Advanced Data Structures

Lecture 07: Packed and Compressed Hash Tables

Florian Kurpicz

New Topic: External Memory Hash Tables

- now hash tables
- first packed and compressed hash table
- presented in January ’23 at ALENEX

Motivation

Setting

- static hash table for objects of variable size
- storing objects in external memory
- ideally retrieve objects in single I/O
- very small internal memory data structure

Objects of Variable Size

External Memory

- only blocks of size B bits can be transferred
- one I/O per block transfer

Space-Efficient Object Stores from Literature

- objects of size 256 bytes
- blocks of size 4096 bytes
- internal space I_b (bits/block)
- (*) consecutive I/O

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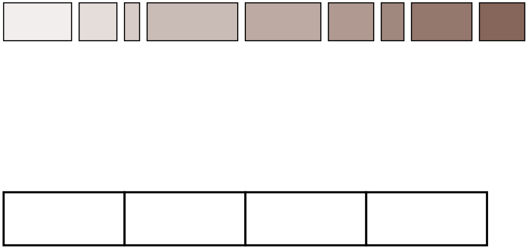
	Method	I_b	load factor	I/Os
fixed	Larson et al. [LR85]	96	$<$96 %	1
	SILT SortedStore [Lim+11]	51	100 %	1
	Linear Separator [Lar88]	8	85 %	1
	Separator [GL88; LK84]	6	98 %	1
	Robin Hood [Cel88]	3	99 %	1.3
	Ramakrishna et al. [RT89]	4	80 %	1
	Jensen, Pagh [JP08]	0	80 %	1.25
	Cuckoo [Aza+94; Pag03]	0	$<$100 %	2
	PaCHash, $a = 1$	2	100 %	2*
	PaCHash, $a = 8$	5	100 %	1.13*
variable	SILT LogStore [Lim+11]	832	100 %	1
	SkimpyStash [DSL11]	32	$\leq$98 %	8
	PaCHash, $a = 1$	2	99.95 %	2.06*
	PaCHash, $a = 8$	5	99.95 %	1.19*

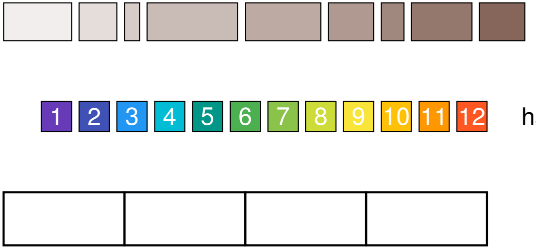
PaCHash Overview



objects of variable size

EM

PaCHash Overview



objects of variable size

hash function $h\colon K \to 1..am$

EM

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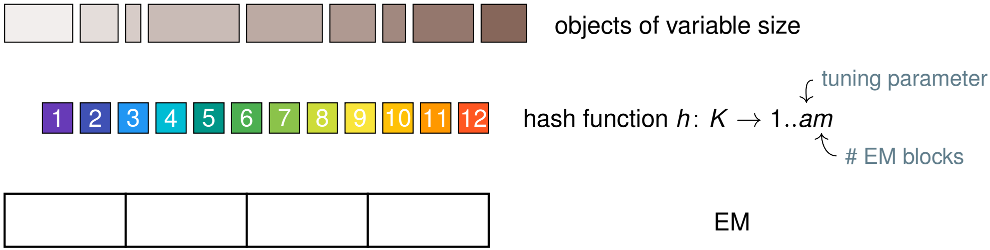

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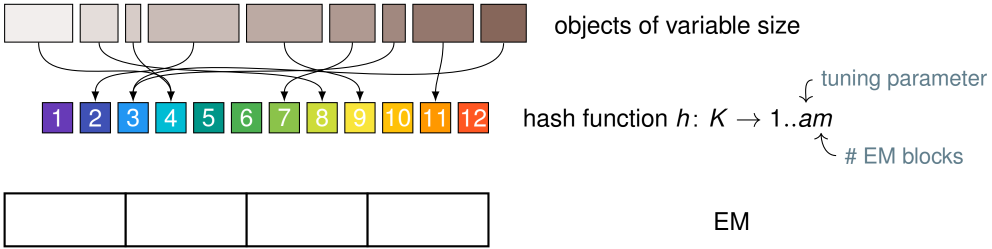

