Functional**

Nodes cannot be modified, only new nodes can be created

# Partial Persistence (1/3)

**Lemma: Making DS Partially Persistent**

Any pointer-machine data structure with $\leq p = O(1)$ pointers to any node can be made partially persistent with

- $O(1)$ amortized factor overhead and
- $O(1)$ additional space per update

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### Proof (Sketch: Idea)

- store original data and pointer (read only)
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  - modification $= (\textit{version}, \textit{field}, \textit{value})$
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- read version $v$
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  - if old version go through old version pointer
- write version
  - if node is not full add modification
  - if node $n$ is full
    - create new node $n'$
    - copy latest version to data fields
    - copy back pointers to $n'$
    - for every node $x$ such that $n$ points to $x$ redirect its pack pointers to $n'$
    - for every node $x$ pointing to $n$ call recursive change of pointer to $n'$

# Partial Persistence (2/3)

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**Proof (Sketch: Space)**

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- $\text{amortizes\_cost}(n) = \text{cost}(n) + \Delta\Phi$

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- amortizes_cost$(n) = \text{cost}(n) + \Delta\Phi$

### Proof (Sketch: Time cnt.)

- potential
  $\Phi = c \cdot \sum \#$modifications in latest version
- change of potential by adding new modification
- change of potential by creating new node
- combined:

$$\text{amortized\_cost} \leq c + c - 2cp + p \cdot \text{recursion}$$

- first $c$: constant time checking
- second $c$: adding new modification
- remaining part if new node is created
- total amortized time: $O(1)$

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- also possible for full persistence? **PINGO**

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## Differences

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- versions are nodes in a tree

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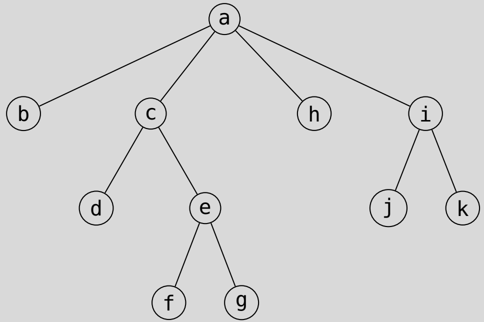

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  **PINGO**

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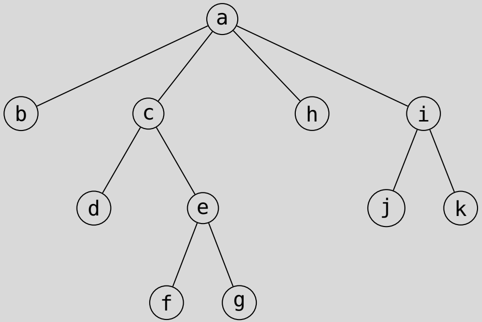

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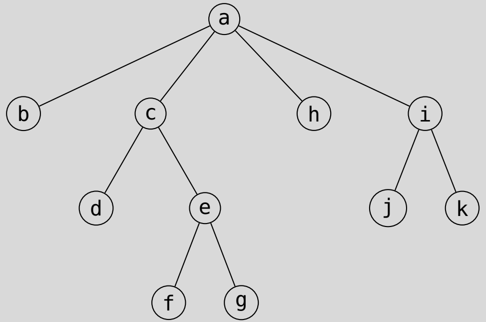


- versions change
- update in constant time?

# Order-Maintenance Data Structure

## Linked List

- insert before or after element in $O(1)$ time
- check if $u$ is predecessor of $v$ in $n$ time

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- linearized version tree in order-maintenance DS
- insert in $O(1)$ time
  - new version $v$ of $u$
  - after $\mathrm{b}_u$
  - before $\mathrm{e}_u$
- check order of versions in $O(1)$ time
- maintain and check linearized version tree in $O(1)$ time
- important for applying modifications to fields

# Full Persistence (2/4)

**Lemma: Making DS Fully Persistent**

Any pointer-machine data structure with $\leq p = O(1)$ pointers to any node can be made fully persistent with

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  - if node $n$ is full
    - split node into two
    - each new node contains half of modifications
    - modifications are tree
    - partition tree
    - apply all modifications to "subtree"
    - recursively update pointers

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- applying versions in $O(1)$ time
- there are $\leq 2(d+p)+1$ recursive pointer updates
- potential

$$\Phi = -c \cdot \sum \#\text{empty modification slots}$$

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**Proof (Sketch: Time cnt.)**

- if node is split $\Delta\Phi = -c \cdot 2(d+p+1)$
- if node is not split $\Delta\Phi = c$
- combined:

$$\begin{aligned}\text{amortized\_cost} = c + c \\ - 2c(d+p+1) \\ + (2(d+p)+1) \cdot \text{recursions}\end{aligned}$$

- if node is split constants cancel each other out

# Full Persistence (4/4)

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- tree has pointers to order-maintenance DS
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- de-amortization is open problem

# Confluent Persistence

- hard because concatenation
- linked list concatenate with itself
- after $u$ version length $2^u$

- more information:
  `https://ocw.mit.edu/courses/6-851-advanced-data-structures-spring-2012/pages/calendar-and-notes/`

# Conclusion and Outlook

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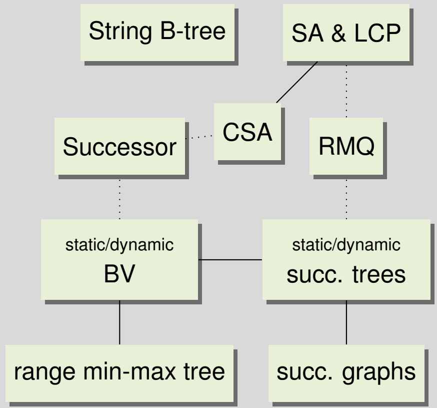
## This Lecture

- partial and full persistent data structures

## Advanced Data Structures



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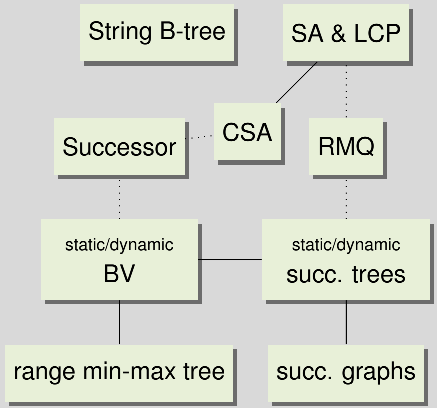
## Next Lecture

- retroactive data structures

## Advanced Data Structures



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[Bro96] Gerth Stølting Brodal. “Partially Persistent Data Structures of Bounded Degree with Constant Update Time”. In: *Nord. J. Comput.* 3.3 (1996), pages 238–255.

[DS87] Paul F. Dietz and Daniel Dominic Sleator. “Two Algorithms for Maintaining Order in a List”. In: *STOC*. ACM, 1987, pages 365–372. DOI: 10.1145/28395.28434.