

# Text Indexing

## Lecture 03: Suffix Trees and Suffix Arrays

Florian Kurpicz

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<https://pingo.scc.kit.edu/289240>

# Recap: Compact Trie

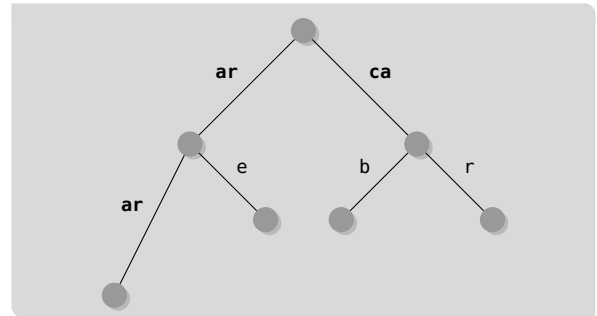
## Definition: Compact Trie

- A compact trie is a trie where all branchless paths are replaced by a single edge.
- The label of the new edge is the concatenation of the replaced edges' labels.

## Next

A full-text index for a text  $T$  is

- a data structure that
- allows to answer queries on  $T$  faster than naive
- we are interested in *pattern matching* queries
- how to use tries to create full-text index

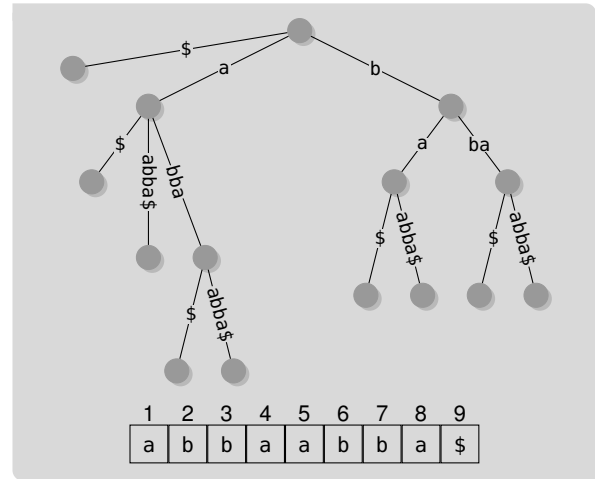


# Suffix Tree (1/4)

## Definition: Suffix Tree [Wei73]

A suffix tree (ST) for a text  $T$  of length  $n$  is a

- compact trie
- over  $S = \{T[1..n], T[2..n], \dots, T[n..n]\}$ 
  - ⓘ suffixes are prefix-free due to sentinel



# Suffix Tree (1/4)

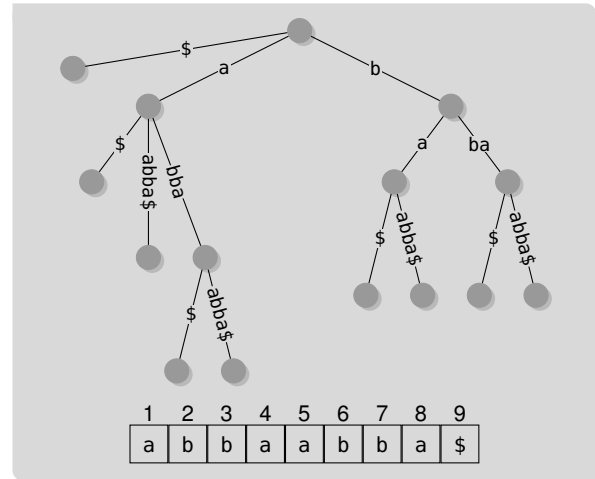
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Let  $G = (V, E)$  be a compact trie with root  $r$  and a node  $v \in V$ , then

- $\lambda(v)$  is the concatenation of labels from  $r$  to  $v$
- $d(v) = |\lambda(v)|$  is the string-depth of  $v$ 
  - ⓘ string depth  $\neq$  depth



# Suffix Tree (1/4)

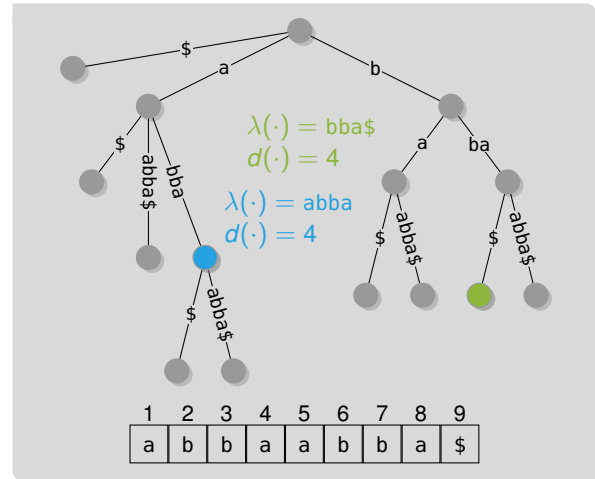
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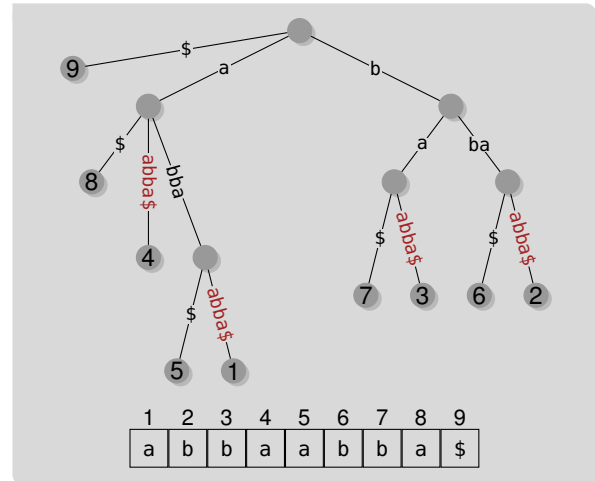
# Suffix Tree (2/4)

## Representing Labels

- explicit edge labels require  $O(n^2)$  words space

## Suffix Information

- label leaves with corresponding suffix
- ⓘ will be important later on



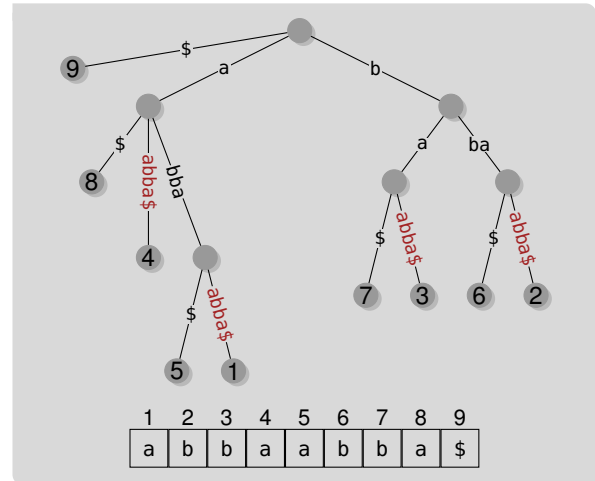
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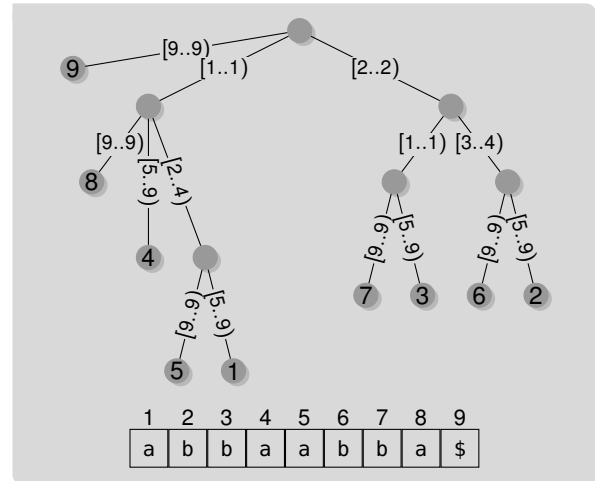
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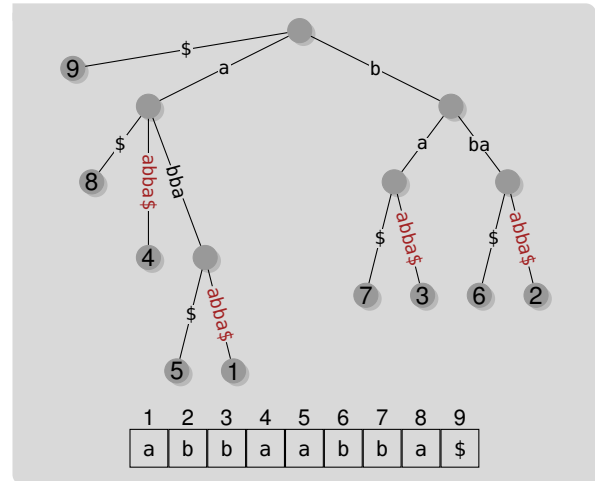
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- for simplicity, we show text

## Suffix Information

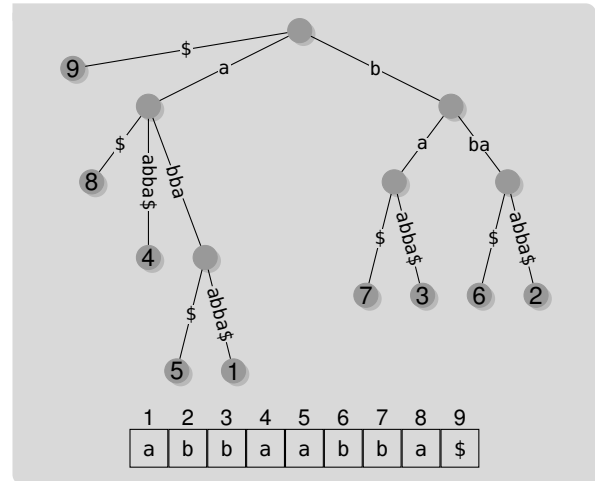
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# Suffix Tree (3/4)

## Pattern Matching using Suffix Trees

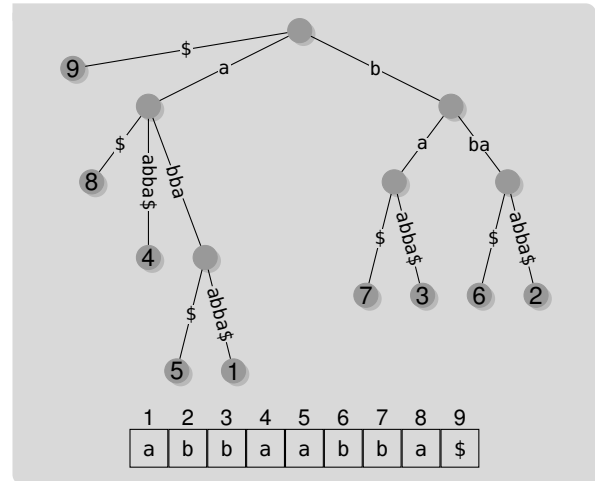
- Pattern  $P[1..m]$
  - start at the root and follow edges
  - query time depends on representation of children
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- $O(m)$  time using  $O(n\sigma)$  words space



