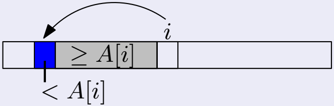
New Solution

Definition (Previous Smaller Values)

For array index i in A, let

$$PSV(i) = \text{argmax}\{k < i : A[k] < A[i]\}$$



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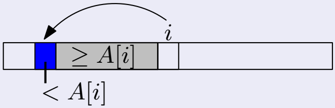

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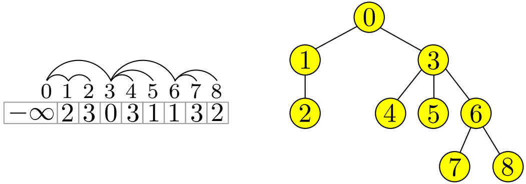
Definition (2d-Min-Heap of array A)

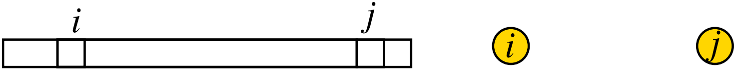
Ordered Tree on nodes $[1, n]$ defined by $parent(i) = PSV(i)$.



Relating 2d-Min-Heaps with RMQs

Lemma

$\mathrm{RMQ}(i,j)$ *is given by the child of* $\mathrm{LCA}(i,j)$ *that is on the path to* j*.*



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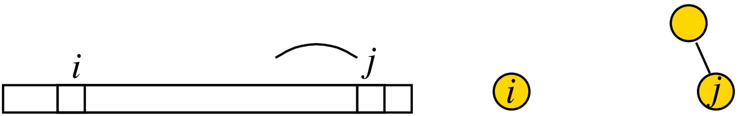

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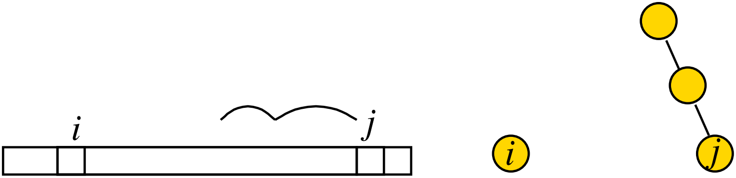

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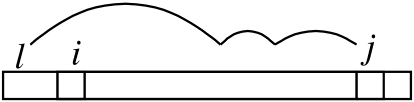

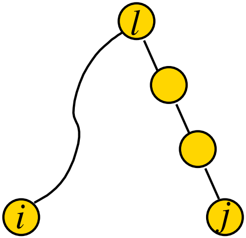

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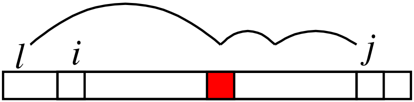
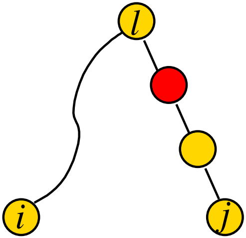
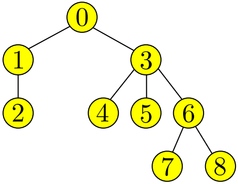




Representation and Construction of 2d-Min-Heaps

- Represent heap succinctly by DFUDS:
  - list degrees of nodes in pre-order:
  - node of out-degree $k \Rightarrow (^k)$
  - $\Rightarrow$ space $2n$ bits
  - $\Rightarrow$ array-index i corresponds to i'th ')'



$((()())((()))(()))$

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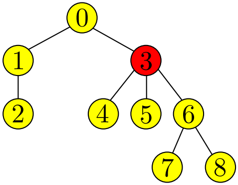

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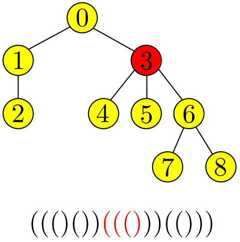

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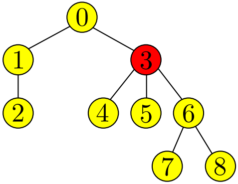


((()())((()))(()))

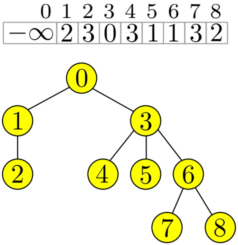
Theorem

There is a preprocessing scheme of optimal size $2n + o(n)$ bits for $\mathcal{O}(1)$-range minimum queries. Workspace is also $O(n)$ bits.

More Functionality: PSV

Definition (2d-Min-Heap)

Ordered Tree defined by $parent(i) = PSV(i)$.



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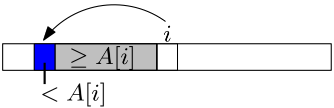

$\Rightarrow$ PSV simple (move to parent in $\mathcal{O}(1)$ time!)

