Algorithm Engineering for Large Graphs

Fast Route Planning

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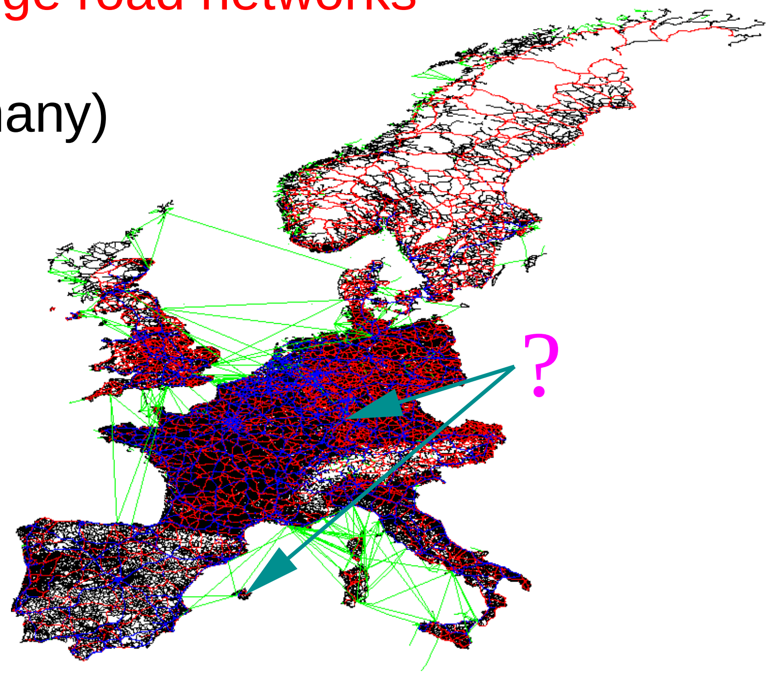
Route Planning

Goals:

- exact shortest (i.e. fastest) paths in large road networks
- fast queries (point-to-point, many-to-many)
- fast preprocessing
- low space consumption
- fast update operations



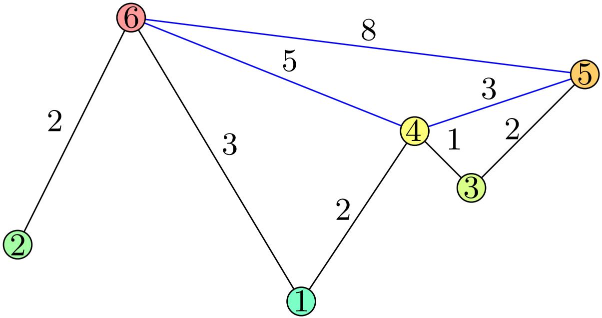
Applications:

- route planning systems in the internet, car navigation systems,
- ride sharing, traffic simulation, logistics optimisation

Overview

- Exact Contraction Hierarchies – a very simple approach
- Transit Node Routing – getting really fast
- Mobile Contraction Hierarchies
- Many-to-many Routing
- Ride Sharing
- Dynamic Scenario
- Time-dependent Contraction Hierarchies
- Future Work

Contraction Hierarchies (CH)



Main Idea

Contraction Hierarchies (CH)

- contract only one node at a time
  $\Rightarrow$ local and cache-efficient operation

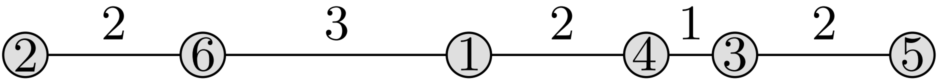
in more detail:

- order nodes by "importance", $V = \{1, 2, \ldots, n\}$
- contract nodes in this order, node v is contracted by

  foreach *pair* (u, v) *and* (v, w) *of edges* **do**
      if $\langle u, v, w\rangle$ *is a unique shortest path* **then**
          add shortcut (u, w) with weight $w(\langle u, v, w\rangle)$

- query relaxes only edges to more "important" nodes
  $\Rightarrow$ valid due to shortcuts

Example: Construction



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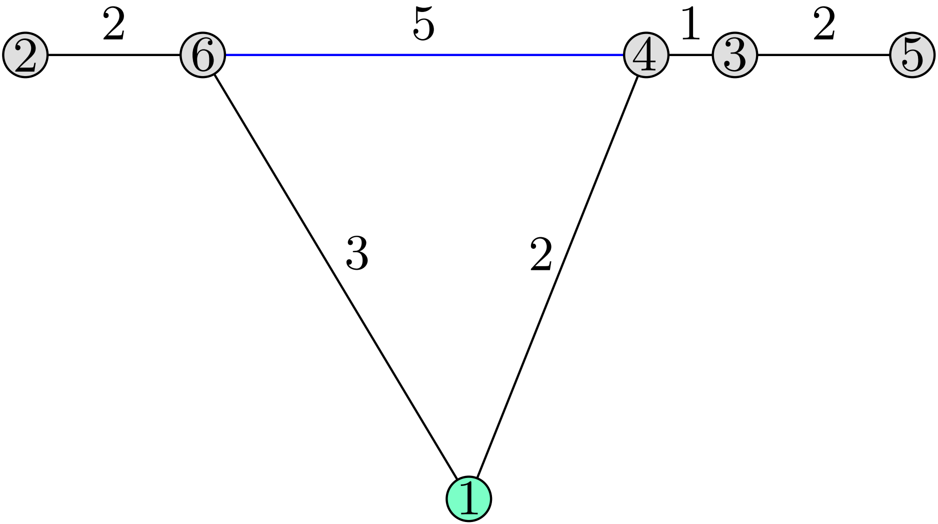

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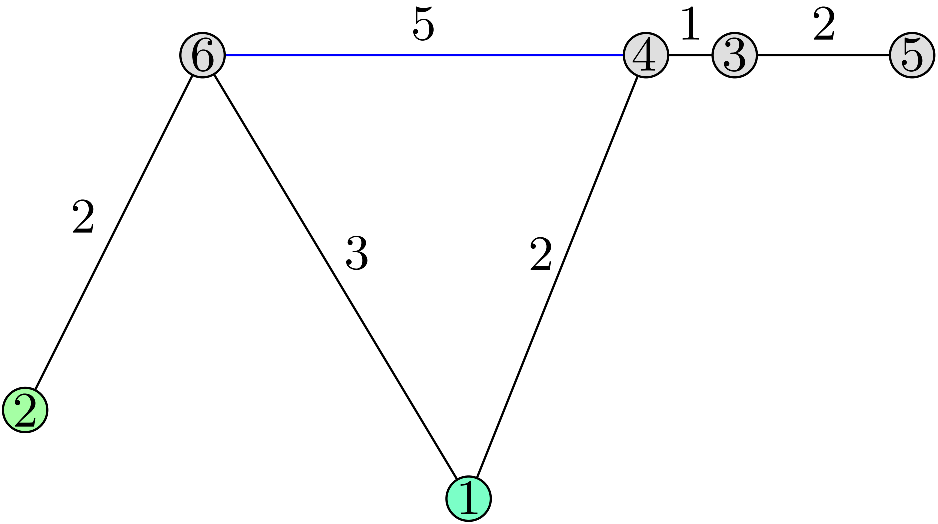

