RESEARCH FOR GRAND CHALLENGES

High Quality (Hyper-)Graph Partitioning

RU 2

Algorithm Engineering for NP-hard Graph Problems

Yaroslav Akhremtsev, Peter Sanders, Sebastian Schlag and Christian Schulz

1. The ε-balanced (Hyper-)Graph Partitioning Problem

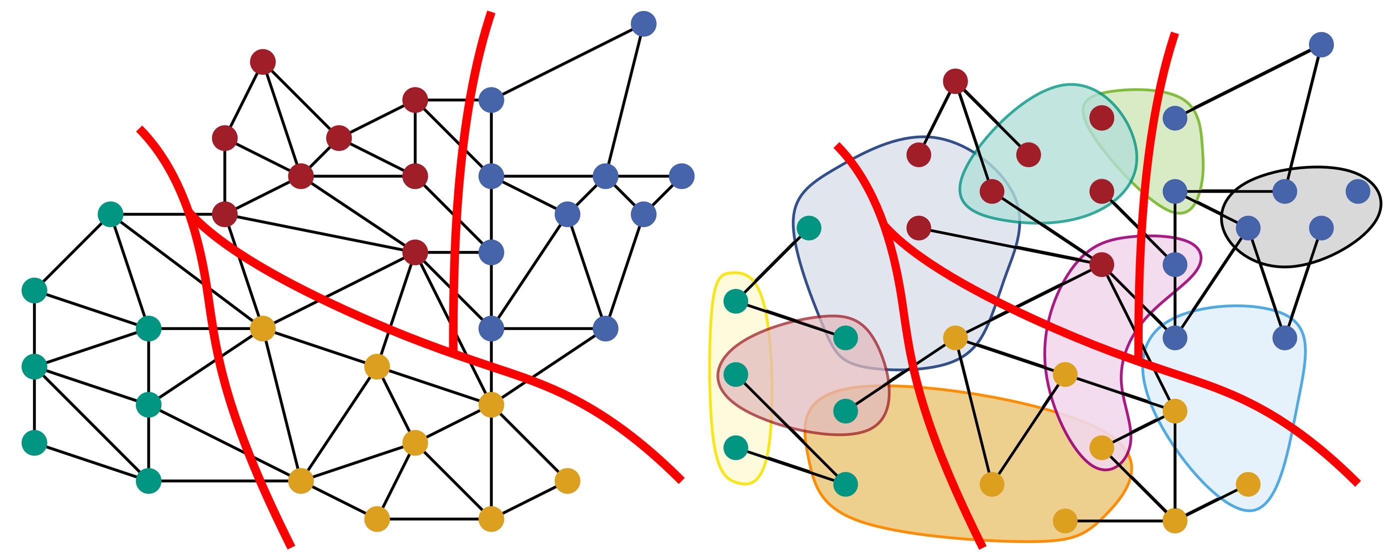
Graph: $G = (V, E)$
- models **relationships** between **objects**
- dyadic (**2-ary**) relationships

Hypergraph: $H = (V, E)$
- generalization $\Rightarrow$ edges connect ≥ 2 vertices
- arbitrary (***d*-ary**) relationships

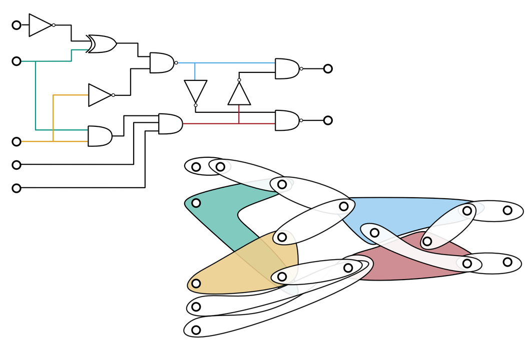
Task:

Partition $G/H = (V, E, c : V \to \mathbb{R}_{>0}, \omega : E \to \mathbb{R}_{>0})$ into **k** disjoint blocks $V_1, \ldots, V_k$ such that