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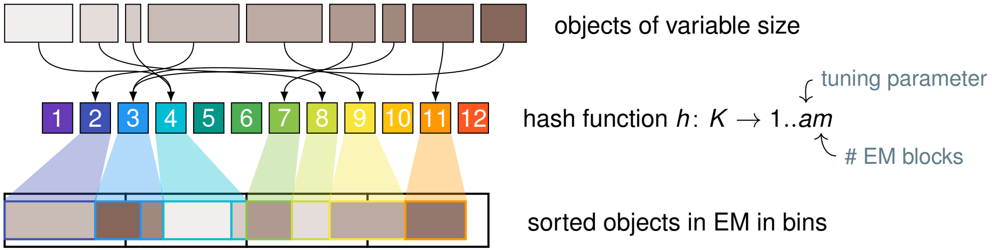

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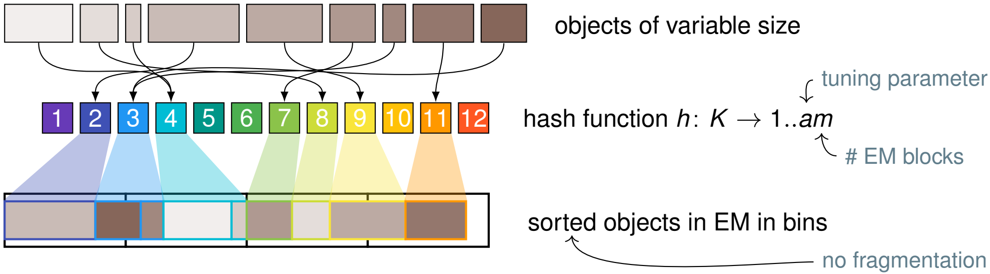

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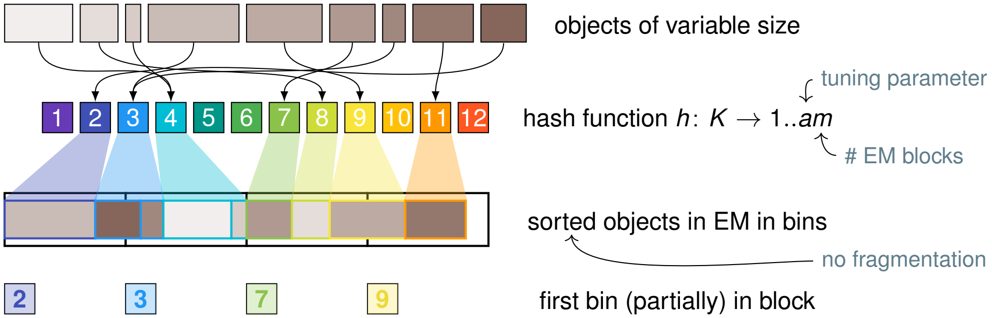

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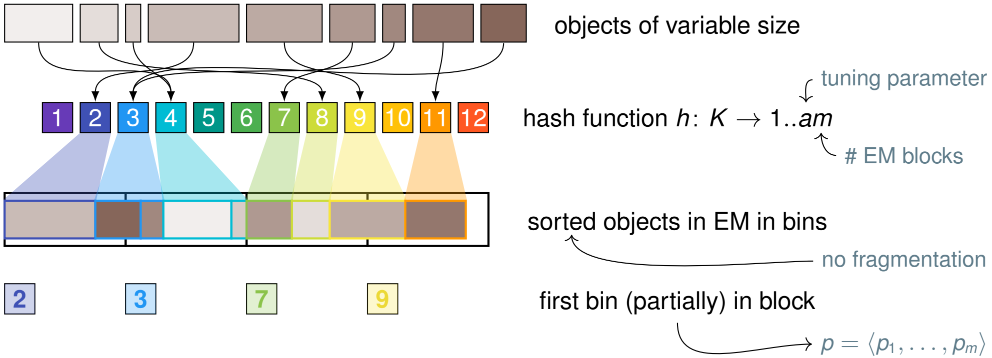