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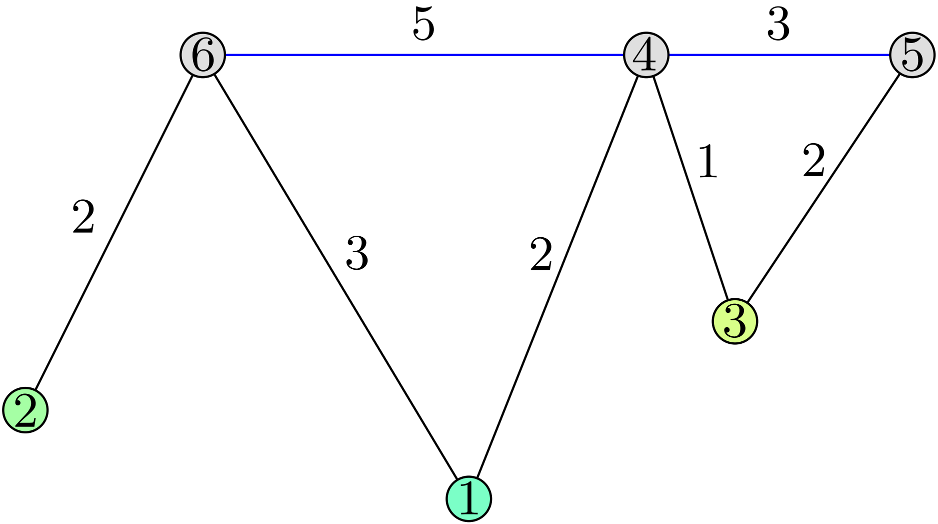

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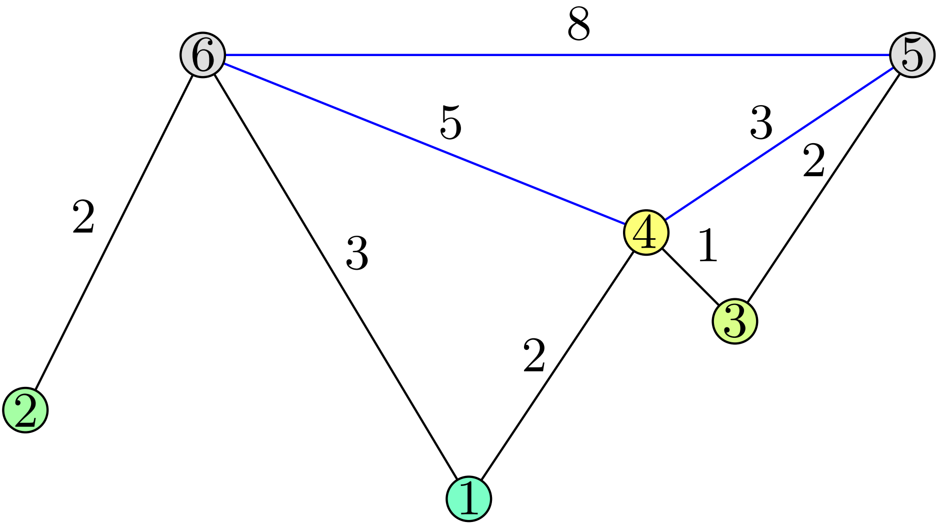

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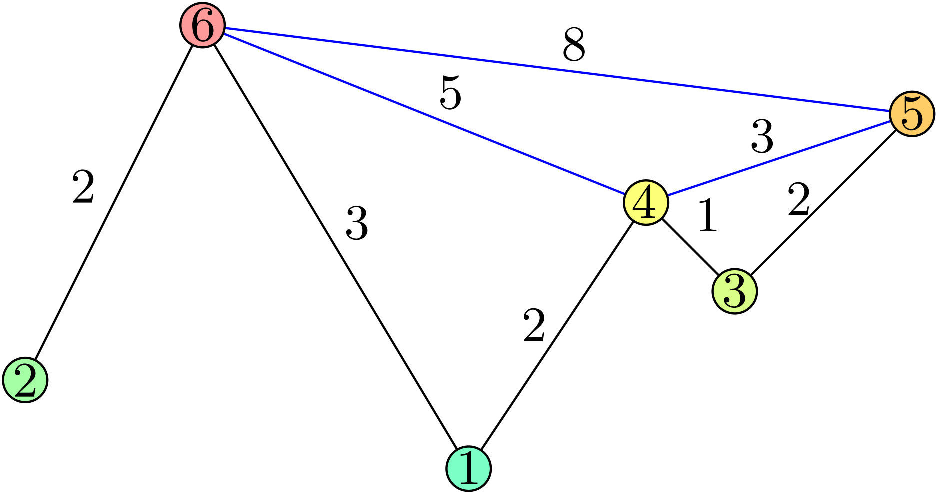
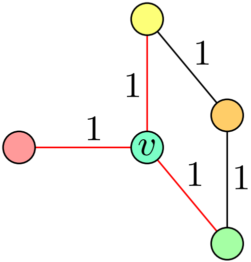


Construction

to identify necessary shortcuts

- local searches from all nodes u with incoming edge (u, v)
- ignore node v at search
- add shortcut (u, w) iff found distance $d(u, w) > w(u, v) + w(v, w)$



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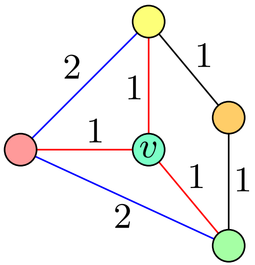
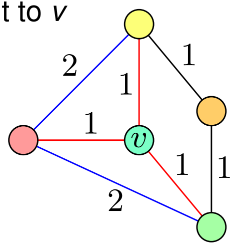


Node Order

use priority queue of nodes, node *v* is weighted with a linear combination of:

- **edge difference** #shortcuts – #edges incident to *v*
- **uniformity** e.g. #deleted neighbors
- ...



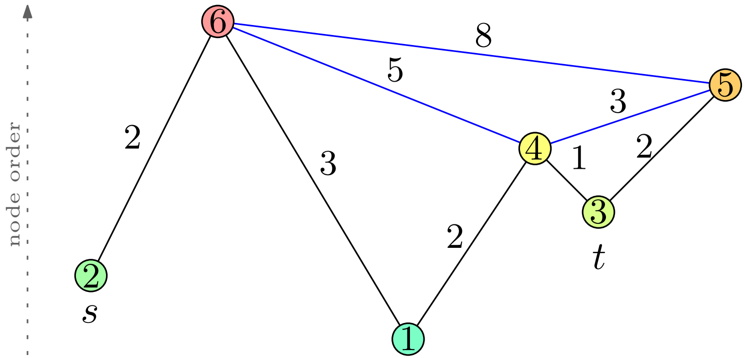
$2 - 3 = -1$

integrated construction and ordering:

1. remove node *v* on top of the priority queue
2. contract node *v*
3. update weights of remaining nodes

Query

- modified bidirectional Dijkstra algorithm
- upward graph $G_{\uparrow} := (V, E_{\uparrow})$ with $E_{\uparrow} := \{(u, v) \in E : u < v\}$
  downward graph $G_{\downarrow} := (V, E_{\downarrow})$ with $E_{\downarrow} := \{(u, v) \in E : u > v\}$
- forward search in $G_{\uparrow}$ and backward search in $G_{\downarrow}$