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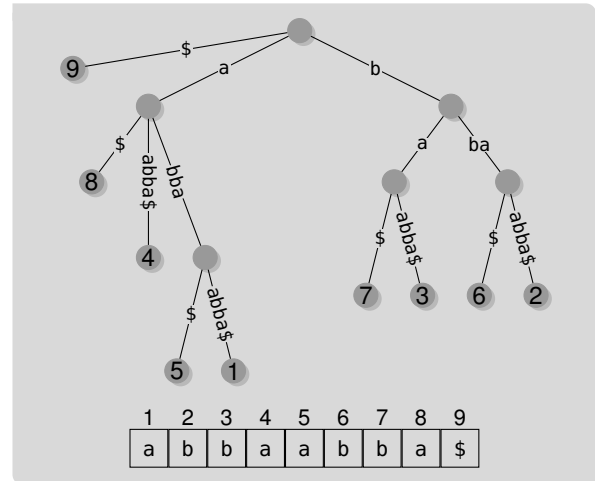


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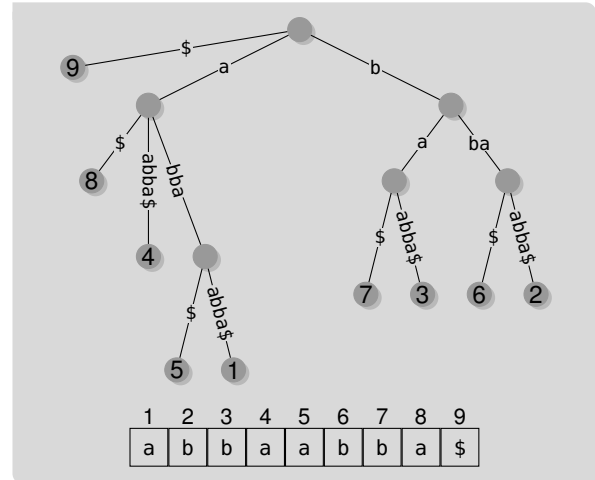
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- $O(m)$  time using  $O(n\sigma)$  words space
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- $O(m + \lg \sigma)$  time with  $O(n)$  words space



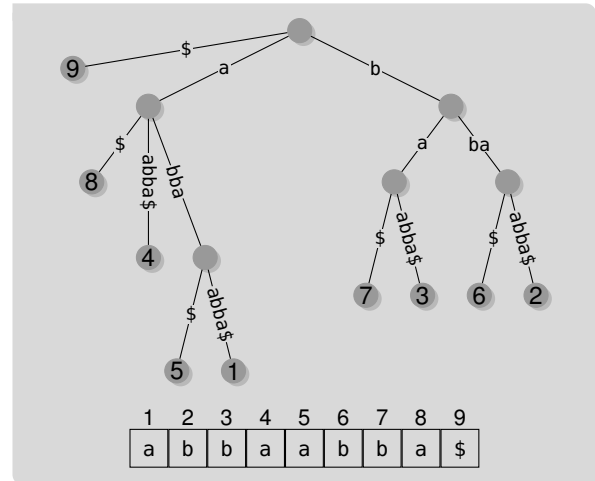
# Suffix Tree (4/4)

- very (most?) powerful text-index



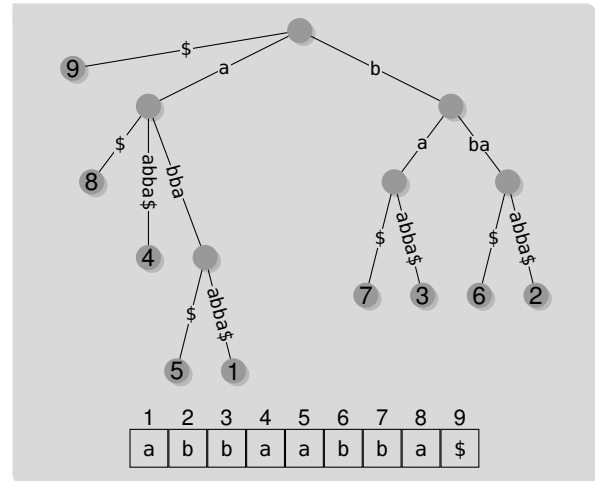
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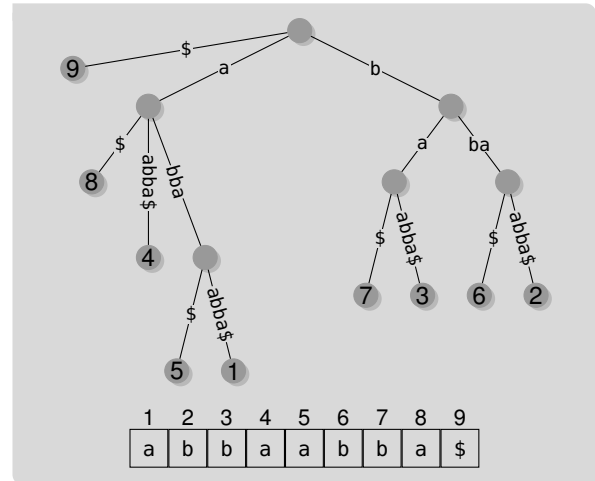
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- efficient direct construction in  $O(n)$  time [Ukk95]
- also possible for integer alphabets [Far97]





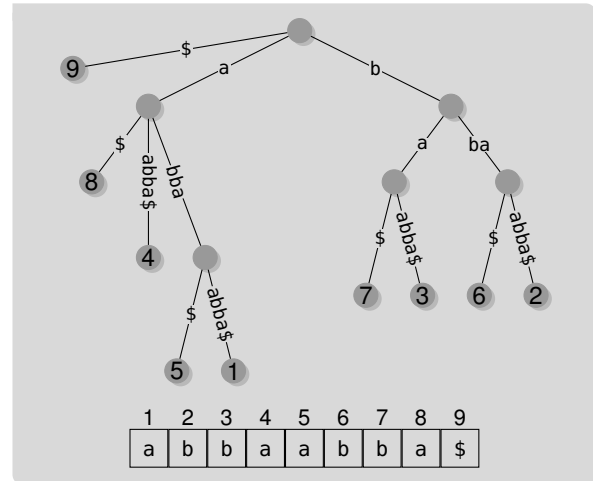
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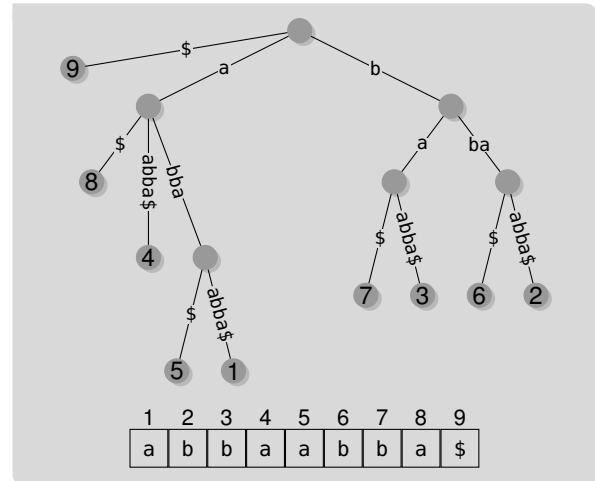


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next, suffix array construction



# Suffix Array and LCP-Array

## Definition: Suffix Array [GBS92; MM93]

Given a text  $T$  of length  $n$ , the **suffix array** (SA) is a permutation of  $[1..n]$ , such that for  $i \leq j \in [1..n]$

$$T[SA[i]..n] \leq T[SA[j]..n]$$

	1	2	3	4	5	6	7	8	9	10	11	12	13
$T$	a	b	a	b	c	a	b	c	a	b	b	a	\$
SA	13	12	1	9	6	3	11	2	10	7	4	8	5
LCP	0	0	1	2	2	5	0	2	1	1	4	0	3

\$	a	a	a	a	a	b	b	b	b	b	c	c
	\$	b	b	b	b	a	a	b	c	c	a	a
		a	b	c	c	\$	b	b	a	a	b	b
		b	a	a	a		c	a	b	b	b	c
		c	\$	b	b		a	b	b	a	\$	a
		a		a	c		c	a	a	a		b
		b		b	a		b	b	\$	b		b
		c		\$	b		a	a		b		a
		a			a		b	b		a		b
		b			b		a	a		\$		a
		b			\$		b	b				\$
		a					a	a				
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## Definition: Longest Common Prefix Array

Given a text  $T$  of length  $n$  and its SA, the **LCP-array** is defined as

$$LCP[i] = \begin{cases} 0 & i = 1 \\ \max\{\ell: T[SA[i]..SA[i] + \ell) = \\ T[SA[i - 1]..SA[i - 1] + \ell)\} & i \neq 1 \end{cases}$$

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\$	a	a	a	a	a	b	b	b	b	b	c	c
\$	\$	b	b	b	b	a	a	b	b	c	a	a
		a	b	c	c	\$	b	a	a	a	b	b
		b	a	a	a		c	c	b	b	b	c
		c	\$	b	b		a	b	a	a	a	a
		a		b	c		b	c	a	a	\$	b
		b		a	a		c	a	\$	b	b	b
		c		\$	b		a	b		a	a	a
		a			b		b	b		b		\$
		b			a		a	a		a		
		b			\$		b	b		\$		
		a					\$	a				
		\$						\$				

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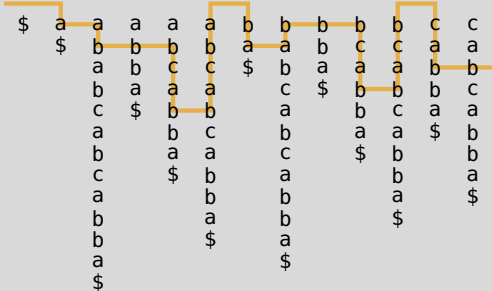
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# Pattern Matching with the Suffix Array (1/2)