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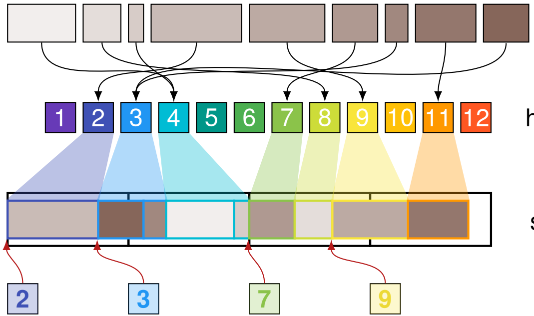


objects of variable size

tuning parameter

hash function $h\colon K \to 1..am$

EM blocks

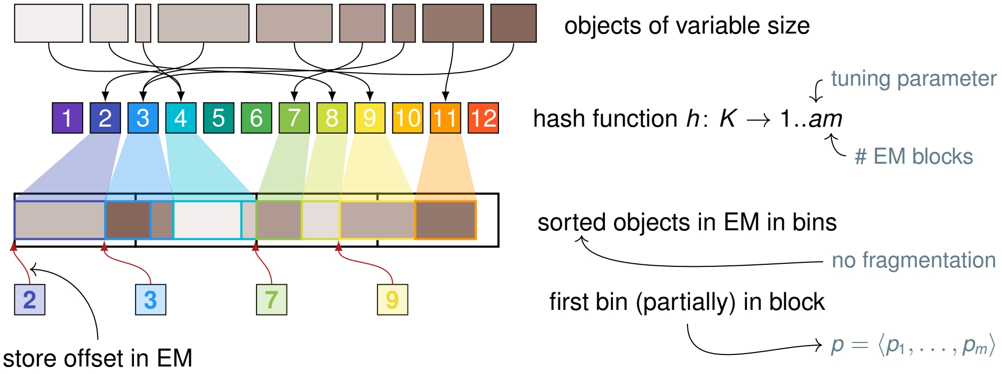
sorted objects in EM in bins

no fragmentation

first bin (partially) in block

$p = \langle p_1, \ldots, p_m \rangle$

PaCHash Overview



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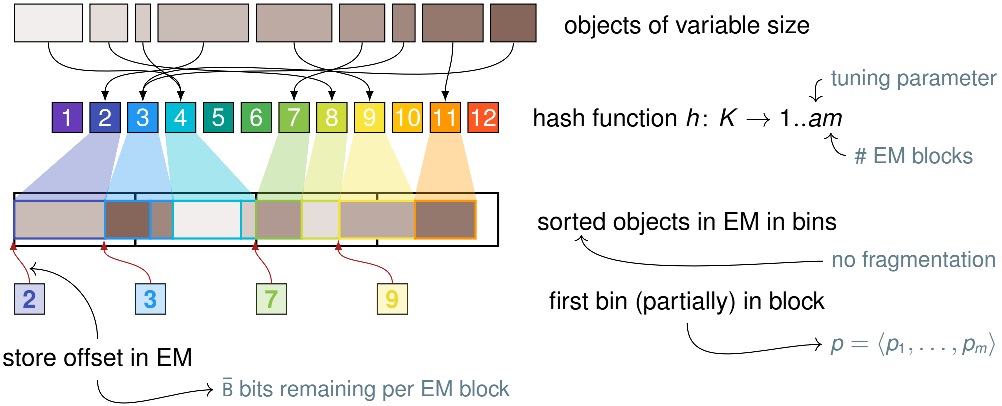
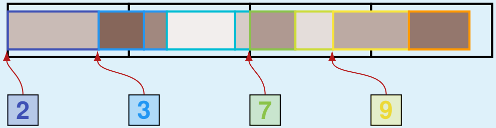


Finding Blocks

Query Algorithm



- $b_x = h(x)$
- find first i with $p_i \leq b_x$
- if $p_i = b_x$ let $i = i - 1$
- find first j with $p_j > b_x$
- return $i..(j-1)$

Finding Blocks

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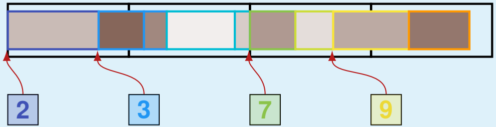

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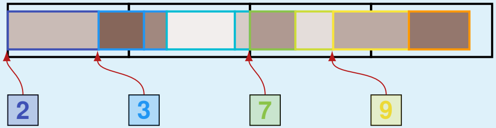
Elias-Fano Coding

- given k monotonic increasing integers in $1..u$
  - store $\log k$ MSBs encoded in bit vector
  - store $\log(u/k)$ LSBs plain
  - $k(2 + \log(u/k)) + 1 + o(k)$ bits in total
- predecessor in $O(k)$ time

Finding Blocks

Query Algorithm



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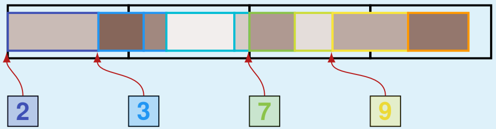
Lemma: Space with Elias-Fano Coding

When using Elias-Fano coding [Eli74; Fan71] to store p, the index needs $2 + \log a + o(1)$ bits of internal memory per block.