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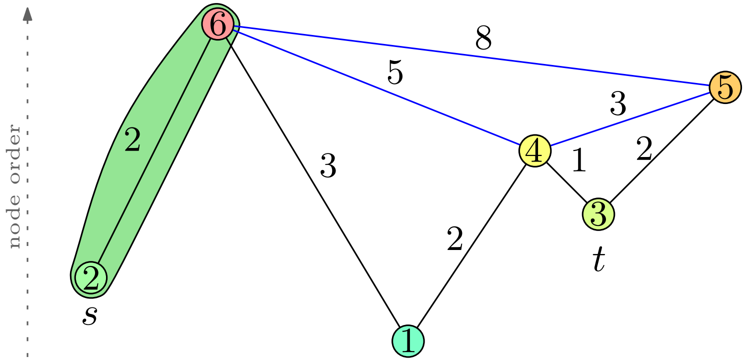

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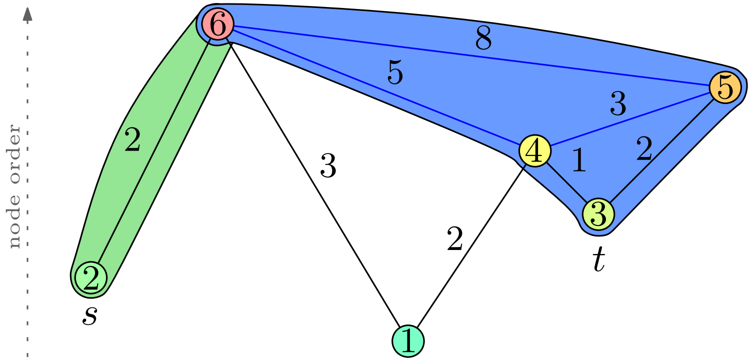

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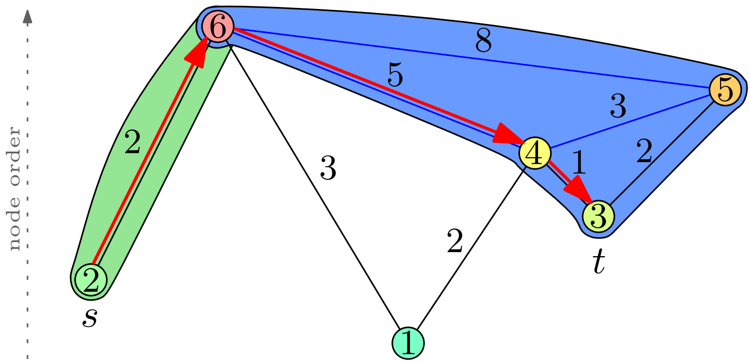
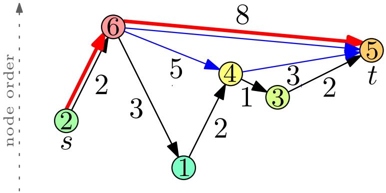
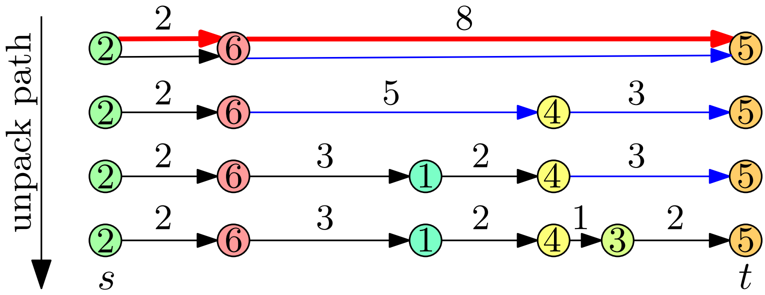


Outputting Paths

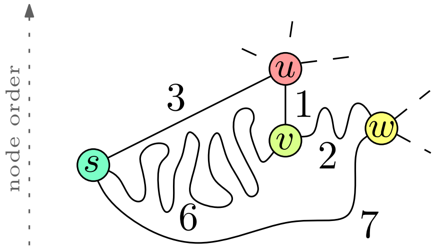
- for a shortcut (u, w) of a path $\langle u, v, w \rangle$, store middle node v with the edge
- expand path by recursively replacing a shortcut with its originating edges





Stall-on-Demand

- v can be "stalled" by u (if $d(u) + w(u, v) < d(v)$)
- stalling can propagate to adjacent nodes
- search is not continued from stalled nodes



- does not invalidate correctness (only suboptimal paths are stalled)

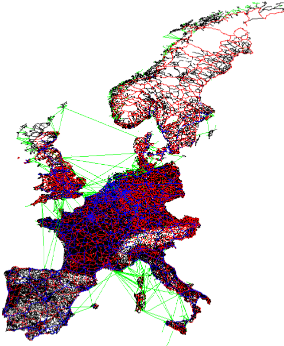
Experiments

environment

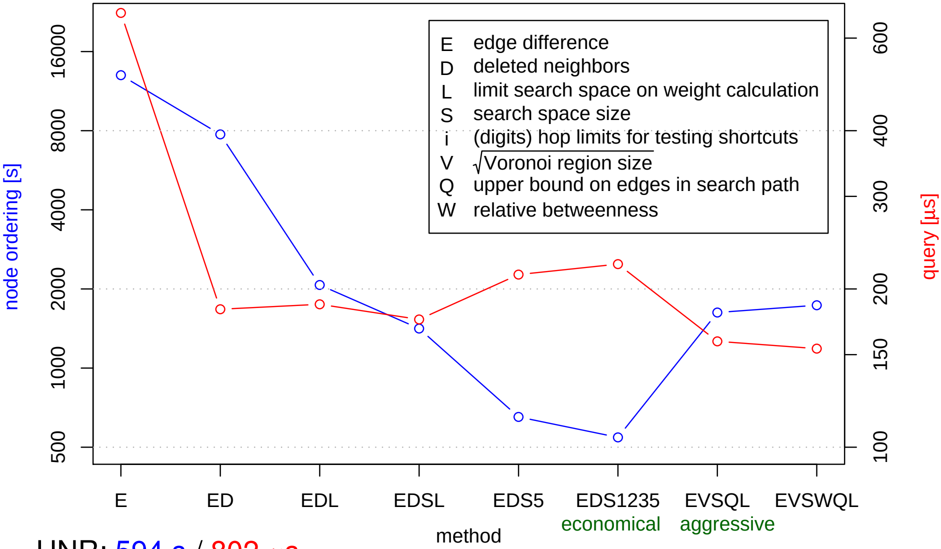
- AMD Opteron Processor 270 at 2.0 GHz
- 8 GB main memory
- GNU C++ compiler 4.2.1

test instance

- road network of Western Europe (PTV)
- 18 029 721 nodes
- 42 199 587 directed edges