**Function**  $\text{SeachSA}(T, SA[1..n], P[1..m]):$

```

1   $l = 1, r = n + 1$ 
2  while  $l < r$  do
3     $i = \lfloor (l + r) / 2 \rfloor$ 
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7   $s = l, l = l - 1, r = n$ 
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■ pattern  $P = abc$

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			a	b	c	c	\$	b	a	a	b	b	b
			b	a	b	b		c	\$	b	a	b	c
			c	\$	b	b		a		b	a	\$	a
			a		b	c		b		a	a		b
			b		a	a		c			\$		b
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			c	\$	b	c		a		b	c	a	a
			a		b	c		b		a	a	\$	b
			b		a	a		c			b		b
			c		\$	b		a			b		a
			a			b		b			a		\$
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			b	a	b	b		c	\$	b	a	b	c
			c	\$	b	c		a		b	a	\$	a
			a		b	a		b		a			b
			b		a	b		c					a
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## Pattern Matching with the Suffix Array (2/2)

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### Lemma: Running Time $\text{SeachSA}$

The  $\text{SeachSA}$  answers counting queries in  $O(m \lg n)$  time and reporting queries in  $O(m \lg n + occ)$  time

### Proof (Sketch)

- two binary searches on the  $SA$  in  $O(\lg n)$  time

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- each comparison requires  $O(m)$  time

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# Pattern Matching with the Suffix Array (2/2)

**Function**  $\text{SeachSA}(T, SA[1..n], P[1..m]):$

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1   $l = 1, r = n + 1$ 
2  while  $l < r$  do
3     $i = \lfloor (l + r) / 2 \rfloor$ 
4    if  $P > T[SA[i]..SA[i] + m]$  then
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6    else  $r = i$ 
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12  return  $[s, r]$ 
  
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The  $\text{SeachSA}$  answers counting queries in  $O(m \lg n)$  time and reporting queries in  $O(m \lg n + occ)$  time

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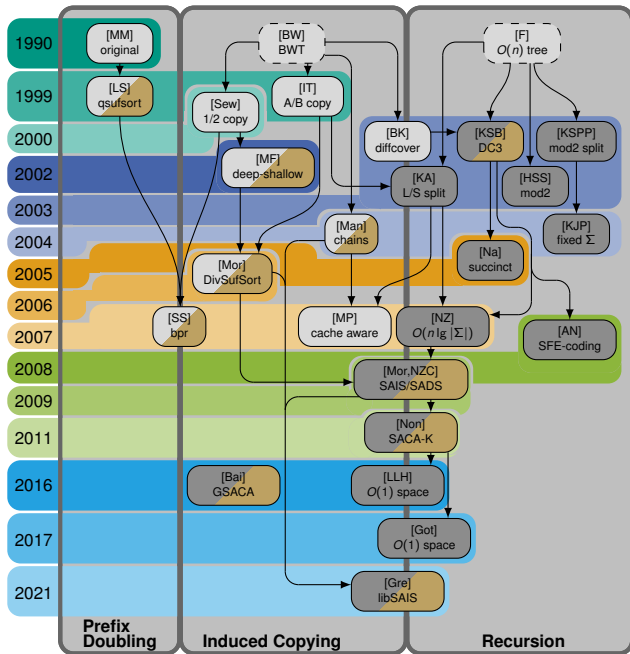
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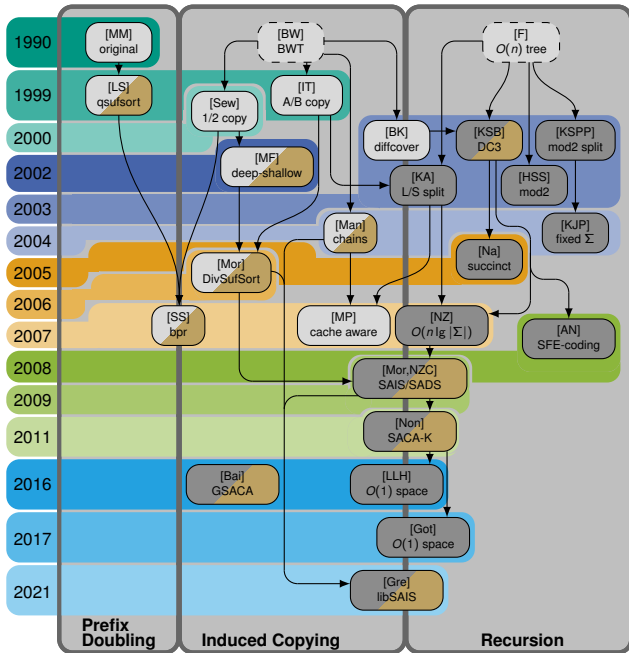
# Preview: Improving Running Time with LCP-Array

- next lecture:  $O(m + \lg n)$  and  $O(m + \lg n + occ)$  time
  - requires additional indices on LCP-array
- 
- now: how to compute the suffix array directly **i** without the suffix tree



## Timeline Sequential Suffix Sorting

- based on [Bah+19; Bin18; Kur20; PST07]
- darker grey: linear running time
- brown: available implementation

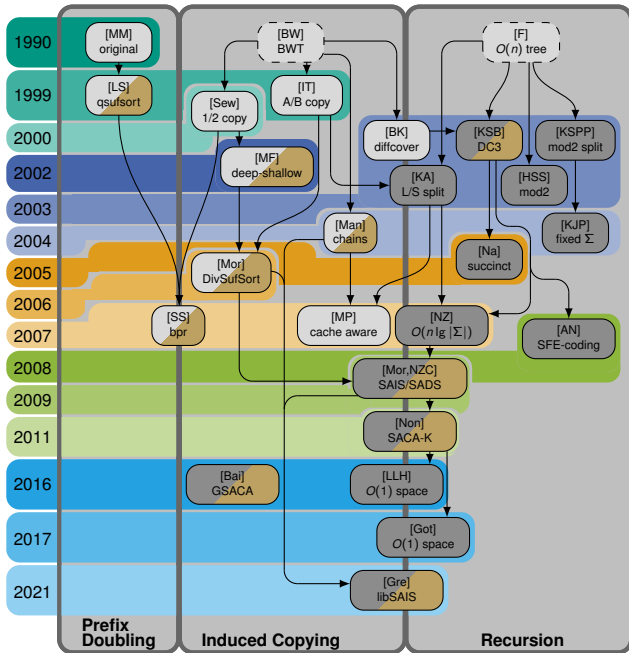


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- DC3 first  $O(n)$  algorithm
- $O(n)$  running time and  $O(1)$  space for integer alphabets possible

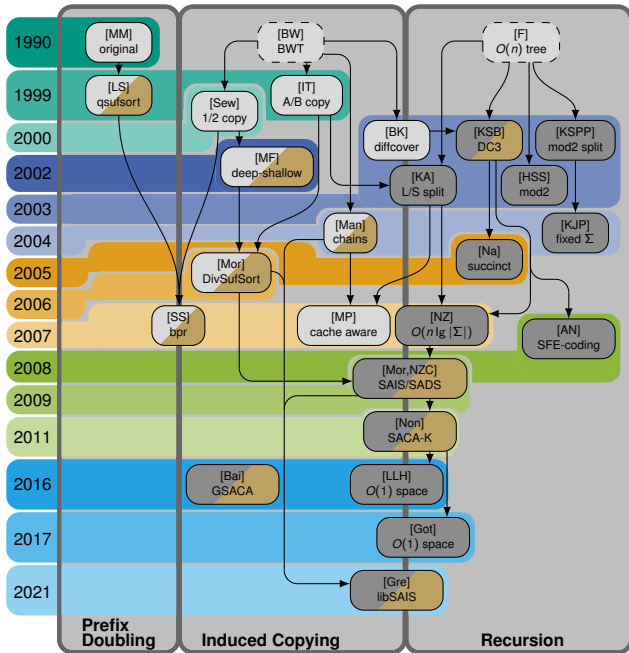


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# Suffix Array Induced Sorting: Overview

## The Idea: Inducing

Given a text  $T$  of length  $n$  and two positions  $i, j \in [1..n]$  with  $T[i] = T[j]$ , then

$$T[i..n] < T[j..n] \iff T[i + 1..n] < T[j + 1..n]$$

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a α

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- classification
- sort special substrings/suffixes recursively
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## Roadmap

- classification
- inducing
- sorting special suffixes

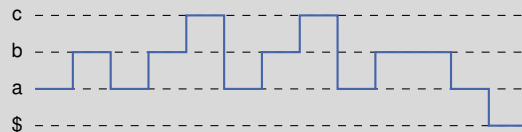
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1	2	3	4	5	6	7	8	9	10	11	12	13
a	b	a	b	c	a	b	c	a	b	b	a	\$



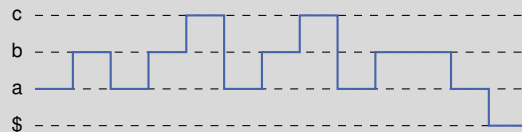
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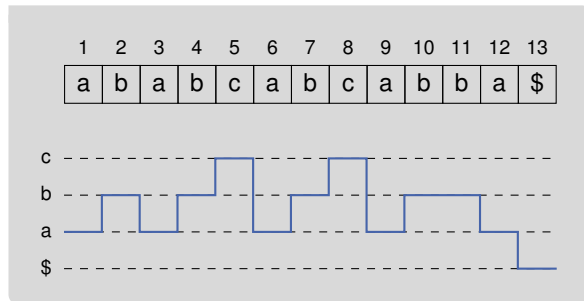


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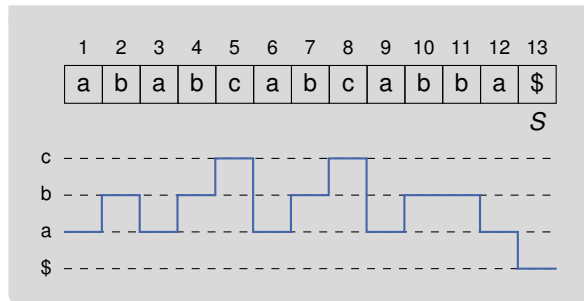


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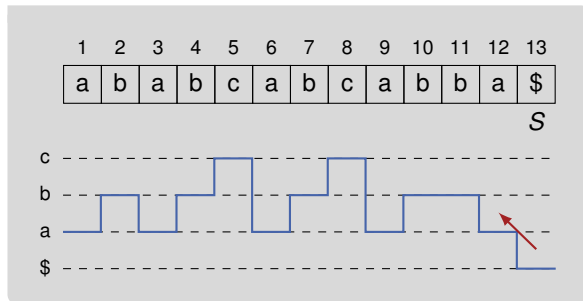


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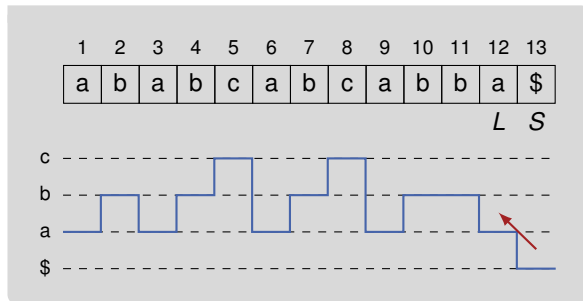


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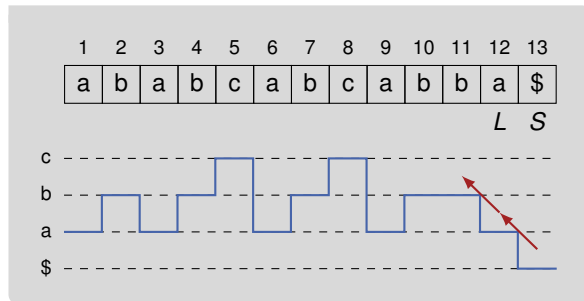


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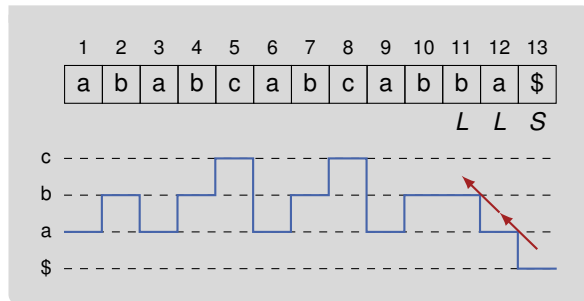


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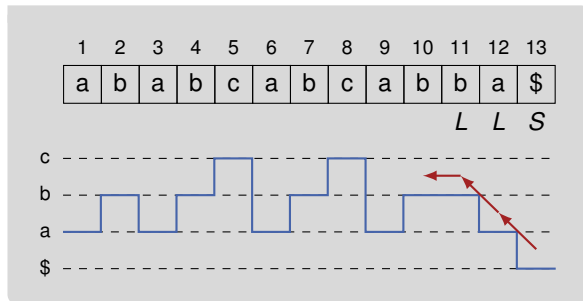


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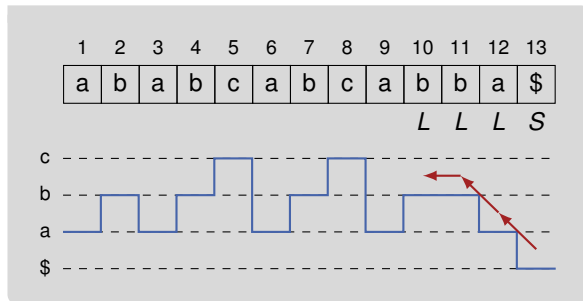


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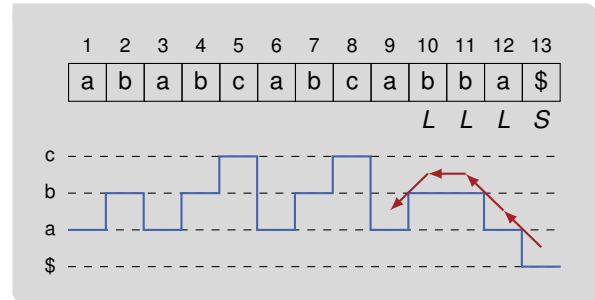


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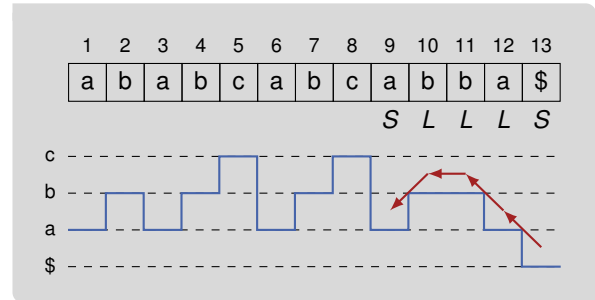


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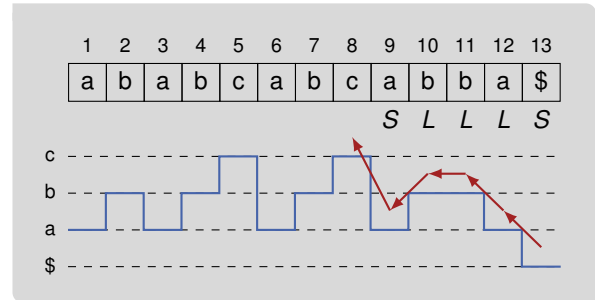


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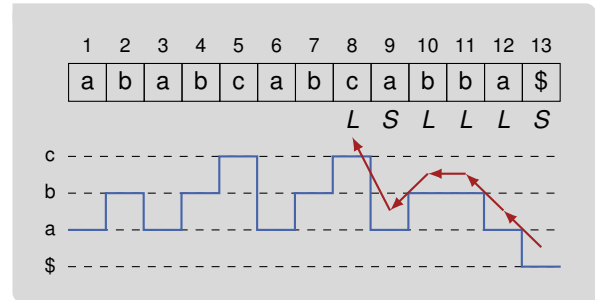


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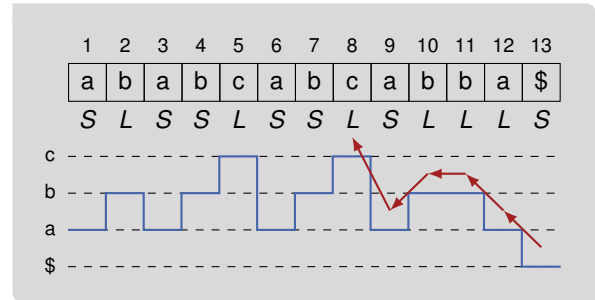


