Karlsruhe Institute of Technology

Advanced Data Structures

Lecture 06: Temporal Data Structures

Florian Kurpicz

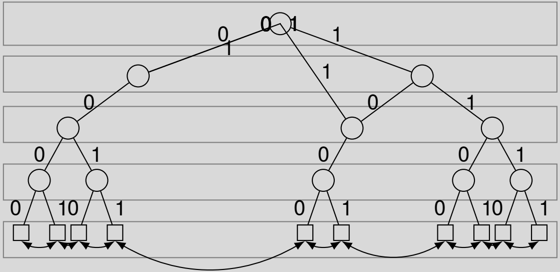
PINGO

https://pingo.scc.kit.edu/551581

Recap: Predecessor and RMQs

Predecessor

- Elias-Fano coding
- y-fast tries



- pointers to *min* and *max* are missing

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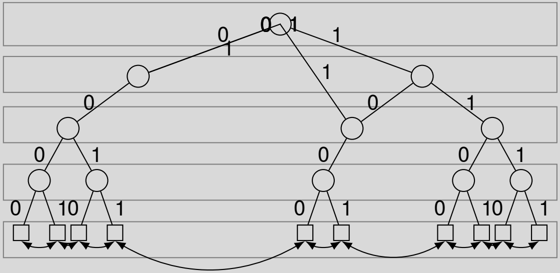
- more to come: learned indices



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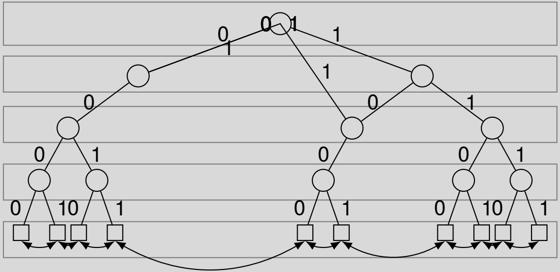

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Range Minimum Queries

- constant time with
  - $O(n^2)$ space
  - $O(n \log n)$ space
  - $O(n)$ space

Temporal Data Structures

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- data structure that allows updates
- queries only on the newest version
- what happens to old versions

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- keep old versions around
- in a “clever” way
- lecture based on: `http://courses.csail.mit.edu/6.851/spring12/lectures/L01`

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Persistence

- change in the past creates new branch
- similar to version control
- everything old/new remains the same

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Retroactivity

- change in the past affects future
- make change in earlier version changes all later versions

Model of Computation

Definition: Pointer Machine

- nodes containing $d = O(1)$ fields
- one root node
- operations in $O(1)$ time
  - new node
  - x = y.field
  - x.field = y
  - x =y+z
- access nodes by root.x.y. . . .

- example on the board

Model of Computation

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- is this a “useful” model? **PINGO**

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- . . .

Persistence

- keep all versions of data structure
- never forget an old version
- updates create new versions ℹ e.g., insert/delete
- all operations are relative to specific version

Definition: Partial Persistence

Only the latest version can be updated

- versions are linearly ordered
- old versions can still be queries

Persistence

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- keep all versions of data structure
- never forget an old version
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Definition: Full Persistence

Any version can be updated

- versions form a tree
- updates on old versions create branch

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Definition: Full Persistence

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Definition: Confluent Persistence

Like full persistence, but two versions can be combined to a new version

Definition: Functional

Nodes cannot be modified, only new nodes can be created

Partial Persistence (1/3)

Lemma: Making DS Partially Persistent

Any pointer-machine data structure with $\leq p = O(1)$ pointers to any node can be made partially persistent with

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Proof (Sketch: Idea)

- store original data and pointer (read only)
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- store $\leq 2p$ modifications to fields
  - modification $= (\mathit{version}, \mathit{field}, \mathit{value})$
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- read version v
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  - if old version go through old version pointer
- write version
  - if node is not full add modification
  - if node n is full
    - create new node n'
    - copy latest version to data fields
    - copy back pointers to n'
    - for every node x such that n points to x redirect its pack pointers to n'
    - for every node x pointing to n call recursive change of pointer to n'

Partial Persistence (2/3)

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- $\text{amortizes\_cost}(n) = \text{cost}(n) + \Delta\Phi$

Proof (Sketch: Time cnt.)

- potential
  $\Phi = c \cdot \sum \#\text{modifications in latest version}$
- change of potential by adding new modification
- change of potential by creating new node
- combined:

$$\text{amortized\_cost} \leq c + c - 2cp + p \cdot \text{recursion}$$

- first c: constant time checking
- second c: adding new modification
- remaining part if new node is created
- total amortized time: $O(1)$

Partial Persistence (3/3)

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- also possible for full persistence? **PINGO**

Full Persistence (1/4)

Differences

- versions are no longer numbers
- versions are nodes in a tree

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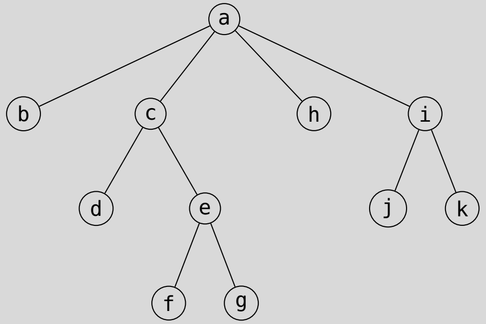