Finding Blocks

Query Algorithm



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Predecessor Query in PaCHash Internal Memory

Karlsruhe Institute of Technology

Lemma: Expected Predecessor Time

When using Elias-Fano coding to store p, the range of blocks containing the bin of an object x can be found in expected constant time.

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Proof (Sketch)

- consider $\lceil \log m \rceil$ MSB
- let bin b_x have MSBs equal to u
- expected size $\mathbb{E}(Y_u)$ of all bins with MSB u that are $< b_x$ is

$$\sum_{y \in S} |y| \cdot \mathbb{P}(h(y) \text{ w/ MSB} = u; h(y) < h(x))$$

$$\leq \sum_{y \in S} |y| \cdot \mathbb{P}(h(y) \text{ w/ MSB} = u)$$

$$= \frac{1}{m} \sum_{y \in S} |y| = \frac{m\bar{\mathrm{B}}}{m} = \bar{\mathrm{B}}$$

- number of entries to scan is $\mathbb{E}(Y_u)/\bar{\mathrm{B}} = 1$

Loading Blocks from External Memory

Karlsruhe Institute of Technology

Lemma: Additional Blocks Loaded

Retrieving an object x of size $|x|$ from a PaCHash data structure loads $\leq 1 + |x|/\overline{\text{B}} + 1/a$ consecutive blocks from the external memory in expectation.

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- expected size of bin $b_x = h(x)$

$$\begin{aligned}\mathbb{E}(|b_x|) &= |x| + \sum_{y \in S, y \neq x} |y| \mathbb{P}(y \in b_x) \\ &\leq |x| + \sum_{y \in S} |y| \mathbb{P}(y \in b_x) \\ &= |x| + \sum_{y \in S} |y| \cdot \frac{1}{am} = |x| + \frac{\bar{\mathrm{B}}}{a}\end{aligned}$$

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Proof (Sketch, cnt.)

- expected number of blocks overlapped by b_x

$$\mathbb{E}(X) = 1 + (\mathbb{E}(|b_x|) - 1)/\bar{\mathrm{B}}$$
$$= 1 + \frac{|x|}{\bar{\mathrm{B}}} + \frac{1}{a} - 1/\bar{\mathrm{B}}$$

- $\mathbb{P}(\text{bin and block border align}) = 1/\bar{\mathrm{B}}$

Experimental Evaluation

Hardware and Software

- Intel i7 11700 (base clock speed: 2.5 GHz)
- 1 TB Samsung 980 Pro NVMe SSD
- Ubuntu 21.10 (Kernel 5.13.0)
- `io_uring` for I/O operations
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- $\mathtt{B} = 4096$ bytes

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- more in the paper (very similar results)