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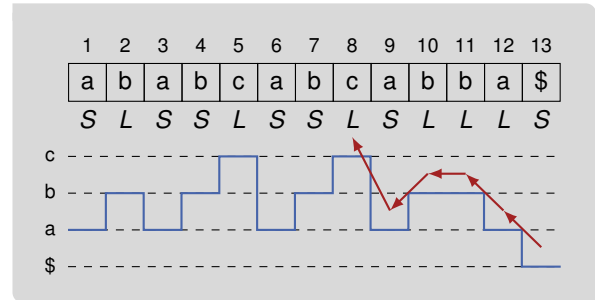
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## Definition: Leftmost $S$ Suffixes

Given a text  $T$  of length  $n$ ,  $i \in [2..n]$  such that  $T[i..n]$  has type  $S$  and  $T[i - 1..n]$  has type  $L$ , then  $T[i..n]$  is called **leftmost  $S$  suffix (LMS)**.

- denoted by  $S^*$



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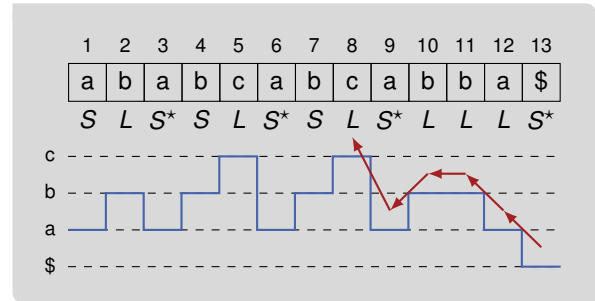
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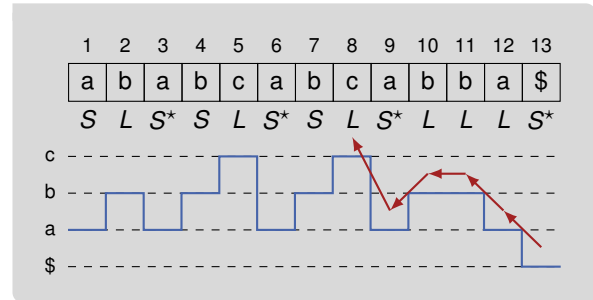
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- $T[i] = T[i + 1] \Rightarrow T[i..n]$  has  $T[i + 1..n]$ 's type.

## Definition: Leftmost $S$ Suffixes

Given a text  $T$  of length  $n$ ,  $i \in [2..n]$  such that  $T[i..n]$  has type  $S$  and  $T[i - 1..n]$  has type  $L$ , then  $T[i..n]$  is called **leftmost  $S$  suffix (LMS)**.

- denoted by  $S^*$



- scan text from right to left
- do not store types explicitly **i** initially, we are only interested in LMS-suffixes

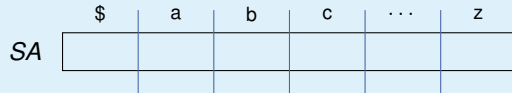
## Suffix Array Induced Sorting: Classification (2/2)

- partition suffix array based text's histogram

SA

## Suffix Array Induced Sorting: Classification (2/2)

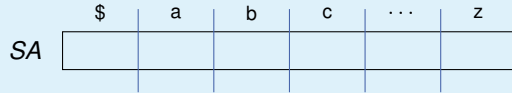
- partition suffix array based text's histogram





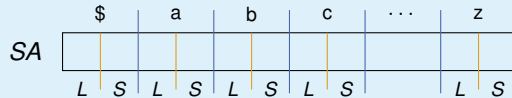
## Suffix Array Induced Sorting: Classification (2/2)

- partition suffix array based text's histogram
- use types of suffixes to partition suffix array



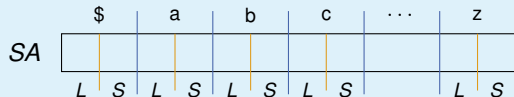
## Suffix Array Induced Sorting: Classification (2/2)

- partition suffix array based text's histogram
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## Suffix Array Induced Sorting: Classification (2/2)

- partition suffix array based text's histogram
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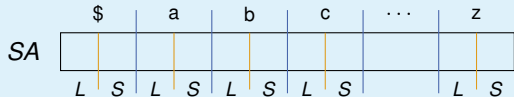
### Lemma: Order of L/S Suffixes

Given a text  $T$  of length  $n$ , a type  $L$  suffixes  $T[i..n]$  and a type  $S$   $T[j..n]$  with  $\alpha = T[i] = T[j]$ , then

$$T[i..n] < T[j..n]$$

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### Lemma: Order of L/S Suffixes

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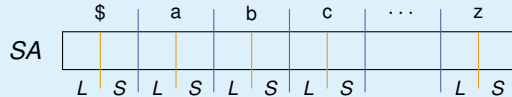
$$T[i..n] < T[j..n]$$

### Proof (Sketch)

- $T[i..n]$  has type  $L$ 
  - $T[i..n] = \alpha \underbrace{\alpha \dots \alpha}_{\ell \geq 0 \text{ times}} \beta \dots \$$
  - with  $\beta < \alpha$

## Suffix Array Induced Sorting: Classification (2/2)

- partition suffix array based text's histogram
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### Lemma: Order of L/S Suffixes

Given a text  $T$  of length  $n$ , a type  $L$  suffixes  $T[i..n]$  and a type  $S$   $T[j..n]$  with  $\alpha = T[i] = T[j]$ , then

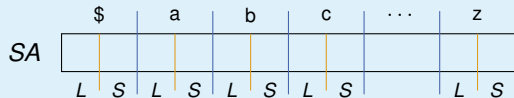
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    - with  $\beta < \alpha$
- $T[j..n]$  has type  $S$ 
  - $T[j..n] = \alpha \underbrace{\alpha \dots \alpha}_{\ell' \geq 0 \text{ times}} \gamma \dots \$$ 
    - with  $\alpha < \gamma$

## Suffix Array Induced Sorting: Classification (2/2)

- partition suffix array based text's histogram
- use types of suffixes to partition suffix array



### Lemma: Order of L/S Suffixes

Given a text  $T$  of length  $n$ , a type  $L$  suffixes  $T[i..n]$  and a type  $S$   $T[j..n]$  with  $\alpha = T[i] = T[j]$ , then

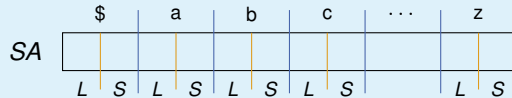
$$T[i..n] < T[j..n]$$

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    - with  $\beta < \alpha$
- $T[j..n]$  has type  $S$ 
  - $T[j..n] = \alpha \underbrace{\alpha \dots \alpha}_{\ell' \geq 0 \text{ times}} \gamma \dots \$$ 
    - with  $\alpha < \gamma$
- if  $\ell < \ell'$  then  $\alpha < \gamma$  and  $T[i..n] < T[j..n]$

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- partition suffix array based text's histogram
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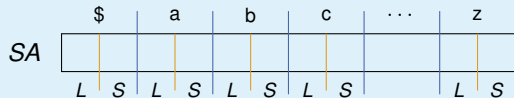
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- $T[i..n]$  has type  $L$ 
  - $T[i..n] = \alpha \underbrace{\alpha \dots \alpha}_{\ell \geq 0 \text{ times}} \beta \dots \$$ 
    - with  $\beta < \alpha$
- $T[j..n]$  has type  $S$ 
  - $T[j..n] = \alpha \underbrace{\alpha \dots \alpha}_{\ell' \geq 0 \text{ times}} \gamma \dots \$$ 
    - with  $\alpha < \gamma$
- if  $\ell < \ell'$  then  $\alpha < \gamma$  and  $T[i..n] < T[j..n]$
- if  $\ell = \ell'$  then  $\beta < \gamma$  and  $T[i..n] < T[j..n]$

## Suffix Array Induced Sorting: Classification (2/2)

- partition suffix array based text's histogram
- use types of suffixes to partition suffix array



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$$T[i..n] < T[j..n]$$