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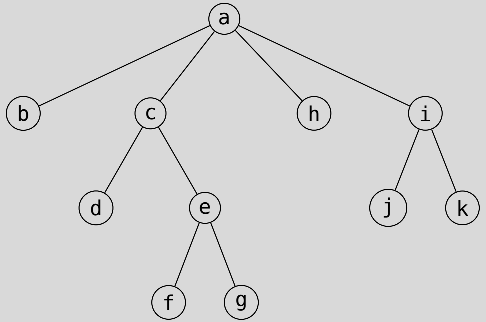
$b_a b_b e_b b_c b_d e_d \ldots$



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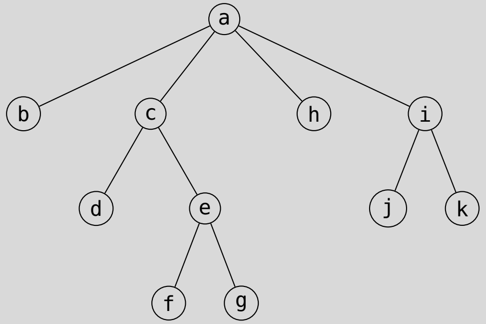

- versions change
- update in constant time?

Order-Maintenance Data Structure

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Linked List

- insert before or after element in $O(1)$ time
- check if u is predecessor of v in n time

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Order-Maintenance DS [DS87]

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Order-Maintenance DS [DS87]

- insert before or after element in $O(1)$ time
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- linearized version tree in order-maintenance DS
- insert in $O(1)$ time
  - new version v of u
  - after b_u
  - before e_u
- check order of versions in $O(1)$ time
- maintain and check linearized version tree in $O(1)$ time
- important for applying modifications to fields

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- store original data and pointer (read only)
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- store $\leq 2(d + p + 1)$ modifications to fields
  - modification $= (\mathit{version}, \mathit{field}, \mathit{value})$
- version v: look at ancestors of v

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  - if node is not full add modification
  - the same if node is full? **PINGO**
  - if node n is full
    - split node into two
    - each new node contains half of modifications
    - modifications are tree
    - partition tree
    - apply all modifications to “subtree”
    - recursively update pointers

Full Persistence (3/4)

Proof (Sketch: Space)

- if no split no additional memory
- if split $O(1)$ memory

Full Persistence (3/4)

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Proof (Sketch: Space)

- if no split no additional memory
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Proof (Sketch: Time)

- applying versions in $O(1)$ time
- there are $\leq 2(d+p)+1$ recursive pointer updates
- potential

$$\Phi = -c \cdot \sum \#\text{empty modification slots}$$

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$$\Phi = -c \cdot \sum \#\text{empty modification slots}$$

Proof (Sketch: Time cnt.)

- if node is split $\Delta\Phi = -c \cdot 2(d+p+1)$
- if node is not split $\Delta\Phi = c$
- combined:

$$\begin{aligned}\text{amortized\_cost} = c + c& \\ -2c(d+p+1)& \\ +(2(d+p)+1) \cdot \text{recursions}&\end{aligned}$$

- if node is split constants cancel each other out

Full Persistence (4/4)

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Full Persistence (4/4)

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- versions are represented by tree
- tree has pointers to order-maintenance DS
- order-maintenance DS has pointers to tree

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- de-amortization is open problem

Confluent Persistence

- hard because concatenation
- linked list concatenate with itself
- after u version length 2^u

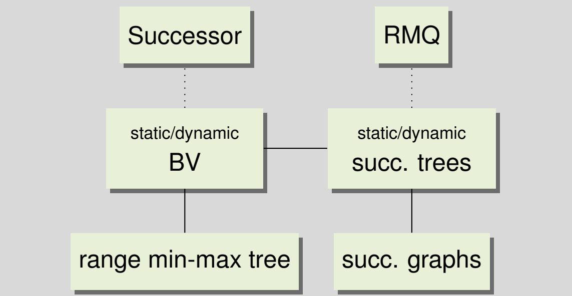
- more information:
  https://ocw.mit.edu/courses/6-851-advanced-data-structures-spring-2012/pages/calendar-and-notes/ 

Conclusion and Outlook

This Lecture

- partial and full persistent data structures

Advanced Data Structures



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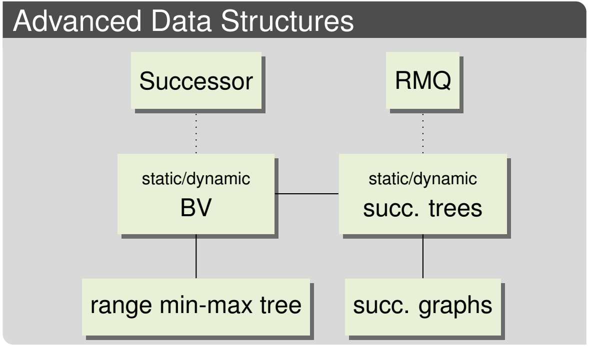
Next Lecture

- substituted by Stefan W.



Project

- bit vectors will have length $> 2^{32}$
- if you want to use a language not listed, write my a mail
  - I will decide by the end of the week
  - depends on the number of additional languages
- time for questions!

Bibliography I

[Bro96] Gerth Stølting Brodal. "Partially Persistent Data Structures of Bounded Degree with Constant Update Time". In: *Nord. J. Comput.* 3.3 (1996), pages 238–255.

[DS87] Paul F. Dietz and Daniel Dominic Sleator. "Two Algorithms for Maintaining Order in a List". In: *STOC*. ACM, 1987, pages 365–372. DOI: 10.1145/28395.28434.