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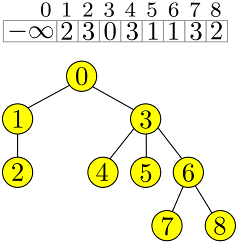

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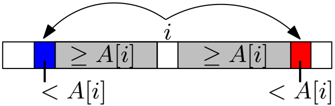

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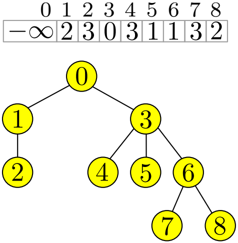

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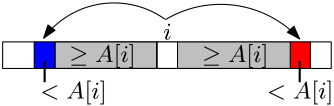
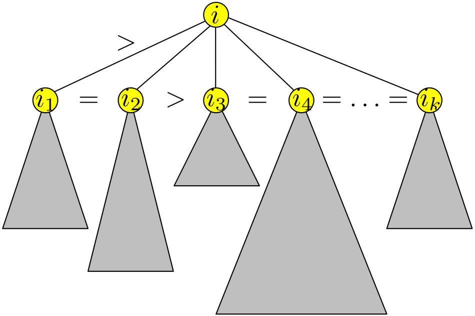
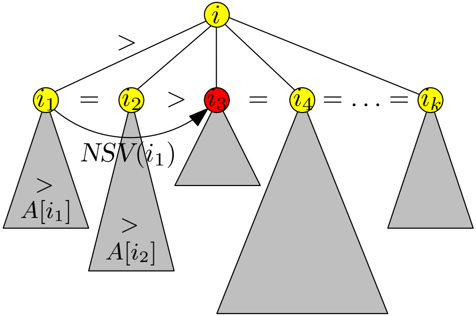


Can we also do NSV???

More Functionality: NSV

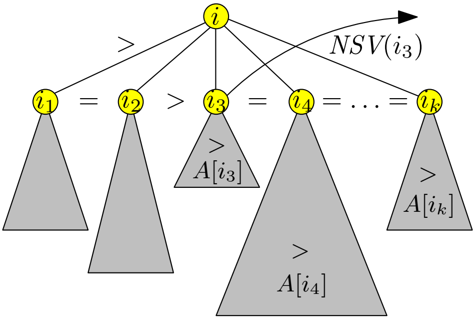


More Functionality: NSV



1. Find leftmost $<$-sibling to the right

More Functionality: NSV



1. Find leftmost $<$-sibling to the right
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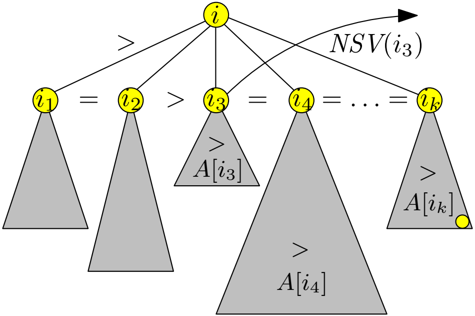

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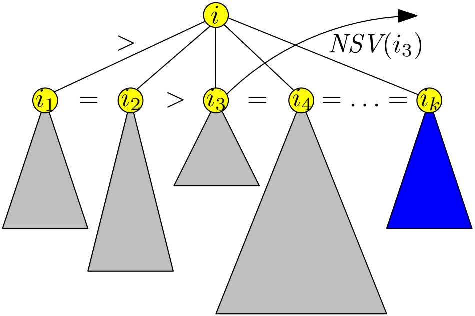


1. Find leftmost $<$-sibling to the right
2. If it does not exist…
3. … $NSV(i_3) = i_k + |T_{i_k}|$

More Functionality: NSV

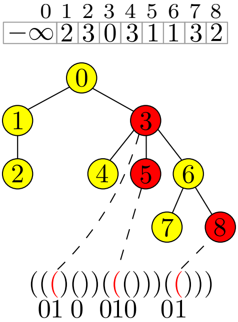
- Distinguish $=$- and $<$-siblings?

$\rightsquigarrow$ Mark $<$-children in additional bit-vector

- Bit-tricks for $\mathcal{O}(1)$-computations

Theorem (Extended 2d-Min-Heap)

$3n + o(n)$ bits suffice to support RMQ, PSV and NSVs in $\mathcal{O}(1)$ time.



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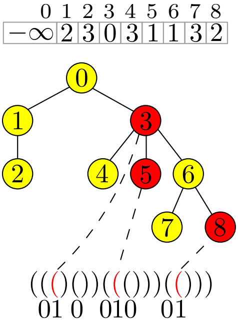

Theorem (Extended 2d-Min-Heap)

$3n + o(n)$ bits suffice to support RMQ, PSV and NSVs in $\mathcal{O}(1)$ time.

- Not necessarily optimal…
- … $\leq 2.54\ldots n$ possible (Schröder Tree!)