### Proof (Sketch)

- $T[i..n]$  has type  $L$ 
  - $T[i..n] = \alpha \underbrace{\alpha \dots \alpha}_{\ell \geq 0 \text{ times}} \beta \dots \$$ 
    - with  $\beta < \alpha$
- $T[j..n]$  has type  $S$ 
  - $T[j..n] = \alpha \underbrace{\alpha \dots \alpha}_{\ell' \geq 0 \text{ times}} \gamma \dots \$$ 
    - with  $\alpha < \gamma$
- if  $\ell < \ell'$  then  $\alpha < \gamma$  and  $T[i..n] < T[j..n]$
- if  $\ell = \ell'$  then  $\beta < \gamma$  and  $T[i..n] < T[j..n]$
- if  $\ell > \ell'$  then  $\beta < \alpha$  and  $T[i..n] < T[j..n]$



# Suffix Array Induced Sorting: Inducing (1/2)

## Lemma: Inducing

If  $T[i + 1..n] < T[j + 1..n]$  and  $T[i] = T[j]$  then

$$T[i..n] < T[j..n]$$

	1	2	3	4	5	6	7	8	9	10	11	12	13
$T$	a	b	a	b	c	a	b	c	a	b	b	a	\$
SA	13	12	1	9	6	3	11	2	10	7	4	8	5
	\$	a	a	a	a	a	b	b	b	b	b	c	c
		\$	b	b	b	b	a	a	b	c	c	a	a
			a	b	c	c	\$	b	a	a	a	b	b
			b	a	a	a		c	b	b	b	b	c
			c	\$	b	b		a	a	a	c	a	a
			a		a	c		b	\$	b	a	\$	b
			b		\$	a		c					b
			c			b		a					a
			a			b		b					b
			b			a		a					a
			b			\$		b					\$
			a					a					
			\$					\$					

# Suffix Array Induced Sorting: Inducing (1/2)

## Lemma: Inducing

If  $T[i + 1..n] < T[j + 1..n]$  and  $T[i] = T[j]$  then

$$T[i..n] < T[j..n]$$

	1	2	3	4	5	6	7	8	9	10	11	12	13
$T$	a	b	a	b	c	a	b	c	a	b	b	a	\$
SA	13	12	1	9	6	3	11	2	10	7	4	8	5
	\$	a	a	a	a	a	b	b	b	b	b	c	c
		\$	b	b	b	b	a	a	b	c	c	a	a
			a	b	c	c	\$	b	a	a	a	b	b
			b	a	a	a		c	b	b	b	b	c
			c	\$	b	b		a	a	a	c	a	a
			a		a	c		b	b	b	a	\$	b
			b		\$	a		c	\$				b
			c			b		a					a
			a			b		b					b
			b			a		a					a
			b			\$		b					b
			a					a					a
			\$					\$					\$

# Suffix Array Induced Sorting: Inducing (1/2)

## Lemma: Inducing

If  $T[i + 1..n] < T[j + 1..n]$  and  $T[i] = T[j]$  then

$$T[i..n] < T[j..n]$$

	1	2	3	4	5	6	7	8	9	10	11	12	13
$T$	a	b	a	b	c	a	b	c	a	b	b	a	\$
SA	13	12	1	9	6	3	11	2	10	7	4	8	5
	\$	a	a	a	a	a	b	b	b	b	b	c	c
	\$	\$	b	b	b	b	a	a	b	b	c	a	a
			a	b	c	c	\$	b	a	a	a	b	b
			b	a	a	a		c	\$	b	b	b	c
			c	\$	b	b		a		a	c	a	a
			a		a	c		b		b	a	\$	b
			b		\$	a		c		\$	b		b
			c			b		a			a		a
			a			a		b			b		b
			b			b		b			a		a
			b			a		a			\$		\$
			a			\$		\$					\$

# Suffix Array Induced Sorting: Inducing (1/2)

## Lemma: Inducing

If  $T[i + 1..n] < T[j + 1..n]$  and  $T[i] = T[j]$  then

$$T[i..n] < T[j..n]$$

	1	2	3	4	5	6	7	8	9	10	11	12	13
$T$	a	b	a	b	c	a	b	c	a	b	b	a	\$
SA	13	12	1	9	6	3	11	2	10	7	4	8	5
	\$	a	a	a	a	a	b	b	b	b	b	c	c
	\$	\$	b	b	b	b	a	a	a	a	c	a	a
			a	a	a	c	\$	b	b	b	a	b	b
			b	b	c	a		c	a	a	b	b	c
			c	\$	b	b		a	\$	b	c	a	a
			a		a	c		b		a	a	\$	b
			b		b	a		c		\$	b		b
			b		a	b		a			a		a
			a		\$	a		b			b		b
			b			b		a			a		a
			a			a		\$			\$		\$

# Suffix Array Induced Sorting: Inducing (1/2)

## Lemma: Inducing

If  $T[i + 1..n] < T[j + 1..n]$  and  $T[i] = T[j]$  then

$$T[i..n] < T[j..n]$$

	1	2	3	4	5	6	7	8	9	10	11	12	13
$T$	a	b	a	b	c	a	b	c	a	b	b	a	\$
SA	13	12	1	9	6	3	11	2	10	7	4	8	5
	\$	a	a	a	a	a	b	b	b	b	b	c	c
	\$	\$	b	b	b	b	b	a	b	c	c	a	a
			a	b	c	c	\$	b	a	a	a	b	b
			b	a	a	a		c	\$	b	b	b	c
			c	\$	b	b		a		b	c	a	a
			a		a	c		b		a	a	\$	b
			b		b	a		c		\$	b		b
			c		a	b		a			a		a
			a		\$	a		b			b		b
			b			b		b			a		a
			b			a		a			\$		\$
			a			\$		\$					

# Suffix Array Induced Sorting: Inducing (1/2)

## Lemma: Inducing

If  $T[i + 1..n] < T[j + 1..n]$  and  $T[i] = T[j]$  then

$$T[i..n] < T[j..n]$$

## Proof (Sketch)

- similar to order of  $L/S$  suffixes
- there is a leftmost character where  $T[i + 1..n]$  and  $T[j + 1..n]$  differ
- $T[i..n]$  and  $T[j..n]$  differ at the same character

	1	2	3	4	5	6	7	8	9	10	11	12	13
$T$	a	b	a	b	c	a	b	c	a	b	b	a	\$
SA	13	12	1	9	6	3	11	2	10	7	4	8	5
	\$	a	a	a	a	a	b	b	b	b	b	c	c
	\$		b	b	b	b	a	a	a	a	c	a	a
		a	a	b	c	c	\$	b	b	a	a	b	b
		b	b	a	a	a		c	a	b	b	b	c
		c	c	\$	b	b		a	\$	b	c	a	a
		a	a		b	c		b		a	a	\$	b
		b	c		a	a		c		\$	b		b
		c			b	b		a			a		a
		a	b			a		b			b		\$
		b	b			\$		a			\$		
		a						\$					

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “–”
  - put *sorted LMS*-suffixes at the end of buckets

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “–”
  - put *sorted LMS*-suffixes at the end of buckets

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
	13	–	–	9	6	3	–	–	–	–	–	–	–



# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “–”
  - put *sorted LMS*-suffixes at the end of buckets

	\$			a			b				c	
1	2	3	4	5	6	7	8	9	10	11	12	13
a	b	a	b	c	a	b	c	a	b	b	a	\$
S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	–	–	9	6	3	–	–	–	–	–	–	–

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	-	9	6	3	-	-	-	-	-	-	-

# Suffix Array Induced Sorting: Inducing (2/2)

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- Initialization
  - initialize each entry in SA with “-”
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  - then put  $SA[i] - 1$  at beginning of bucket