- blocks V_i are **roughly equal-sized**: $c(V_i) \leq (1+\varepsilon)\left\lceil \frac{c(V)}{k} \right\rceil$
- **objective** fct. on edges is **minimized**: **cut**$=\sum_{e \in \text{cut}} \omega(e)$, **connectivity**$=\sum_{e \in \text{cut}} (\lambda - 1)\, \omega(e)$, where λ = # blocks connected by edge e

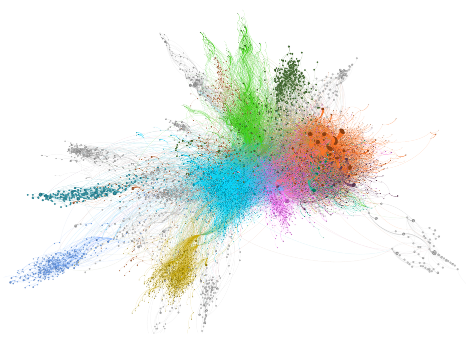


2. Applications



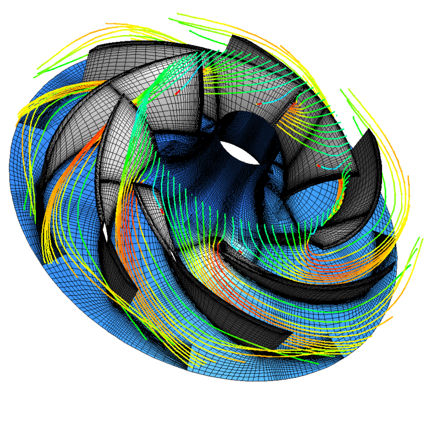
VLSI Design

Warehouse Optimization

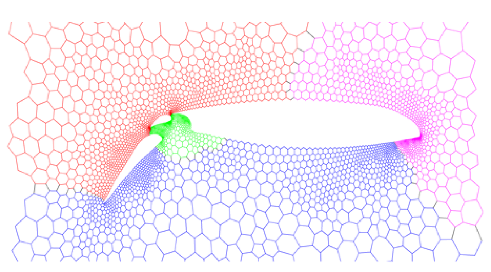


Complex Networks

Route Planning



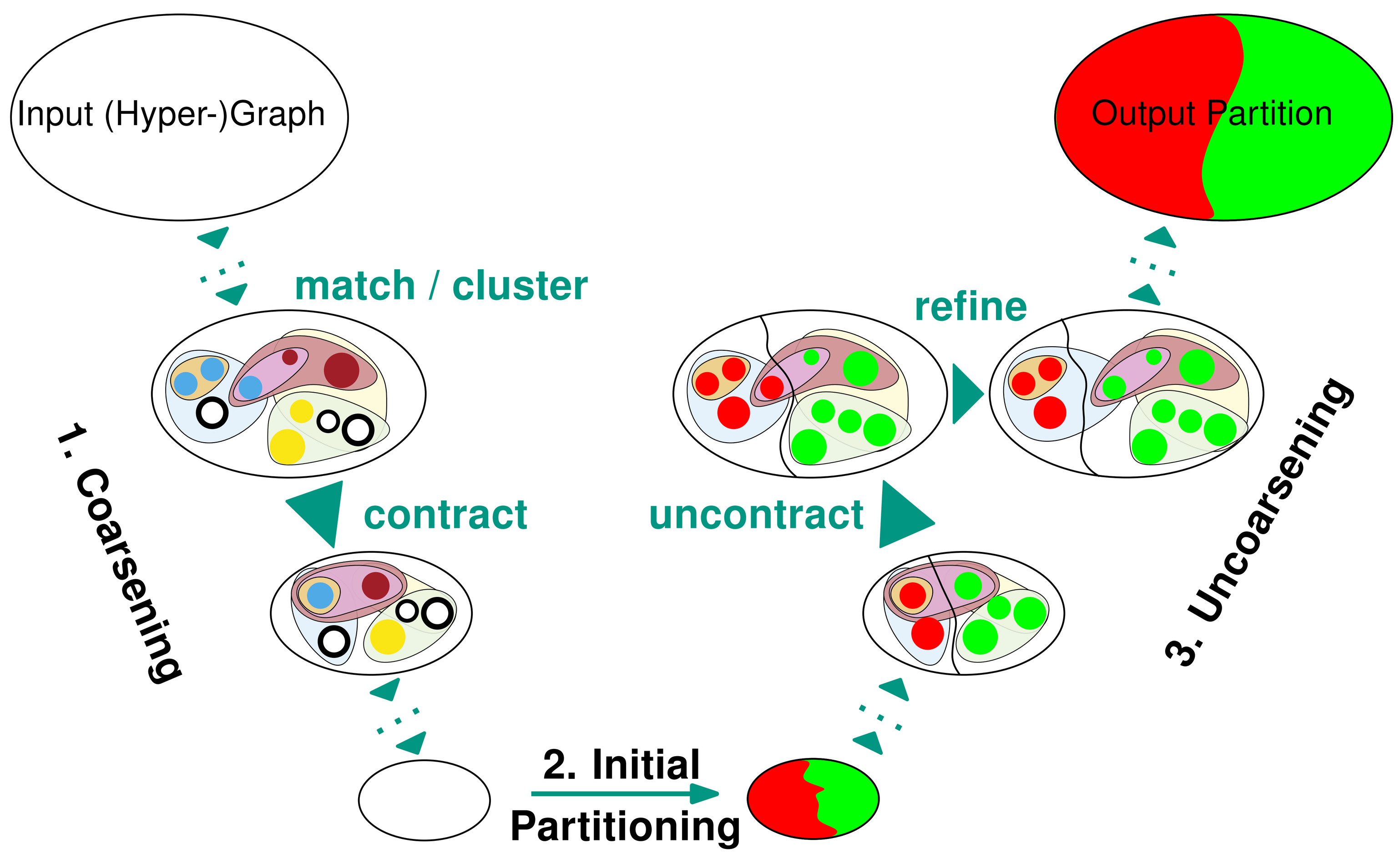
Simulation



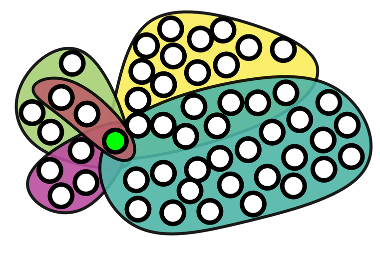
$\mathbb{R}^{n \times n} \ni Ax = b \in \mathbb{R}^n$

Scientific Computing

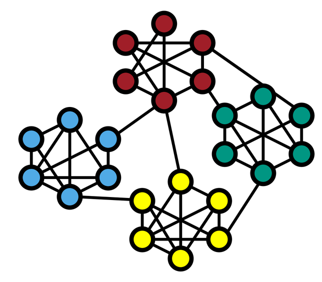
3. Multilevel (Hyper-)Graph Partitioning