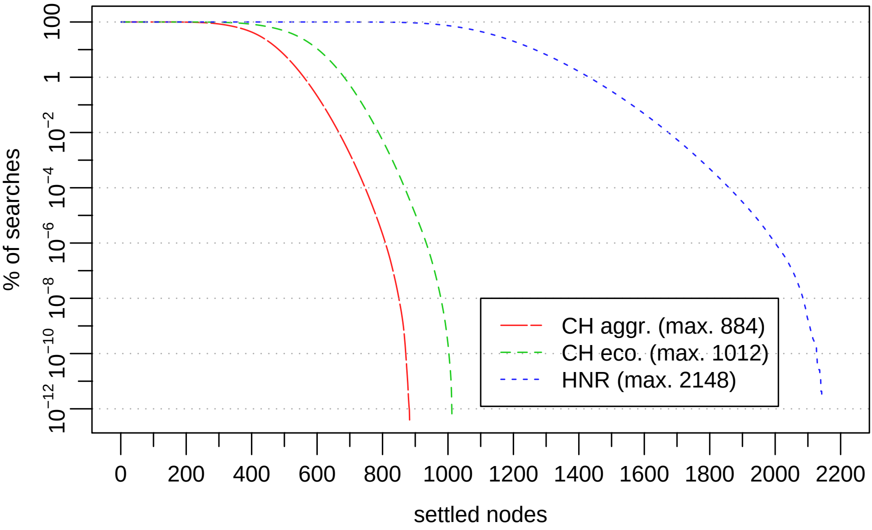


Performance

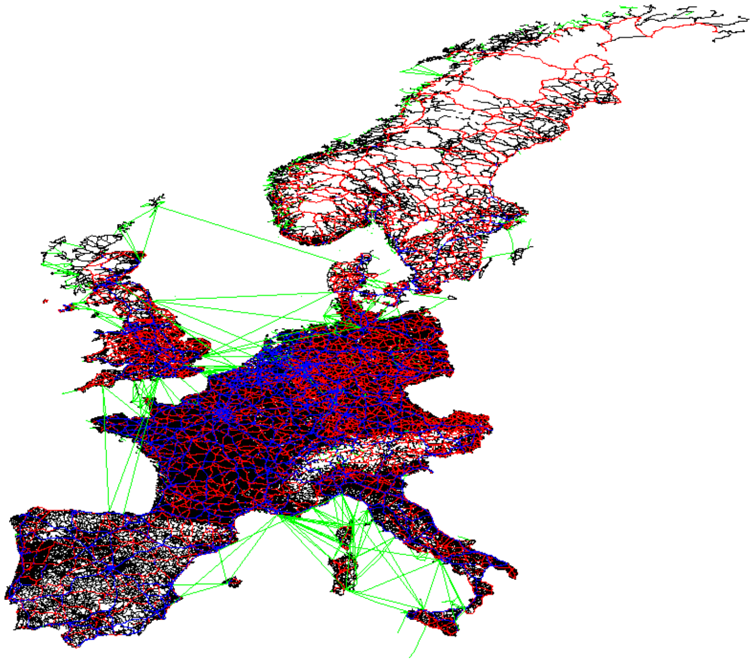


HNR: 594 s / 802 μs

Worst Case Costs



Contraction Hierarchies



- foundation for our other methods
- conceptually very simple
- handles dynamic scenarios

Static scenario:

- 7.5 min preprocessing
- 0.21 ms to determine the path length
- 0.56 ms to determine a complete path description
- little space consumption (23 bytes/node)

Transit-Node Routing

[DIMACS Challenge 06, ALENEX 07, Science 07]

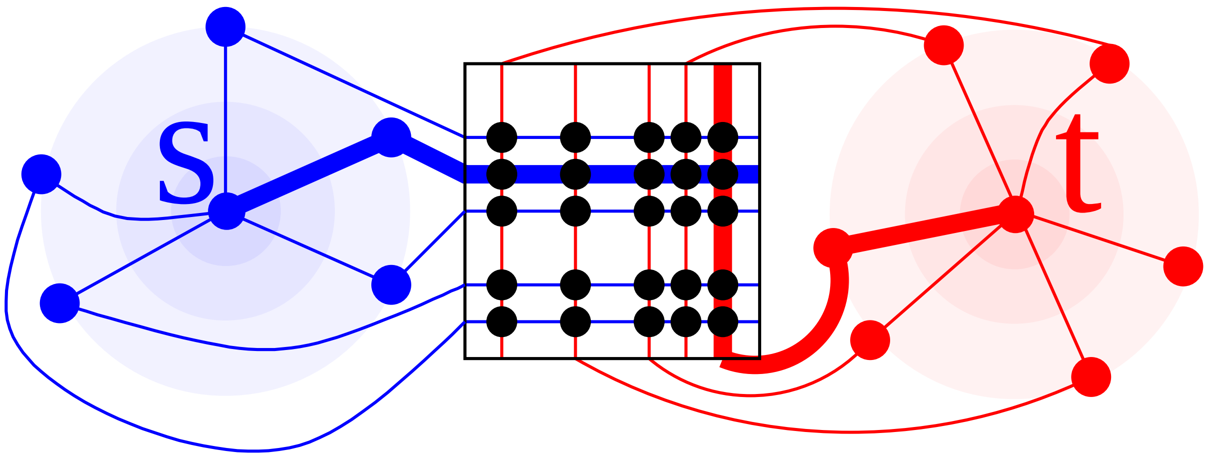
joint work with H. Bast, S. Funke, D. Matijevic

- *very* fast queries

  (down to 1.7 μs, 3 000 000 times faster than DIJKSTRA)

- winner of the 9th DIMACS Implementation Challenge

- more preprocessing time (2:37 h) and space (263 bytes/node) needed



SciAm50 Award

Mobile Contraction Hierarchies

[ESA 08]

- preprocess data on a personal computer
- highly compressed blocked graph representation 8 bytes/node
- compact route reconstruction data structure + 8 bytes/node

experiments on a Nokia N800 at 400 MHz

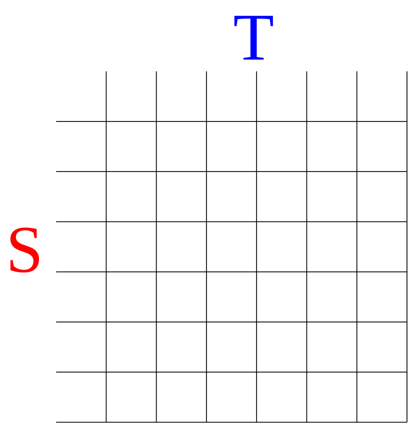
- cold query with empty block cache 56 ms
- compute complete path 73 ms
- recomputation, e.g. if driver took the wrong exit 14 ms