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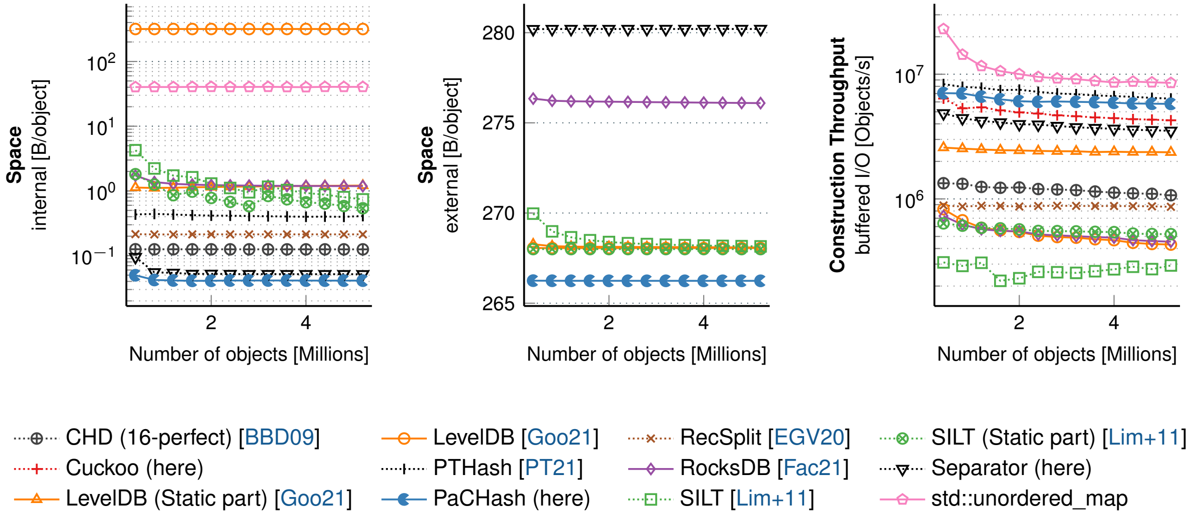
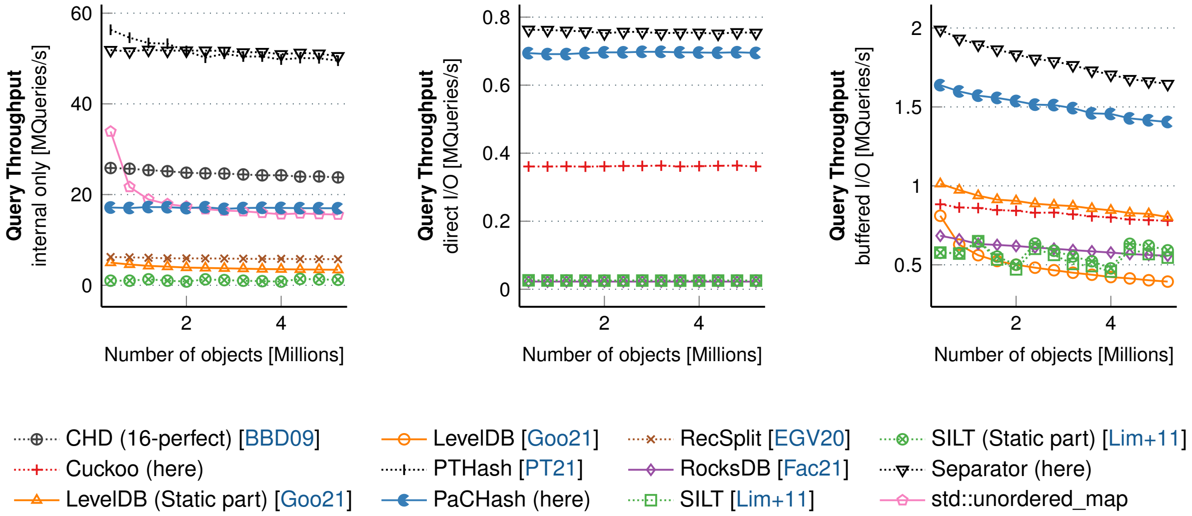
Competitors

- LevelDB [Goo21]
- RocksDB [Fac21]
- SILT [Lim+11].
- `std::unordered_map`
- RecSplit [EGV20]
- CHD [BBD09; CR+12]
- PTHash [PT21]