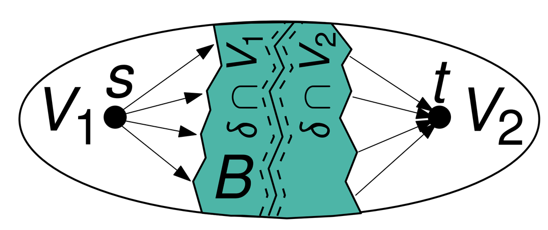
4. Algorithmic Innovations



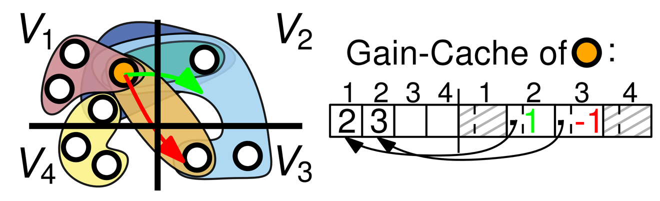
Min-Hash Based Sparsification



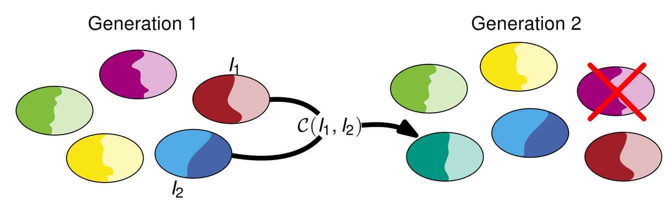
Community-Aware Coarsening



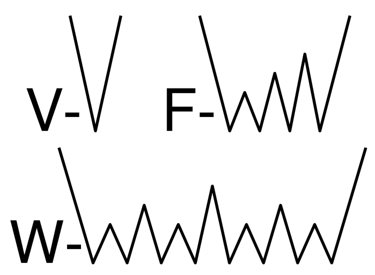
Max-Flow Min-Cut Refinement



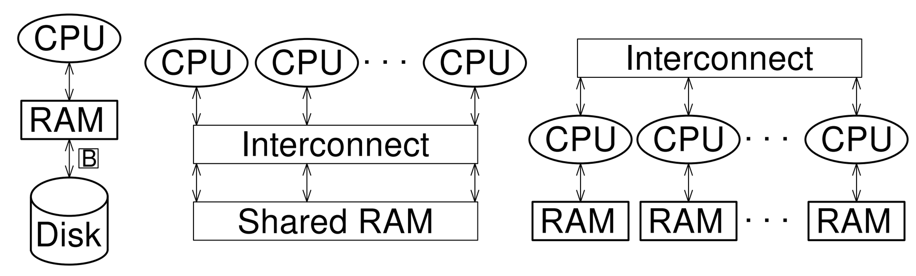
Engineered FM Local Search



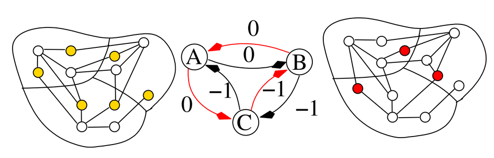
Evolutionary Algorithms



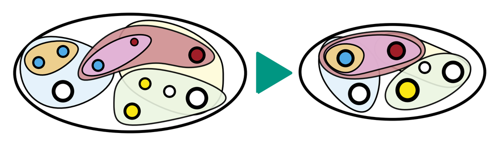
Global Search Strategies



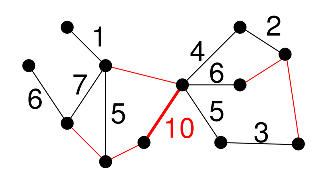
Advanced Memory Models / Parallelization