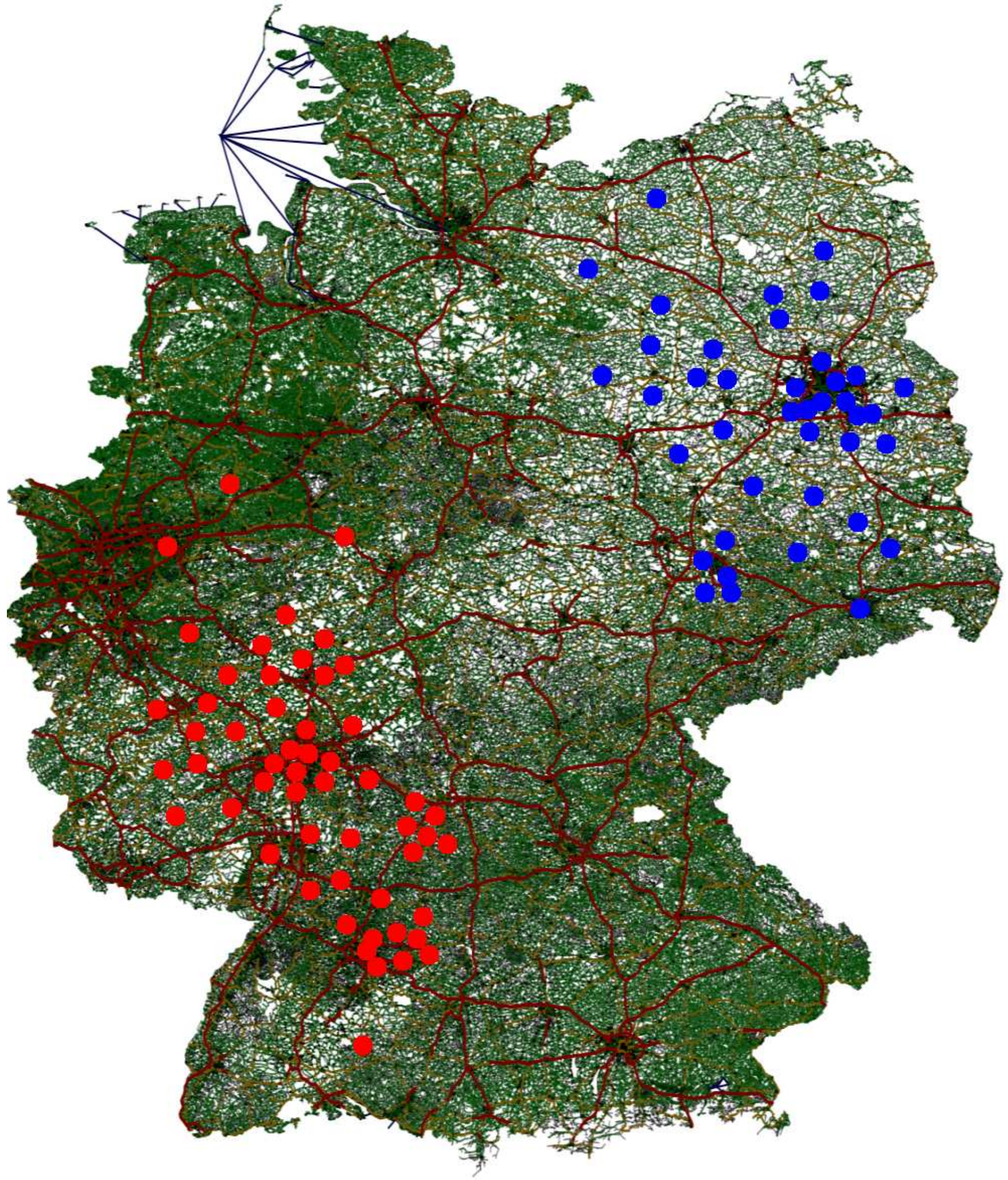
Many-to-Many Shortest Paths

joint work with S. Knopp, F. Schulz, D. Wagner

[ALENEX 07]

- efficient many-to-many variant of hierarchical bidirectional algorithms
- 10 000 × 10 000 table in 10s





Ride Sharing

Current approaches:

- match only ride offers with identical start/destination (perfect fit)
- sometimes radial search around start/destination

Our approach:

- driver picks passenger up and gives him a ride to his destination
- find the driver with the minimal detour (reasonable fit)

Efficient algorithm:

- adaption of the many-to-many algorithm

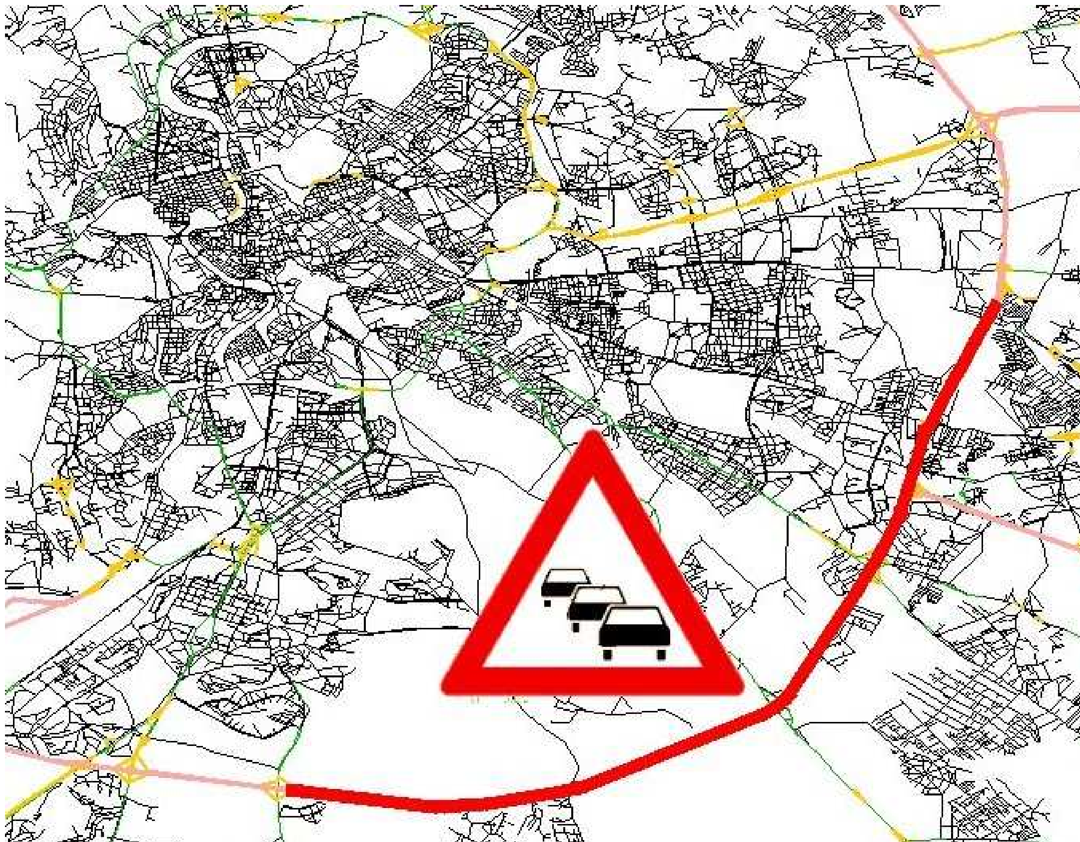
Highway-Node Routing

[WEA 07]