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	-	9	6	3	-	-	-	-	-	-	-

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
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  - then put  $SA[i] - 1$  at beginning of bucket

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	-	9	6	3	-	-	-	-	-	-	-
12	12	-	-	9	6	3	-	-	-	-	-	-	-

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	-	9	6	3	-	-	-	-	-	-	-
12	12	-	-	9	6	3	-	-	-	-	-	-	-

# Suffix Array Induced Sorting: Inducing (2/2)

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- Initialization
  - initialize each entry in SA with “-”
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- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket

	a			b			c					
\$	2	3	4	5	6	7	8	9	10	11	12	13
a	b	a	b	c	a	b	c	a	b	b	a	\$
S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	9	6	3	-	-	-	-	-	-	-
12	12	-	9	6	3	-	-	-	-	-	-	-
13	12	-	9	6	3	11	-	-	-	-	-	-

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	-	9	6	3	-	-	-	-	-	-	-
12	-	-	-	9	6	3	-	-	-	-	-	-	-
13	12	-	-	9	6	3	11	-	-	-	-	-	-

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
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- Scan Left to Right ( $i = 1, 2, \dots, n$ )
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  - then put  $SA[i] - 1$  at beginning of bucket

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	-	9	6	3	-	-	-	-	-	-	-
12	-	-	-	9	6	3	-	-	-	-	-	-	-
13	12	-	-	9	6	3	11	-	-	-	-	-	-
13	12	-	-	9	6	3	11	-	-	-	-	8	-



# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	-	9	6	3	-	-	-	-	-	-	-
12	12	-	-	9	6	3	-	-	-	-	-	-	-
13	12	-	-	9	6	3	11	-	-	-	-	-	-
13	12	-	-	9	6	3	11	-	-	-	-	8	-

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	9	6	3	-	-	-	-	-	-	-	-
12	12	-	9	6	3	-	-	-	-	-	-	-	-
11	12	-	9	6	3	11	-	-	-	-	-	-	-
10	12	-	9	6	3	11	-	-	-	-	8	-	-
9	12	-	9	6	3	11	-	-	-	-	8	5	-

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	9	6	3	-	-	-	-	-	-	-	-
12	12	-	9	6	3	-	-	-	-	-	-	-	-
11	12	-	9	6	3	11	-	-	-	-	-	-	-
10	12	-	9	6	3	11	-	-	-	-	8	-	-
9	12	-	9	6	3	11	-	-	-	8	5	-	-

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	9	6	3	-	-	-	-	-	-	-	-
12	12	-	9	6	3	-	-	-	-	-	-	-	-
11	12	-	9	6	3	11	-	-	-	-	-	-	-
10	12	-	9	6	3	11	-	-	-	-	8	-	-
9	12	-	9	6	3	11	-	-	-	-	8	5	-
8	12	-	9	6	3	11	2	-	-	-	8	5	-

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	9	6	3	-	-	-	-	-	-	-	-
12	12	-	9	6	3	-	-	-	-	-	-	-	-
11	12	-	9	6	3	11	-	-	-	-	-	-	-
10	12	-	9	6	3	11	-	-	-	-	-	8	-
9	12	-	9	6	3	11	-	-	-	-	-	8	5
8	12	-	9	6	3	11	2	-	-	-	-	8	5

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket

	\$	a					b					c	
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	9	6	3	-	-	-	-	-	-	-	-
12	-	-	9	6	3	-	-	-	-	-	-	-	-
11	-	-	9	6	3	-	-	-	-	-	-	-	-
10	-	-	9	6	3	-	-	-	-	-	-	-	-
9	-	-	9	6	3	-	-	-	-	-	-	-	-
8	-	-	9	6	3	-	-	-	-	-	-	-	-
7	-	-	9	6	3	-	-	-	-	-	-	-	-
6	-	-	9	6	3	-	-	-	-	-	-	-	-
5	-	-	9	6	3	-	-	-	-	-	-	-	-
4	-	-	9	6	3	-	-	-	-	-	-	-	-
3	-	-	9	6	3	-	-	-	-	-	-	-	-
2	-	-	9	6	3	-	-	-	-	-	-	-	-
1	-	-	9	6	3	-	-	-	-	-	-	-	-

Diagram illustrating the induction step in SAIS. The table shows the suffix array (SA) and the corresponding text (T) for the string "ababacbababac\$". The SA is initially filled with dashes (-). The process involves scanning left to right and placing the index of the current character (SA[i]) at the beginning of the bucket corresponding to the character T[SA[i]-1]. The diagram shows the following steps:

- Step 1: SA[1] = 13, T[13] = '\$', bucket '\$' contains 13.
- Step 2: SA[2] = 12, T[12] = 'a', bucket 'a' contains 12.
- Step 3: SA[3] = 9, T[9] = 'b', bucket 'b' contains 9.
- Step 4: SA[4] = 6, T[6] = 'a', bucket 'a' contains 6, 12.
- Step 5: SA[5] = 3, T[3] = 'b', bucket 'b' contains 3, 9.
- Step 6: SA[6] = 11, T[11] = 'b', bucket 'b' contains 11, 3, 9.
- Step 7: SA[7] = 8, T[8] = 'c', bucket 'c' contains 8.
- Step 8: SA[8] = 5, T[5] = 'a', bucket 'a' contains 5, 6, 12.
- Step 9: SA[9] = 10, T[10] = 'b', bucket 'b' contains 10, 11, 3, 9.
- Step 10: SA[10] = 13, T[13] = '\$', bucket '\$' contains 13, 8.
- Step 11: SA[11] = 12, T[12] = 'a', bucket 'a' contains 12, 5, 6, 12.
- Step 12: SA[12] = 9, T[9] = 'b', bucket 'b' contains 9, 10, 11, 3, 9.
- Step 13: SA[13] = 6, T[6] = 'a', bucket 'a' contains 6, 12, 5, 6, 12.
- Step 14: SA[14] = 3, T[3] = 'b', bucket 'b' contains 3, 9, 10, 11, 3, 9.

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	9	6	3	-	-	-	-	-	-	-	-
12	-	-	9	6	3	-	-	-	-	-	-	-	-
11	-	-	9	6	3	-	-	-	-	-	-	-	-
10	-	-	9	6	3	-	-	-	-	-	-	-	-
9	-	-	9	6	3	-	-	-	-	-	-	-	-
8	-	-	9	6	3	-	-	-	-	-	-	-	-
7	-	-	9	6	3	-	-	-	-	-	-	-	-
6	-	-	9	6	3	-	-	-	-	-	-	-	-
5	-	-	9	6	3	-	-	-	-	-	-	-	-
4	-	-	9	6	3	-	-	-	-	-	-	-	-
3	-	-	9	6	3	-	-	-	-	-	-	-	-
2	-	-	9	6	3	-	-	-	-	-	-	-	-