Negative Cycle Detection



Fast *n*-Level Coarsening



Approx. Weighted Matching

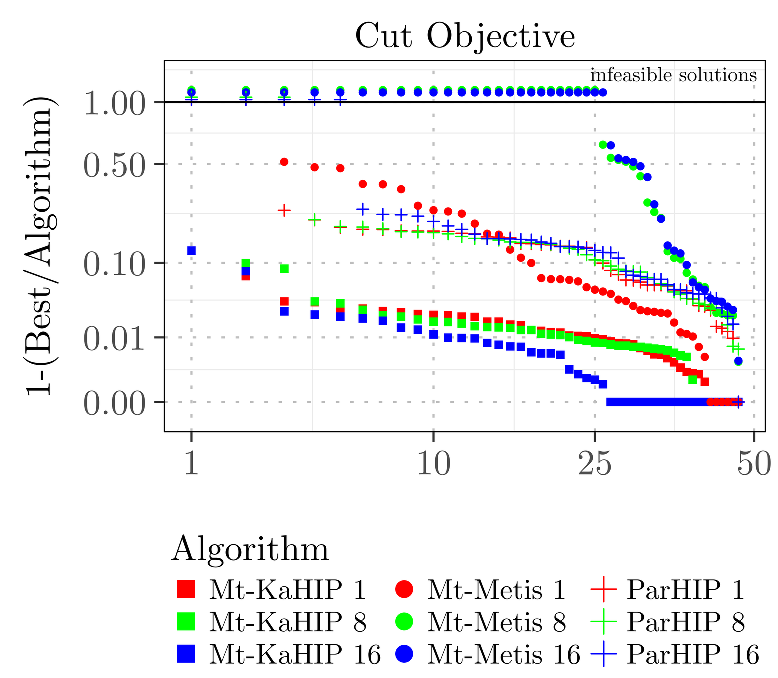
5. Open Source (Hyper-)Graph Partitioning Software

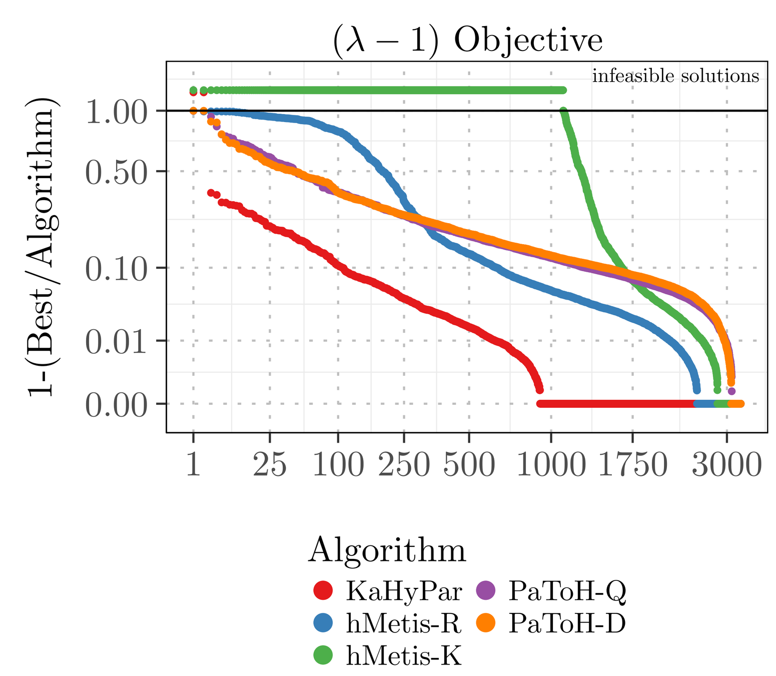
KaHIP - **Ka**rlsruhe **Hi**gh Quality **P**artitioning:
- `http://algo2.iti.kit.edu/kahip/`
- multilevel graph partitioning algorithms
- parallel evolutionary algorithm
- perfectly balanced partitioning

KaHyPar - **Ka**rlsruhe **Hy**pergraph **Par**titioning:
- `http://www.kahypar.org`
- *n*-Level approach
- cut and connectivity optimization
- recursive bisection & direct *k*-way

6. Experimental Results

Extensive experiments on large benchmark sets of graphs and hypergraphs confirm that KaHIP and KaHyPar compute high-quality solutions – outperforming competing state-of-the-art tools.





Selected References

[1] T. Heuer and **S. Schlag**. Improving Coarsening Schemes for Hypergraph Partitioning by Exploiting Community Structure. *SEA*, 2017.

[2] P. Sanders and C. Schulz. Think Locally, Act Globally: Highly Balanced Graph Partitioning. *SEA*, 2013.

Sebastian Schlag
Institute of Theoretical Informatics, Algorithms II
sebastian.schlag@kit.edu