- generalization of contraction hierarchies
- allow multiple nodes in the same ‘importance’-level

  i.e., select node sets $S_1 \supseteq S_2 \supseteq S_3 \ldots$

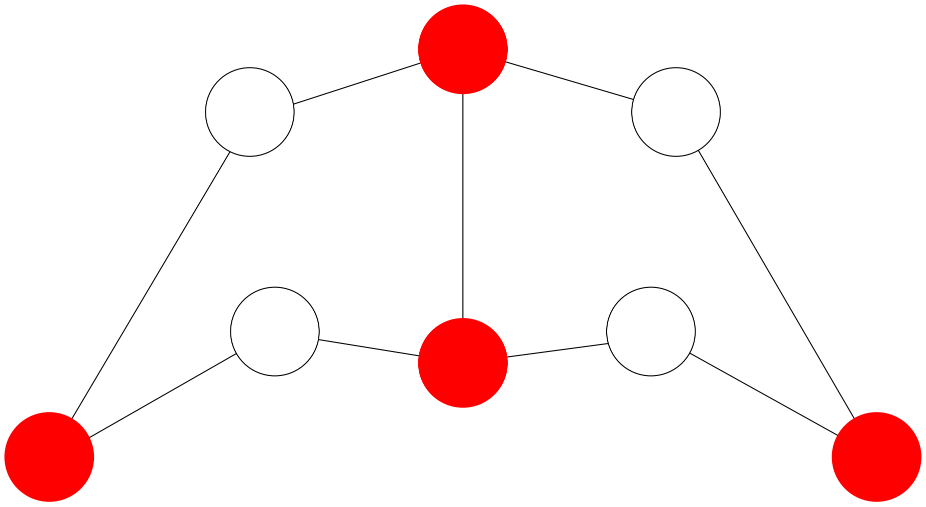
- construct multi-level overlay graph
- perform multi-level query
- designed for dynamic scenarios



Overlay Graph

[Holzer, Schulz, Wagner, Weihe, Zaroliagis 2000–2007]

- graph $G = (V, E)$ is given
- select node subset $S \subseteq V$



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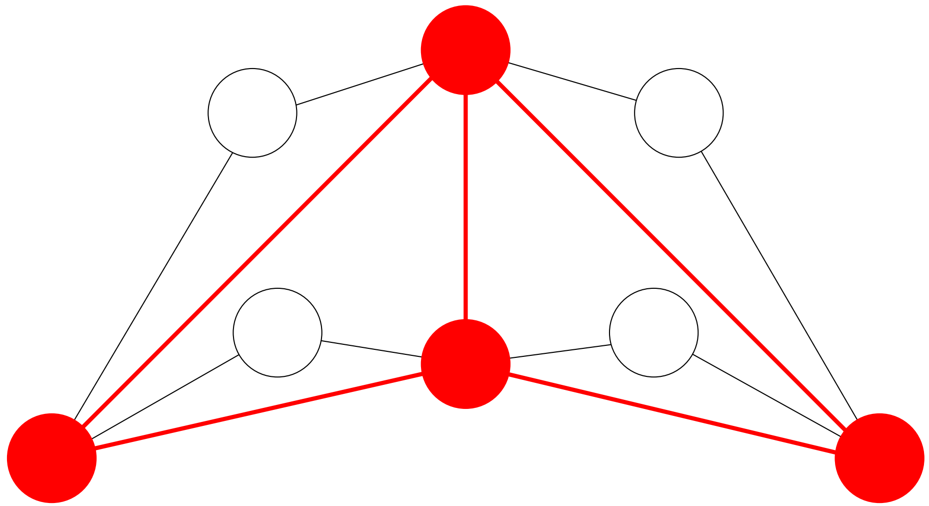


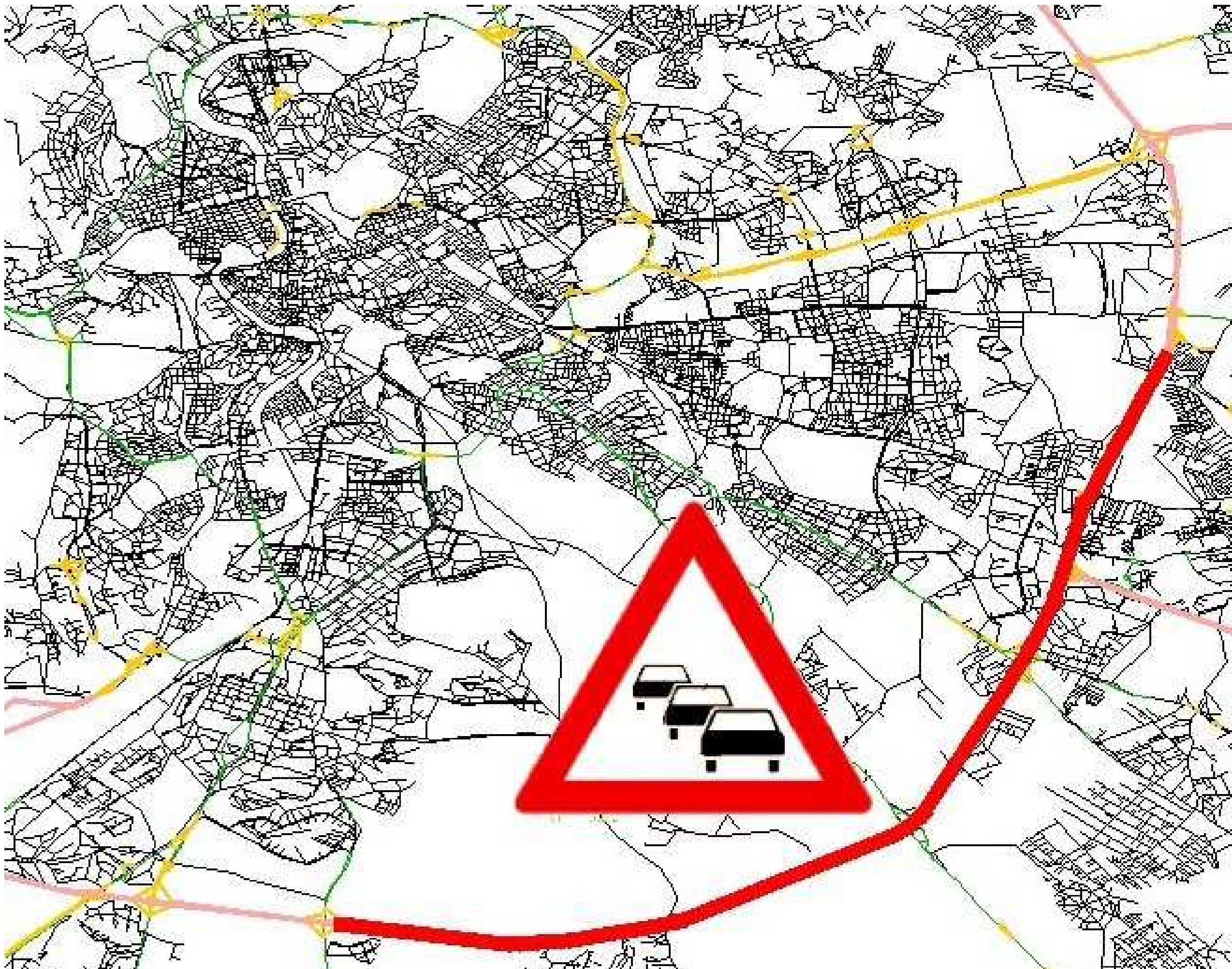
- overlay graph $G' := (S, E')$ where

$$E' := \{(s,t) \in S \times S \mid \text{no inner node of the shortest } s\text{-}t\text{-path belongs to } S\}$$

Dynamic Scenarios

- change entire cost function
  (e.g., use different speed profile)