1	-	-	9	6	3	-	-	-	-	-	-	-	-

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	-	9	6	3	-	-	-	-	-	-	-
12	-	-	-	9	6	3	-	-	-	-	-	-	-
11	-	-	-	9	6	3	-	-	-	-	-	-	-
10	-	-	-	9	6	3	-	-	-	-	-	-	-
9	-	-	-	9	6	3	-	-	-	-	-	-	-
8	-	-	-	9	6	3	-	-	-	-	-	-	-
7	-	-	-	9	6	3	-	-	-	-	-	-	-
6	-	-	-	9	6	3	-	-	-	-	-	-	-
5	-	-	-	9	6	3	-	-	-	-	-	-	-
4	-	-	-	9	6	3	-	-	-	-	-	-	-
3	-	-	-	9	6	3	-	-	-	-	-	-	-
2	-	-	-	9	6	3	-	-	-	-	-	-	-
1	-	-	-	9	6	3	-	-	-	-	-	-	-

Arrows indicate the scanning process from right to left. The value 10 is highlighted in red.



# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	-	9	6	3	-	-	-	-	-	-	-
12	-	-	-	9	6	3	-	-	-	-	-	-	-
11	-	-	-	9	6	3	-	-	-	-	-	-	-
10	-	-	-	9	6	3	-	-	-	-	-	-	-
9	-	-	-	9	6	3	-	-	-	-	-	-	-
8	-	-	-	9	6	3	-	-	-	-	-	-	-
7	-	-	-	9	6	3	-	-	-	-	-	-	-
6	-	-	-	9	6	3	-	-	-	-	-	-	-
5	-	-	-	9	6	3	-	-	-	-	-	-	-
4	-	-	-	9	6	3	-	-	-	-	-	-	-
3	-	-	-	9	6	3	-	-	-	-	-	-	-
2	-	-	-	9	6	3	-	-	-	-	-	-	-
1	-	-	-	9	6	3	-	-	-	-	-	-	-

Diagram illustrating the induction step in SAIS. The table shows the suffix array (SA) and the corresponding text (T) for indices 1 to 13. The text is "ababacbababac\$". The SA is "SLS\*SL S\*SL S\*L S\*L L L L S\*". The diagram shows the process of moving the suffix starting at index 11 (suffix "abac\$") to the beginning of the bucket for 'a' (index 4), resulting in the new SA entry 8 at index 4. Arrows indicate the movement of the suffix and the resulting SA entries.

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	-	9	6	3	-	-	-	-	-	-	-
12	-	-	-	9	6	3	-	-	-	-	-	-	-
11	-	-	-	9	6	3	-	-	-	-	-	-	-
10	-	-	-	9	6	3	-	-	-	-	-	8	-
9	-	-	-	9	6	3	-	-	-	-	-	8	-
8	-	-	-	9	6	3	-	-	-	-	-	8	5
7	-	-	-	9	6	3	-	-	-	-	-	8	5
6	-	-	-	9	6	3	-	-	-	-	-	8	5
5	-	-	-	9	6	3	-	-	-	-	-	8	5
4	-	-	-	9	6	3	-	-	-	-	-	8	5
3	-	-	-	9	6	3	-	-	-	-	-	8	5
2	-	-	-	9	6	3	-	-	-	-	-	8	5
1	-	-	-	9	6	3	-	-	-	-	-	8	5

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

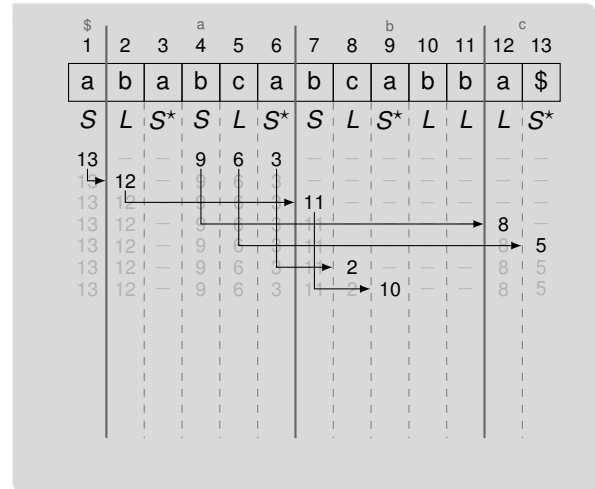
- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket

	\$			a			b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
	a	b	a	b	c	a	b	c	a	b	b	a	\$
	S	L	S*	S	L	S*	S	L	S*	L	L	L	S*
13	-	-	9	6	3	-	-	-	-	-	-	-	-
12	-	-	9	6	3	-	-	-	-	-	-	-	-
11	-	-	9	6	3	-	-	-	-	-	-	-	-
10	-	-	9	6	3	-	-	-	-	-	-	-	-
9	-	-	9	6	3	-	-	-	-	-	-	-	-
8	-	-	9	6	3	-	-	-	-	-	-	-	-
7	-	-	9	6	3	-	-	-	-	-	-	-	-
6	-	-	9	6	3	-	-	-	-	-	-	-	-
5	-	-	9	6	3	-	-	-	-	-	-	-	-
4	-	-	9	6	3	-	-	-	-	-	-	-	-
3	-	-	9	6	3	-	-	-	-	-	-	-	-
2	-	-	9	6	3	-	-	-	-	-	-	-	-
1	-	-	9	6	3	-	-	-	-	-	-	-	-

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket
- Scan Right to Left ( $i = n, n - 1, \dots, 1$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *S*-type
  - then put  $SA[i] - 1$  at end of bucket



# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

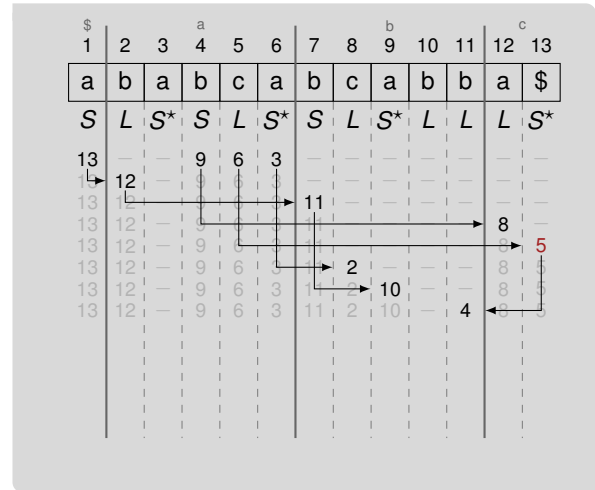
- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket
- Scan Right to Left ( $i = n, n - 1, \dots, 1$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *S*-type
  - then put  $SA[i] - 1$  at end of bucket

	\$		a				b				c		
	1	2	3	4	5	6	7	8	9	10	11	12	13
a	b	a	b	c	a	b	c	a	b	b	a	\$	
S	L	S*	S	L	S*	S	L	S*	L	L	L	S*	
13	-	-	9	6	3	-	-	-	-	-	-	-	-
12	-	-	9	6	3	-	-	-	-	-	-	-	-
11	-	-	9	6	3	-	-	-	-	-	-	-	-
10	-	-	9	6	3	-	-	-	-	-	-	-	-
9	-	-	9	6	3	-	-	-	-	-	-	-	-
8	-	-	9	6	3	-	-	-	-	-	-	-	-
7	-	-	9	6	3	-	-	-	-	-	-	-	-
6	-	-	9	6	3	-	-	-	-	-	-	-	-
5	-	-	9	6	3	-	-	-	-	-	-	-	-
4	-	-	9	6	3	-	-	-	-	-	-	-	-
3	-	-	9	6	3	-	-	-	-	-	-	-	-
2	-	-	9	6	3	-	-	-	-	-	-	-	-
1	-	-	9	6	3	-	-	-	-	-	-	-	-

# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

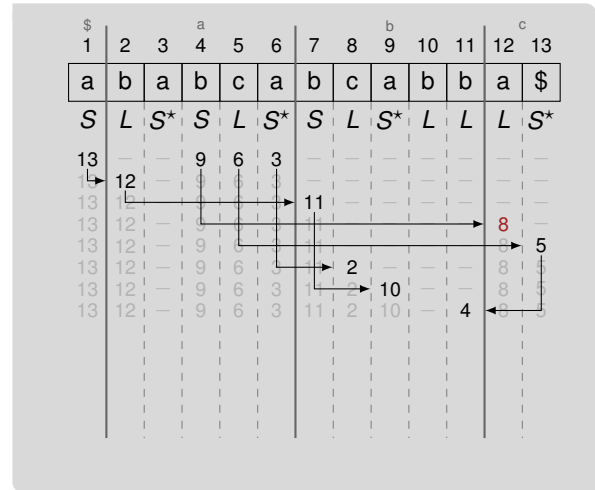
- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket
- Scan Right to Left ( $i = n, n - 1, \dots, 1$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *S*-type
  - then put  $SA[i] - 1$  at end of bucket



# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

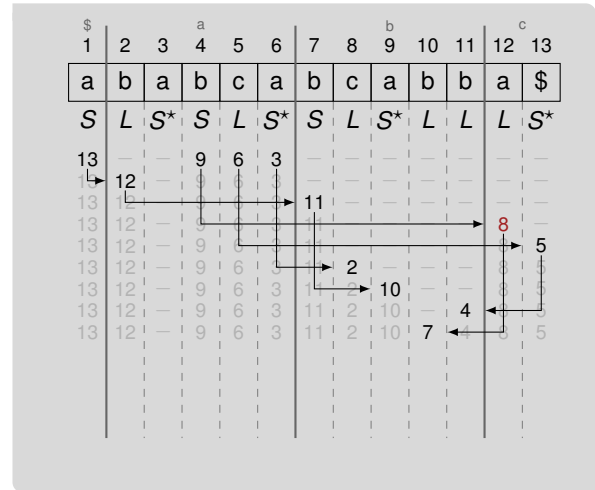
- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket
- Scan Right to Left ( $i = n, n - 1, \dots, 1$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *S*-type
  - then put  $SA[i] - 1$  at end of bucket



# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

- Initialization
  - initialize each entry in SA with “-”
  - put *sorted LMS*-suffixes at the end of buckets
- Scan Left to Right ( $i = 1, 2, \dots, n$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *L*-type
  - then put  $SA[i] - 1$  at beginning of bucket
- Scan Right to Left ( $i = n, n - 1, \dots, 1$ )
  - if  $SA[i] \neq -$  and  $T[SA[i] - 1..n]$  is *S*-type
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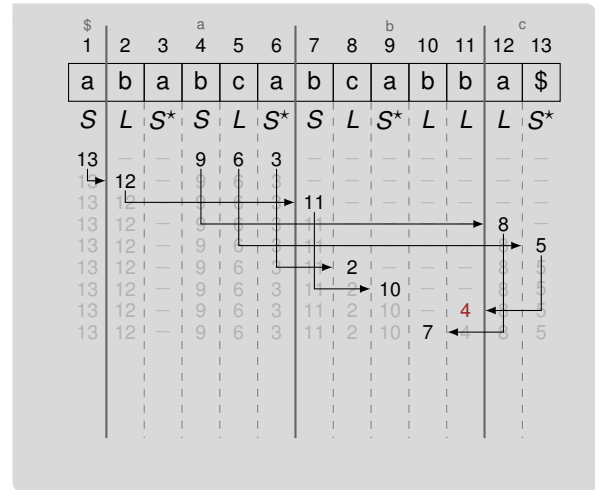




# Suffix Array Induced Sorting: Inducing (2/2)

## Inducing in SAIS

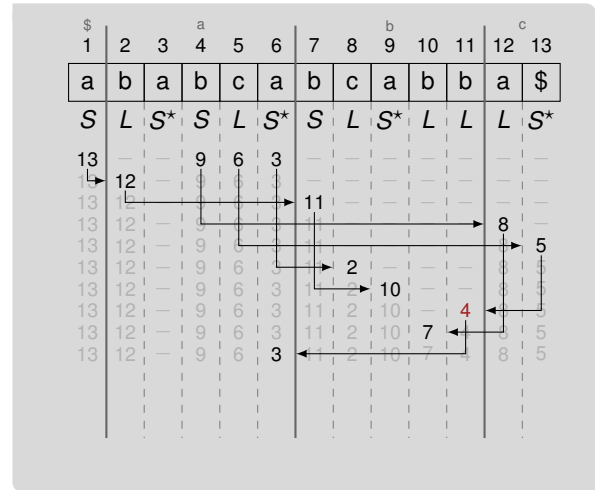
