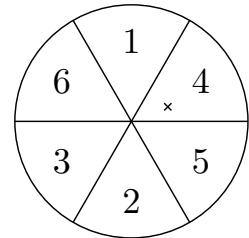# Exercise Sheet 1 – Basic Concepts

Probability and Computing

Exercise 1 – I probably still know it . . .

A darts player throws a dart at a dartboard. The board is divided into 6 equally sized segments, each assigned a different score from $\{1, \ldots, 6\}$. Since the player is still in training, the dart lands uniformly at random on the board (but never off target). In the example on the right, the throw scored 4 points.

(a) Model the random experiment of the throw (not the resulting score) by describing the sample space and a probability measure of a continous probability space.

(b) We are now particularly interested in the following properties of a throw:

- the distance of the dart tip from the center of the board
- the resulting score
- the resulting score is even
- the resulting score modulo 2

Define the corresponding random variables and events.

(c) Determine the cumulative distribution function of the distance from part (b).

In the following, we are no longer interested in the position of the dart tip, but only in the resulting score.

(d) Model this random experiment by specifying a suitable *discrete* probability space.

(e) Let X be the random variable representing the score. Determine the following values:

- The expectation of X
- The variance of X
- The expectation of $X \cdot \mathbb{1}_{X \text{ is odd}}$. This is the expectation of the score in a game variant where only odd numbers count.
- The expectation of a throw that produced an odd score.

Exercise 2 – Analogies to the calculation rules

Let Ω be the set of inhabitants of the distant land Omegon. Consider the following four statements and identify which of the five calculation rules from the slides each one is analogous to. For the remaining rule, come up with your own analogy. Argue (formally or intuitively, as you wish) why the calculation rules hold.

1. Let h be the proportion of dog owners, k the proportion of cat owners, and t the proportion of inhabitants with a dog or a cat. Then: $t \leq h + k$.

2. Suppose 40% of the inhabitants live in the west, the rest in the east. If g_1 is the average height of westerners and g_2 the average height of easterners, then $g_1 \cdot 0.4 + g_2 \cdot 0.6$ is the average height in Omegon.

3. Suppose 40% of the inhabitants live in the west, the rest in the east. Let k_1 be the proportion of cat owners among westerners and k_2 the proportion of cat owners among easterners. Then the total proportion of cat owners is $k = k_1 \cdot 0.4 + k_2 \cdot 0.6$.

4. If an inhabitant eats on average w white and b brown chicken eggs per year, then on average they eat $w + b$ chicken eggs per year.