- change a few edge weights
  (e.g., due to a traffic jam)



Constancy of Structure

Assumption:

- structure of road network does not change

  (no new roads, road removal = set weight to ∞)

  $\leadsto$ not a significant restriction

- classification of nodes by 'importance' might be slightly perturbed, but not completely changed

  (e.g., a sports car and a truck both prefer motorways)

  $\leadsto$ performance of our approach relies on that

  (not the correctness)

Dynamic Highway-Node Routing

change entire cost function

- keep the node sets $S_1 \supseteq S_2 \supseteq S_3 \ldots$
- recompute the overlay graphs

speed profile	default	fast car	slow car	slow truck	distance
constr. [min]	1:40	1:41	1:39	1:36	3:56
query [ms]	1.17	1.20	1.28	1.50	35.62
#settled nodes	1 414	1 444	1 507	1 667	7 057

Dynamic Highway-Node Routing

change a few edge weights

- **server scenario:** if something changes,
  - update the preprocessed data structures
  - answer many subsequent queries very fast

- **mobile scenario:** if something changes,
  - it does not pay to update the data structures
  - perform single 'prudent' query that takes changed situation into account

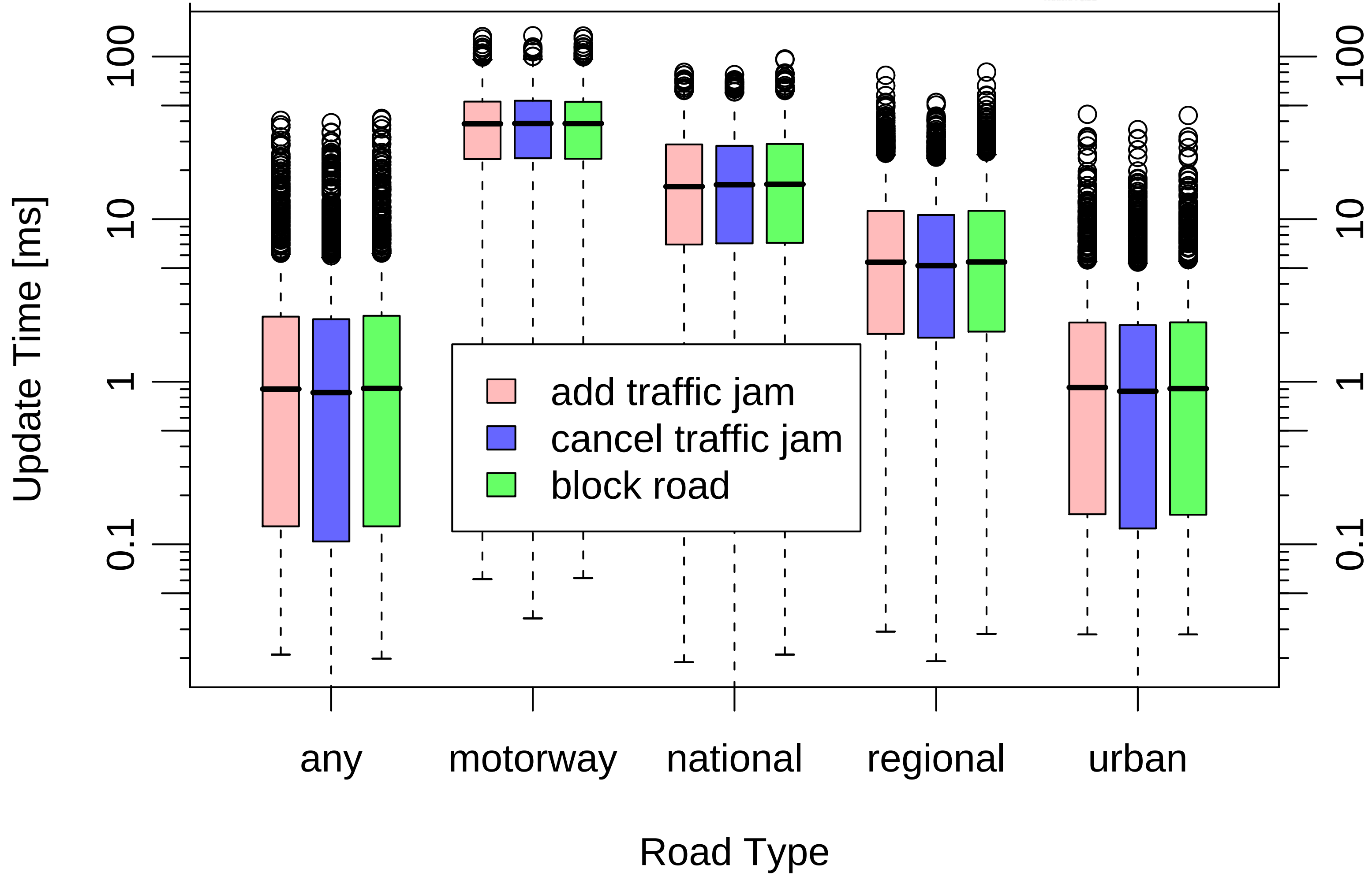
Dynamic Highway-Node Routing

change a few edge weights, server scenario

- keep the node sets $S_1 \supseteq S_2 \supseteq S_3 \ldots$
- recompute only possibly affected parts of the overlay graphs
  - the computation of the level-ℓ overlay graph consists of $|S_\ell|$ local searches to determine the respective covering nodes
  - if the initial local search from $v \in S_\ell$ has not touched a now modified edge (u,x), that local search need not be repeated
  - we manage sets $A_u^\ell = \{v \in S_\ell \mid v$'s level-$\ell$ preprocessing might be affected when an edge (u,x) changes$\}$