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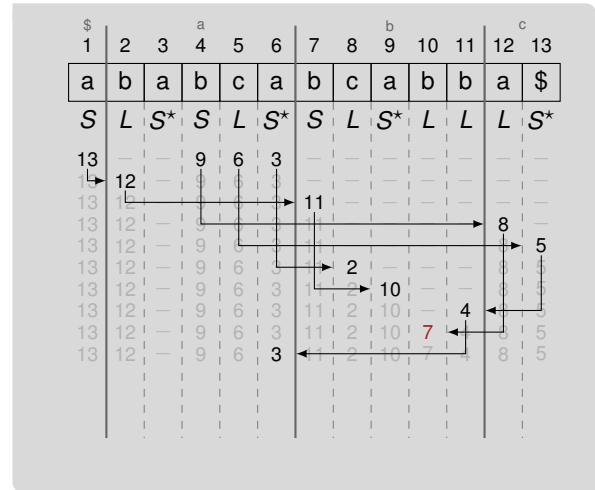
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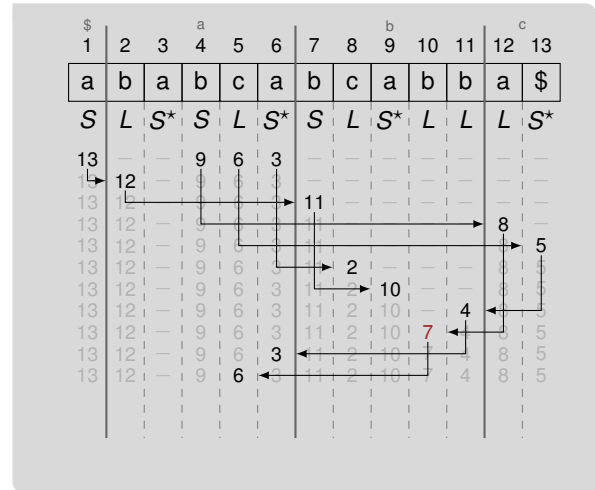
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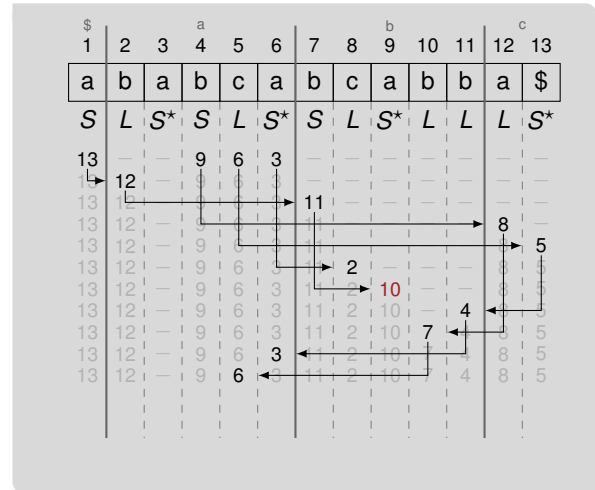
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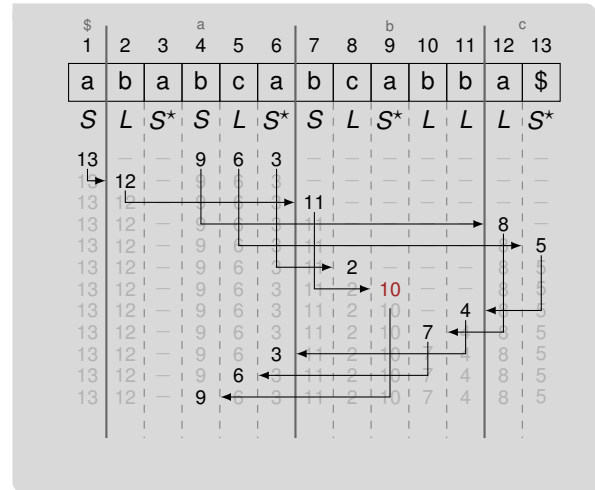
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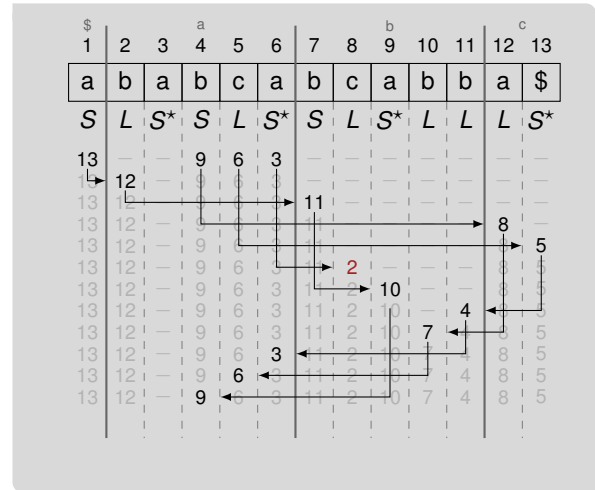
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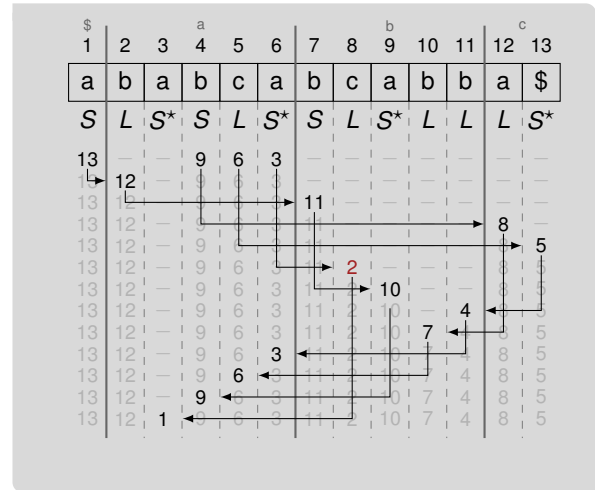
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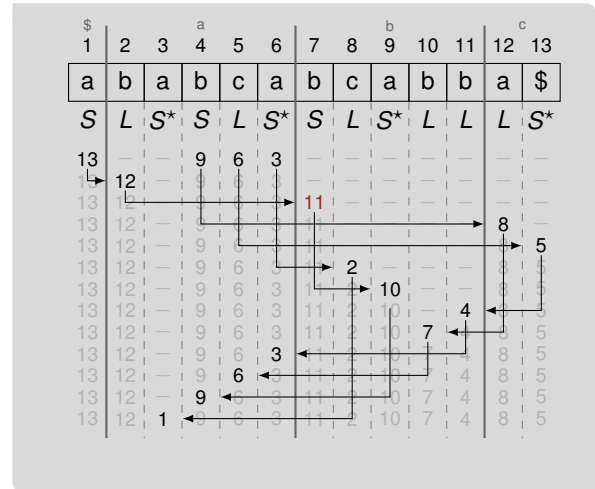




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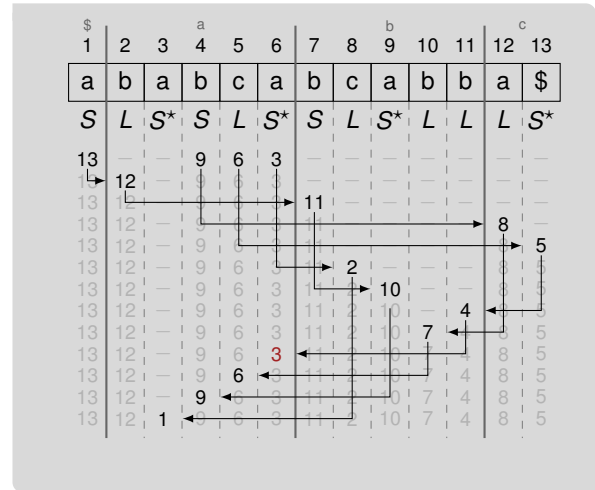
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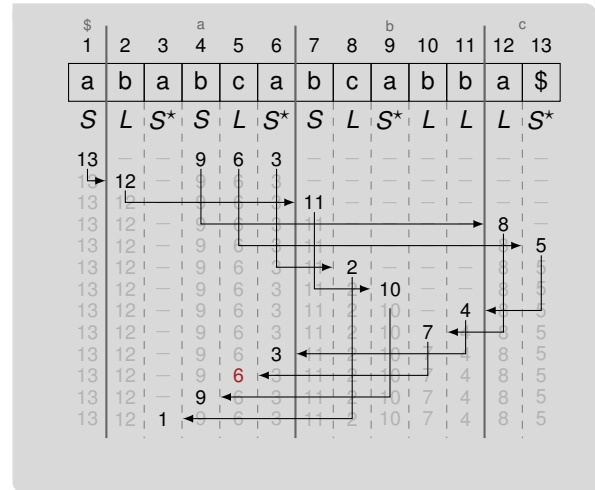
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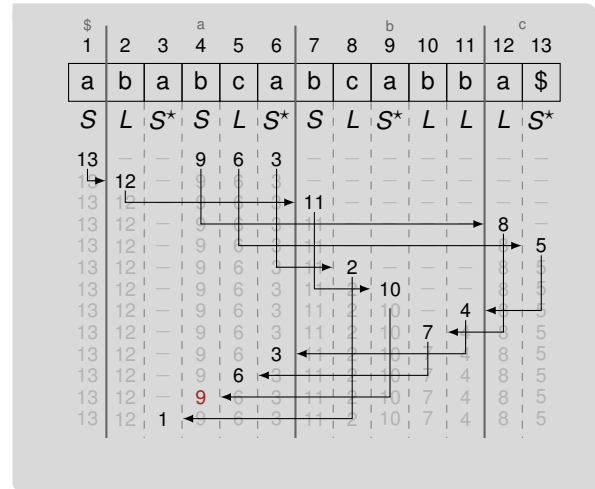
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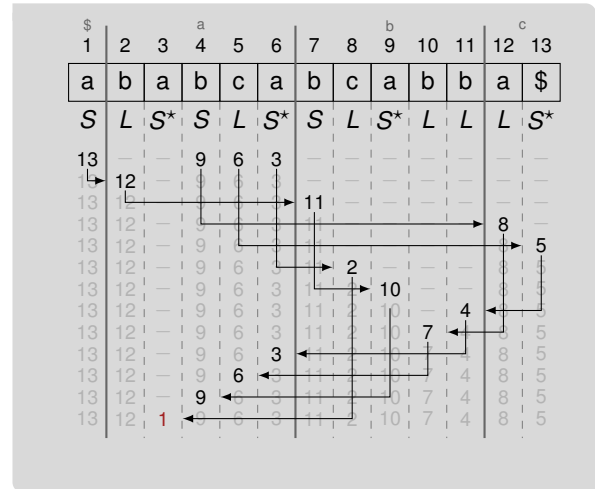
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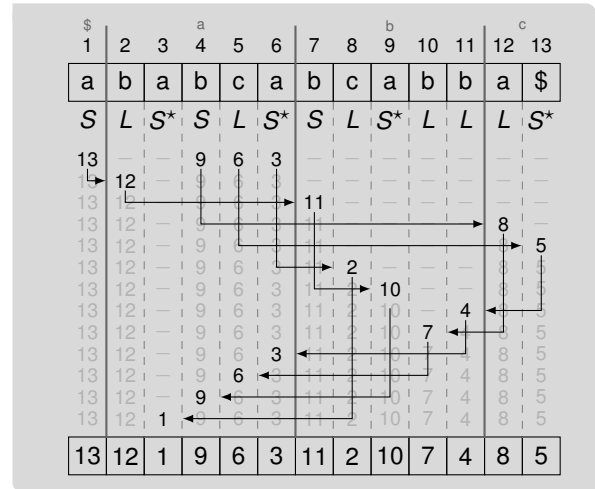
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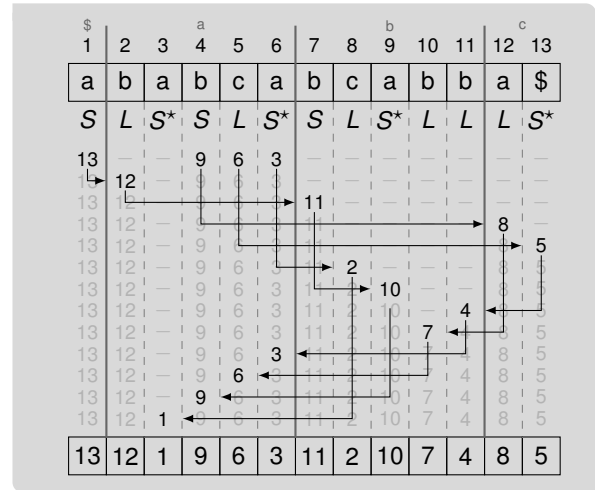
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- 
- are all suffixes induced?

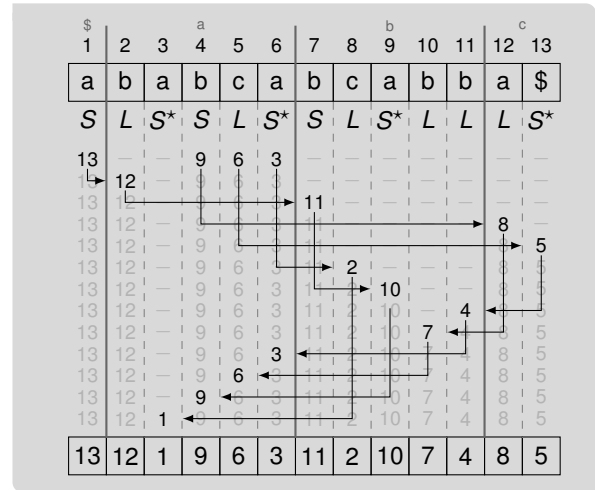


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  - then put  $SA[i] - 1$  at beginning of bucket
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	1	2	3	4	5	6	7	8	9	10	11	12	13
$T$	a	b	a	b	c	a	b	c	a	b	b	a	\$
SA	13	12	1	9	6	3	11	2	10	7	4	8	5
	\$	a	a	a	a	a	b	b	b	b	b	c	c
		\$	b	b	b	b	a	a	b	b	c	a	a
			a	b	c	c	\$	b	a	a	a	b	b
			b	a	a	a		c	\$	b	b	a	c
			c	\$	b	b		a		a	a	\$	a
			a		b	c		b		b	b		b
			b		a	a		c		a	a		a
			c		\$	b		a		\$	b		b
			a			b		b			a		a
			b			a		b			\$		\$
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Computing  $T'$  requires  $O(n)$  time

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		\$	b	b	b	b	a	a	b	c	c	a	a
			a	b	c	c	\$	b	a	a	a	b	b
			b	a	a	a		c	\$	b	b	b	c
			c	\$	b	b		a	b	a	a	a	a
			a		a	c		b			b	\$	b
			b		\$	a		a			a		a
			c			b		b			b		b
			a			a		a			a		a
			b			b		b			\$		\$
			a			a		a					\$

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			a	b	c	c	\$	b	a	a	a	b	b
			b	a	b	b		c	b	b	b	a	b
			c	\$	a	c		a	a	a	a	\$	a
			a		\$	a		b		\$	b		b
			b			b		a			a		a
			a			b		b			a		\$
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			b	a	b	b		c	\$	b	b	a	b
			c	\$	a	c		a		a	a	\$	c
			a		b	a		b		b	b		b
			b		\$	b		a		a	a		a
			c			a		b					\$
			a			b		b					
			b			a		a					
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SA	13	12	1	9	6	3	11	2	10	7	4	8	5
	\$	a	a	a	a	a	b	b	b	b	b	c	c
		\$	b	b	b	b	a	a	b	c	c	a	a
			a	b	c	c	\$	b	a	a	a	b	b
			b	a	b	b		c	\$	b	b	a	b
			c	\$	a	c		a		a	a	\$	c
			a		b	a		b		b	b		b
			b		\$	b		a		a	a		a
			c			a		b					\$
			a			b		b					
			b			a		a					
			a			\$		b					
			\$					\$					

# Suffix Array Induced Sorting: Recursion

## Lemma: Running Time Computation $T'$

Computing  $T'$  requires  $O(n)$  time

## Proof (Sketch)

- find LMS-substrings in  $O(1)$  time  save  $S$ -buckets
  - scan each LMS-substring twice
  - each character is in at most two LMS-substrings
- 
- construct text  $T'$  using ranks of LMS-substrings
  - compare LMS-substrings character-wise

	1	2	3	4	5	6	7	8	9	10	11	12	13
$T$	a	b	a	b	c	a	b	c	a	b	b	a	\$
SA	13	12	1	9	6	3	11	2	10	7	4	8	5
	\$	a	a	a	a	a	b	b	b	b	b	c	c
		\$	b	b	b	b	a	a	b	c	c	a	a
			a	b	c	c	\$	b	a	a	a	b	b
			b	a	b	b		c	b	b	b	a	b
			c	\$	b	c		a	a	a	a	\$	a
			a		a	a		b	b	\$	b	b	b
			b		\$	b		a	b		a	a	a
			b			a		b	b		\$		\$
			a			\$		\$					\$

■  $T' = 0122\$$

# Suffix Array Induced Sorting: Running Time

## Lemma: SAIS Time Complexity

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## Space Requirements

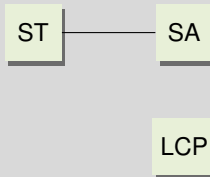
- naive:  $O(n \lg n)$  bits
- better:  $n \lceil \lg n \rceil + 2\sigma \lceil \lg n \rceil$  bits 

# Conclusion and Outlook

## This Lecture

- suffix trees and suffix arrays
- linear time suffix array construction

## Linear Time Construction



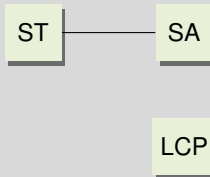
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- suffix trees require  $\approx 8\text{--}20$  bytes per character
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  - currently fastest implementation available at <https://github.com/IlyaGrebnev/libsa>



## Linear Time Construction



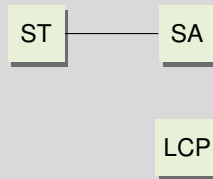
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## Linear Time Construction



## Next Lecture

- linear time LCP-array construction
- interesting properties of LCP-array
- computing suffix trees using suffix array and LCP-array

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