Construction

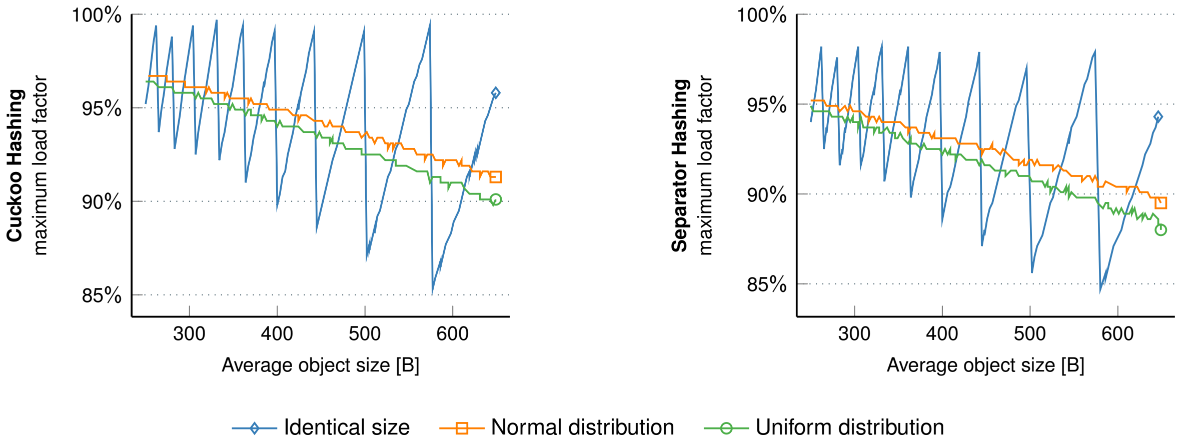


Queries



Query Throughput
internal only [MQueries/s]
60
40
20
0
2
4
Number of objects [Millions]
Query Throughput
direct I/O [MQueries/s]
0.8
0.6
0.4
0.2
0
2
4
Number of objects [Millions]
Query Throughput
buffered I/O [MQueries/s]
2
1.5
1
0.5
2
4
Number of objects [Millions]
CHD (16-perfect) [BBD09]
Cuckoo (here)
LevelDB (Static part) [Goo21]
LevelDB [Goo21]
PTHash [PT21]
PaCHash (here)
RecSplit [EGV20]
RocksDB [Fac21]
SILT [Lim+11]
SILT (Static part) [Lim+11]
Separator (here)
std::unordered_map

Maximum Load Factor of Competitors



Alternative Internal Memory Data Structures

Lemma: Space with Succincter

When using Succincter [Pat08] to store p, the index needs $1.44 + \log(a + 1) + o(1)$ bits of internal memory per block.

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Structure of Bit Vector

- runs of `0`s and `10`s
- sometimes additional `1`s

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- encode blocks of size 8, 16, 32, or 64

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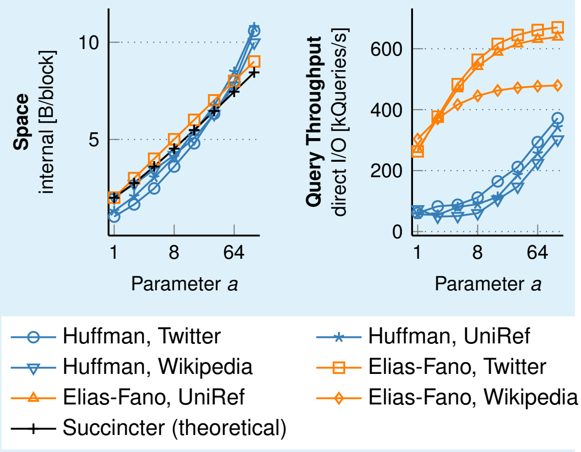
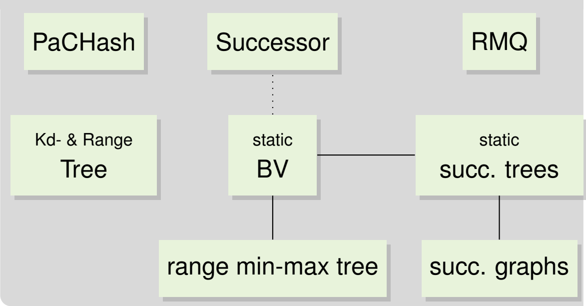


Conclusion and Outlook

This Lecture

- PaCHash

Advanced Data Structures



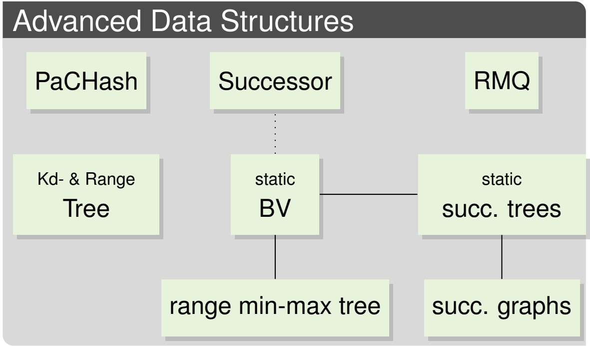
Conclusion and Outlook

This Lecture

- PaCHash

Next Lecture

- more on hashing



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