Dynamic Highway-Node Routing

change a few edge weights, server scenario



Dynamic Highway-Node Routing

change a few edge weights, mobile scenario

1. keep the node sets $S_1 \supseteq S_2 \supseteq S_3 \ldots$

2. keep the overlay graphs

3. $C :=$ all changed edges

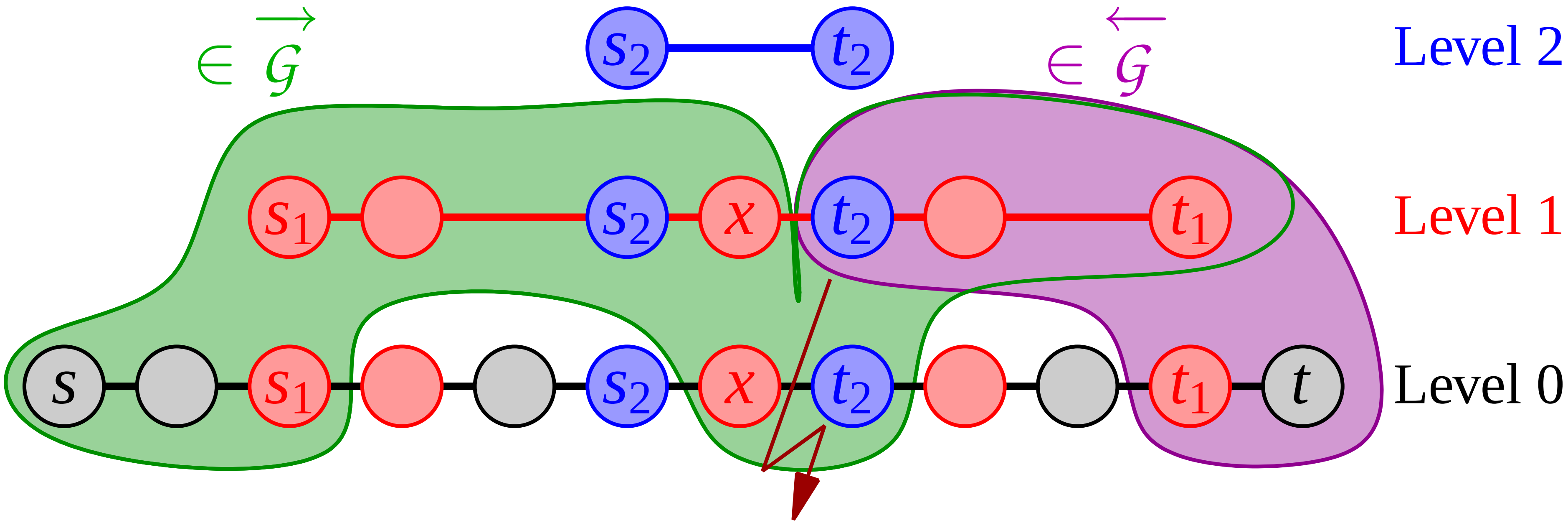
4. use the sets A_u^ℓ (considering edges in C) to determine for each node v a reliable level $r(v)$

5. during a query, at node v

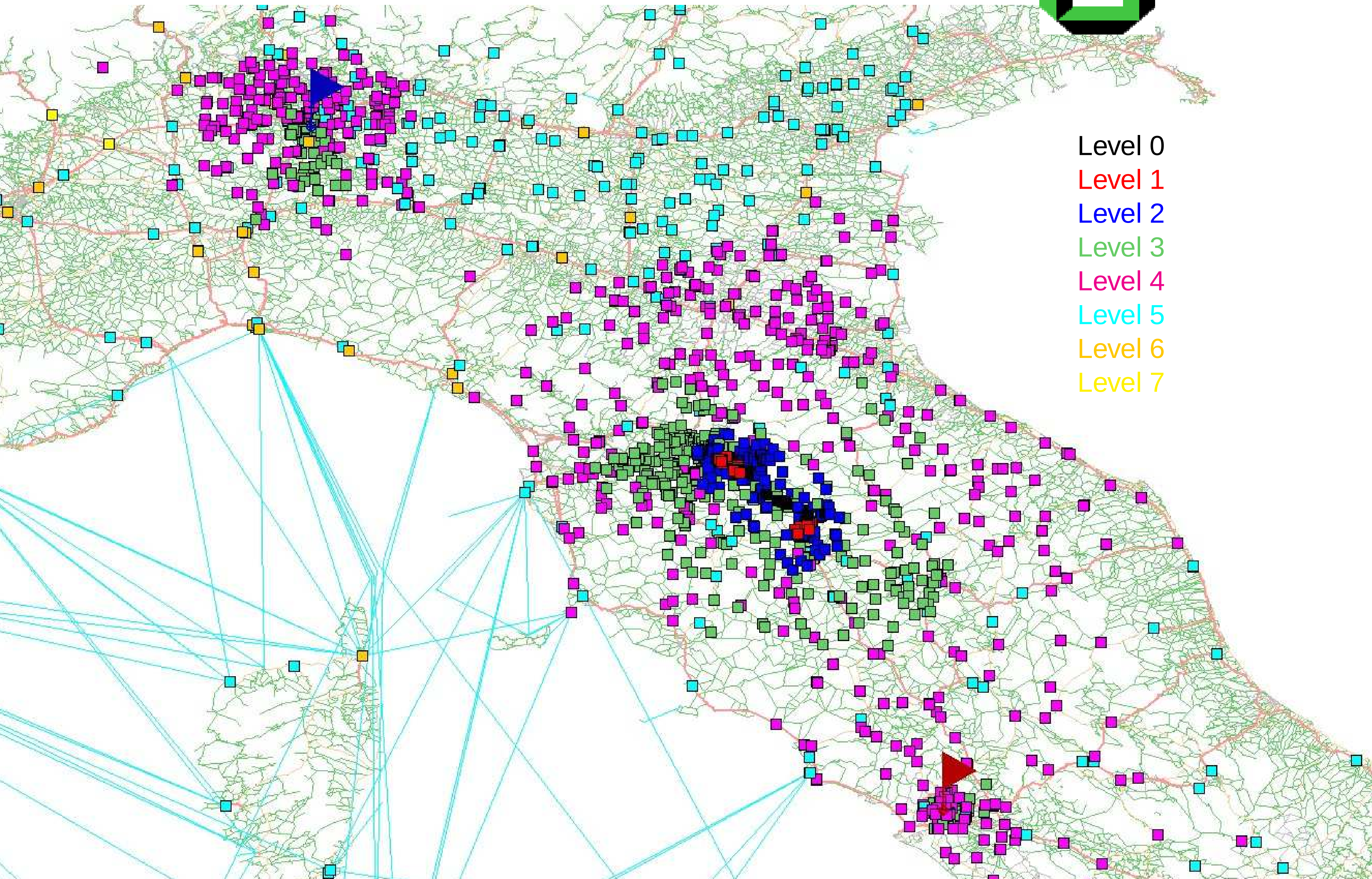
   - do not use edges that have been created in some level $> r(v)$
   - instead, downgrade the search to level $r(v)$ (forward search only)

Dynamic Highway-Node Routing

change a few edge weights, mobile scenario



reliable levels: $r(x) = 0, \quad r(s_2) = r(t_2) = 1$


Level 0
Level 1
Level 2
Level 3
Level 4
Level 5
Level 6
Level 7

Dynamic Highway-Node Routing

change a few edge weights, mobile scenario

iterative variant (provided that only edge weight increases allowed)

1. keep everything (as before)
2. $C := \emptyset$
3. use the sets A_u^ℓ (considering edges in C) to determine for each node v a reliable level $r(v)$ (as before)
4. 'prudent' query (as before)
5. if shortest path P does not contain a changed edge, we are done
6. otherwise: add changed edges on P to C, **repeat** from 3.

Dynamic Highway-Node Routing

change a few edge weights, mobile scenario

|change set|	affected	single pass	iterative		
		query time	query time	#iterations	
(motorway edges)	queries	[ms]	[ms]	avg	max
1	0.4 %	2.3	1.5	1.0	2
10	5.8 %	8.5	1.7	1.1	3
100	40.0 %	47.1	3.6	1.4	5
1 000	83.7 %	246.3	25.3	2.7	9

Summary

static routing in road networks is easy

⇝ applications that require massive amount or routing

⇝ instantaneous mobile routing

⇝ techniques for advanced models

⇝ updating a few edge weights is OK

Current / Future Work

- Time-dependent edge weights

  challenge: backward search impossible (?)
- Multiple objective functions and restrictions (bridge height,. . .)
- Multicriteria optimization (cost, time,. . .)
- Integrate individual and public transportation
- Other objectives for time-dependent travel
- Routing driven traffic simulation