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Probability and Computing – Basic Notions and Notation

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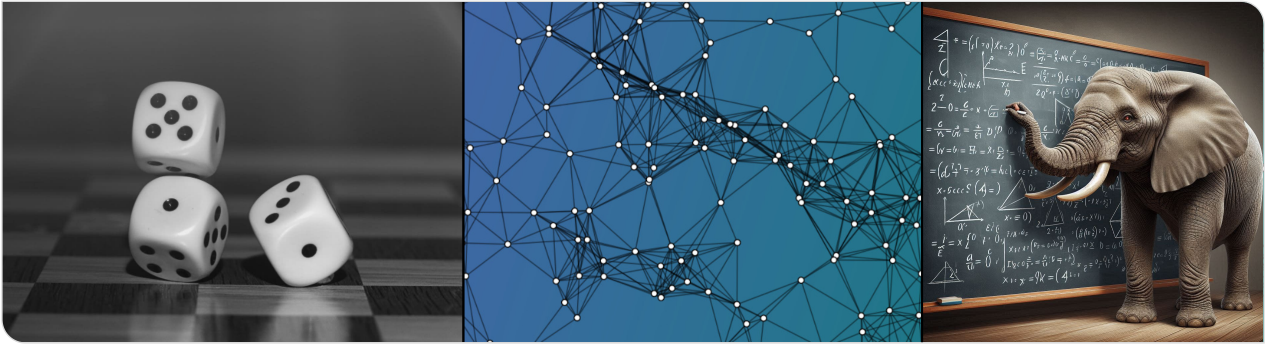

Basic Notions from Probability Theory

The following holds for *discrete* probability spaces.[1] Gray = typically supressed in notation.

German	English	standard notation & meaning
Ergebnismenge	sample space	set Ω
Ergebnis	outcome	element $\omega \in \Omega$
Wahrscheinlichkeitsfunktion	probability mass function	$p : \Omega \to [0,1]$ with $\sum_{\omega \in \Omega} p(\omega) = 1$
Wahrscheinlichkeitsraum	probability space	(Ω, p)
Ereignis	event	subset $E \subseteq \Omega$
Wahrscheinlichkeit	probability	$\Pr[E] = \sum_{\omega \in E} p(\omega)$ for event E
(reellwertige) Zufallsvariable	(real-valued) random variable	$X : \Omega \to \mathbb{R}$, a “random real number”: “$X \geq 2$” means “$X(\omega) \geq 2$”
Erwartungswert	expectation	$\mathbb{E}[X] = \sum_{\omega \in \Omega} p(\omega) \cdot X(\omega) = \sum_x x \cdot \Pr[X = x]$
Varianz	variance	$Var(X) = \mathbb{E}[(X - \mathbb{E}[X])^2]$
bedingte Wahrscheinlichkeit	conditional probability	$\Pr[E \mid C] = \Pr[E \cap C] / \Pr[C]$
bedingter Erwartungswert	conditional expectation	$\mathbb{E}[X \mid C] = \sum_x x \cdot \Pr[X = x \mid C]$
Verteilungsfunktion	cumulative distribution function (CDF)	$x \mapsto \Pr[X \leq x]$

[1] Continuous probability spaces are more complicated. The probability mass function is often a probability density function and sums become integrals. We use continuous probability spaces informally in this lecture.

Some Calculation Rules for Probabilities

Linearity of Expectation

$$\mathbb{E}[X_1 + \ldots + X_n] \overset{\text{lin.}}{=} \mathbb{E}[X_1] + \ldots + \mathbb{E}[X_n].$$

Tail Sum Formula

If $X : \Omega \to \mathbb{N}_0$ is a random variable assuming natural numbers then

$$\mathbb{E}[X] = \sum_{i=1}^{\infty} \Pr[X \geq i].$$

Union Bound

For any events $E_1, \ldots, E_n \subseteq \Omega$

$$\Pr[E_1 \cup \ldots \cup E_n] \overset{\text{UB}}{\leq} \Pr[E_1] + \ldots + \Pr[E_n].$$

Law of Total Probability

If $E_1, \ldots, E_n \subseteq \Omega$ are disjoint events with $E_1 \cup \ldots \cup E_n = \Omega$ and F is any event then

$$\Pr[F] \overset{\text{LTP}}{=} \sum_{i=1}^{n} \Pr[F \mid E_i] \cdot \Pr[E_i].$$

Law of Total Expectation

If $E_1, \ldots, E_n \subseteq \Omega$ are disjoint events with $E_1 \cup \ldots \cup E_n = \Omega$ and X is a random variable then

$$\mathbb{E}[X] \overset{\text{LTE}}{=} \sum_{i=1}^{n} \mathbb{E}[X \mid E_i] \cdot \Pr[E_i].$$