Karlsruhe Institute of Technology

Stefan Walzer, Stefan Hermann
ITI Prof. Sanders
winter term 2025/2026

# Exercise Sheet 5 – Randomized Complexity Classes

## Probability and Computing

## Exercise 1 – Relations between Complexity Classes

Justify the following inclusions:

(i) **ZPP** $\subseteq$ **RP** and **ZPP** $\subseteq$ co–**RP**

(ii) **P** $\subseteq$ **ZPP**

(iii) **RP** $\subseteq$ **NP** and co–**RP** $\subseteq$ co–**NP**

(iv) **RP** $\subseteq$ **BPP** and co–**RP** $\subseteq$ **BPP**

(v) **BPP** $\subseteq$ **PP**

## Exercise 2 – Las Vegas Algorithm for $L$ implies $L \in$ ZPP

Let **LV** (for Las Vegas) denote the class of all languages $L$ for which there exists a probabilistic Turing machine $T$ with the following properties:

- $T$ decides $L$ (that is, $T$ outputs 1 for all $x \in L$ and 0 for all $x \notin L$).
- There exists a polynomial $p(n)$ such that the *expected* runtime of $T$ on input $x$ is bounded by $p(|x|)$.

In the lecture we proved that **ZPP** $\subseteq$ **LV**. Show that also **LV** $\subseteq$ **ZPP** holds. Thus, the two classes are identical.

**Hint:** Definition of **ZPP**, Markov inequality.

## Exercise 3 – Bonus: Probability Amplification for BPP

Look up on Wikipedia what the complexity class **P**/**poly** means. Show that **BPP** $\subseteq$ **P**/**poly**. This insight is also known as *Adleman's Theorem*.

**Hint:** Make the error probability smaller than $2^{-n}$. Transform the PTM into a DTM and show that there exists a random string that yields the correct result for all inputs.